PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON.

From December 1913 *to August* 1914.

VOL. XXVI.

LONDON:
"THE ELECTRICIAN" PRINTING & PUBLISHING COMPANY, LTD.,
1, 2 & 3, SALISBURY COURT, FLEET STREET, E.C.
MDCCCCXIV.

LONDON:
PRINTED BY GEORGE TUCKER, SALISBURY COURT, FLEET STREET, E.C.

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VOL. XXVI.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1913–1914.

October 24th, 1913.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Ice Calorimeter, with remarks on the Density of Ice. By Mr. Ezer Griffiths.
2. An Electrostatic Oscillograph. By Messrs. H. Ho and S. Koto.

November 14th, 1913.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Thermal Conductivity of Mercury by the Impressed Velocity Method. By Mr. H. Redmayne Nettleton.

2. Polarisation and Energy Losses in Dielectrics. By Dr. A. W. Ashton.

3. A Lecture Experiment to illustrate Ionisation by Collision and to show Thermoluminescence. By Mr. F. J. Harlow.

November 28th, 1913.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Expansion of Silica. By Prof. H. L. Callendar.

2. The Thermal Expansions of Mercury and Fused Silica. By Mr. F. J. Harlow.

3. An Experimental Method for the Production of Vibrations on Strings. By Prof. J. A. Fleming.

December 16th, 1913.

The Ninth Annual Exhibition of Physical Apparatus was held at the Imperial College of Science from 3 p.m. to 6 p.m. in the afternoon and from 7 p.m. to 10 p.m. in the evening.

At both the afternoon and evening meetings a discourse was given by Mr. Lewis Brennan, C.B., on "The 'Iridiscope' and some Experiments on Soap Films," and Prof. J. A. Fleming gave a short demonstration of his method for producing Vibrations on Loaded and Unloaded Strings.

Experimental Demonstrations were given by Mr. W. E. Curtis, Mr. F. J. Harlow, Dr. G. W. C. Kaye and Mr. E. A. Owen, Prof. J. T. Morris and Mr. J. F. Forrest, Mr. Clifford C. Paterson and Mr. B. P. Dudding, Mr. F. S. Phillips, and Dr. W. Watson, F.R.S.

The following firms exhibited apparatus : The Bausch & Lomb Optical Co., The Cambridge Scientific Instrument Co. (Ltd.), A. C. Cossor (Ltd.), Crompton & Co. (Ltd.), J. H. Dallmeyer (Ltd.), Elliott Bros., Evershed & Vignoles (Ltd.), Foster Instrument Co., A. Gallenkamp & Co. (Ltd.), Gambrell Bros. (Ltd.),

F. Harrison Glew, Graham & Latham, John J. Griffin & Sons (Ltd.), Phillip Harris & Co. (Ltd.), Adam Hilger (Ltd.), Isenthal & Co., E. Leitz, the Ludgate Wireless Co., Marconi's Wireless Telegraph Co. (Ltd.), Muirhead & Co. (Ltd.), Nalder Bros. & Thompson, Newton & Co., Robt. W. Paul, the Record Electrical Co. (Ltd.), James Swift & Son, the Synchronome Co. (Ltd.), H. Tinsley & Co., Townson & Mercer, the Weston Electrical Instrument Co., and Carl Zeiss (Ltd.).

January 23rd, 1914.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. Some Characteristic Curves and Sensitiveness Tests of Crystal and other Detectors. By Mr. P. R. COURSEY.

2. A Water Model of the Musical Electric Arc. By Mr. W. DUDDELL.

3. Further Experiments with Liquid Drops and Globules. By Mr. C. R. DARLING.

4. Note on Aberration in a Dispersive Medium and Airy's Experiment. By Mr. J. WALKER.

Annual General Meeting.

February 13th, 1914.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The Report of the Council was taken as read.

It has been felt that the Report of the Council should, in future, deal with the same period as that covered by the Report of the Treasurer, namely, from January 1st to December 31st in each year. The present

Report of the Council, therefore, covers only the period from the last Annual General Meeting up to December 31, 1913.

During this period there have been held 10 ordinary meetings and two informal meetings. One ordinary meeting was held at King's College and another at University College. The informal meetings were the Annual Exhibition and a visit to the National Physical Laboratory. The average attendance, excluding informal meetings, was 38.

Owing to the improved financial position the Council feel that the Society's field of activity should be increased, and careful consideration has, therefore, been given, during the past year, to the possibility of introducing new features. It has been thought that expenditure might profitably be made upon the issue, from time to time, of reports upon certain subjects of general interest. The first subject selected for the purpose is Radiation. Mr. J. H. Jeans, F.R.S., has expressed his willingness to write the report upon this subject, and to have it complete during the summer.

The Council have also felt that many Fellows would appreciate an occasional or annual lecture by some eminent physicist, and they have accordingly arranged with Prof. R. W. Wood, of Johns Hopkins University, Baltimore, to give the first of these lectures at an early date. This series of lectures will be known as the Guthrie Lectures, in memory of the late Prof. F. Guthrie, through whose efforts the Society was founded.

During the period under review a Committee has been appointed by the Council to consider questions in regard to Nomenclature and Symbols and allied matters, and consists of Prof. H. L. Callendar, Mr. A. Campbell, Dr. C. Chree, Dr. W. Eccles, Prof. G. Carey Foster, Sir George Greenhill, Dr. A. Russell, Prof. the Hon. R. J. Strutt, Prof. S. P. Thompson and Prof. W. Watson, with Dr. Eccles as secretary and convener. At present the Committee is discussing electric and magnetic quantities; but reports on Mathematical and Mechanical Nomenclature and Symbols, so far as these concern physicists, and on Heat are also projected. The reports will be submitted to the Council and will be published for the use of the Fellows.

The Ninth Annual Exhibition of Apparatus by manufacturers was held on December 16th, in the afternoon and evening, the number of Fellows and visitors present being about 650. The number of firms exhibiting was 30. An experimental discourse on "The 'Iridiscope' and Some Experiments on Soap Films" was given by Mr. Louis Brennan, C.B. Prof. J. A. Fleming repeated his experiments on the Vibrations of Loaded and Unloaded Strings, and for the first time a number of experiments were shown by Fellows and others.

The Society has to thank Prof. A. Schuster for a gift of valuable books.

The number of ordinary Fellows on the roll at December 31, 1913, as distinct from Honorary Fellows, was 442 ; nine new Fellows have been elected.

The Society has to mourn the loss by death of one member of Council. Prof. P. V. Bevan, and three other Fellows, namely, Sir W. H. Preece, G. B. Finch and Prof. J. G. McGregor.

The Report was adopted by the meeting.

The Report of the Treasurer and the Balance-Sheet were presented by the Treasurer.

The total income of the Society again shows a slight increase over the preceding year, but this is mainly due to the Income Tax recovered and to the increased sale of publications. The subscriptions for the year actually show a slight decrease.

The expenditure for the year has decreased. This is accounted for by the fact that in 1912 the Society issued a special number containing the Papers read at the joint meeting with the Optical Convention.

By comparing the balance brought forward on the 1st January, 1913 (less two cheques)—namely, £140. 12s. 2d.—with the balance carried forward (less four cheques) on the 1st January, 1914—namely, £271. 6s. 3d.—it will be seen that the Society has increased its assets during the year by £130. 14s. 1d. This, I think, shows that the finances of the Society are in a very sound condition.

In spite of the above excess of income over expenditure, the total assets of the Society again show a decrease, mainly due to continued depreciation in the market value of the securities.

The Society has again to thank the Manager of Parr's Bank for kindly valuing the securities at the 30th December, 1913, and for supplying the figures which appear in these accounts.

The liabilities on account of the Life Composition Funds have decreased during the year owing to the deaths of four life fellows and to no new fellows having compounded for their subscriptions. The balance available in the General Fund of the Society is slightly less than last year.

It will be noted that I have reduced the valuations of the publications of the Society since my estimate of last year. The main reason for this is that, on a re-examination of the stock of the Society, it has been found that one of the early parts appears to be missing. If this proves to be the case, it may be necessary to reprint it in order to perfect the complete sets of the "Proceedings of the Society." I have, therefore, deducted a sum in making my estimate which should cover this cost.

The Report of the Treasurer was adopted.

The Treasurer in Account with the Physical Society, from January, 1913, to December 31st, 1913.

Dr.	£	s.	d.	£	s.	d.
Entrance Fees	10	10	0			
Subscriptions	493	8	6			
Extra Subscriptions	22	1	0			
Arrears	10	15	0			
Subscriptions paid in Advance	8	19	0			
Students' Subscriptions	4	4	0			
Subscriptions for "Science Abstracts" and Advanced Proofs	13	17	0			
				563	14	6
Composition Fees					nil	
Dividends:—						
Furness Debenture Stock	15	1	2			
Midland Railway	37	13	4			
Metropolitan Board of Works	6	12	0			
Lancaster Corporation Stock	11	6	0			
New South Wales	8	7	6			
London, Brighton & South Coast Railway	24	14	5			
Great Eastern Railway	18	16	8			
				122	11	1
Interest on deposit account				17	17	1
Income Tax recovered				22	17	0
Sales of Publications ("The Electrician" Printing & Publishing Co.)				108	6	4
				835	6	0
Balance at Bank, January 1, 1913	209	14	0			
Less two cheques	69	1	10			
				140	12	2
				£975	18	2

WILLIAM DUDDELL, *Honorary Treasurer.*

Cr.	£	s.	d.	£	s.	d.
"Science Abstracts"				278	13	0
Printing and Distributing Publications of the Society:—						
"The Electrician" Printing & Publishing Co.—						
Proceedings	229	2	4			
Bulletin	20	8	9			
Distribution	40	18	10			
General	36	19	3			
	327	9	2			
Less credit for use of type	1	8	0			
				326	1	2
Periodicals				6	11	0
Reporting				35	0	0
Refreshments and attendance at meetings				31	10	0
Petty cash				12	14	7
Bank commission				0	3	0
Royal Asiatic Society				2	2	0
International Catalogue				5	16	2
Donation to the Annual Table of Constants				5	0	0
Royal Society				1	1	0
				704	11	11
Balance at Bank, January 1, 1914	98	14	0			
Less four Cheques	27	7	9			
				71	6	3
Transferred to Deposit Account				200	0	0
				£975	18	2

Audited and found correct,

HARRY M. ELDER.
THOMAS H. BLAKESLEY.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, December 31st, 1913.

Assets.

	£	s.	d.
Subscriptions due, Treasurer's estimate	26	5	0
£533 Furness Ry. Co. 3 per cent. Debenture Stock	357	0	0
£1,600 Midland Railway $2\frac{1}{2}$ per cent. Preference Stock	956	0	0
£200 Metropolitan Board of Works $3\frac{1}{2}$ per cent. Consolidated Stock	190	0	0
£400 Lancaster Corporation 3 per cent. Redeemable Stock	302	0	0
£254. 2s. 9d. New South Wales $3\frac{1}{2}$ per cent. Inscribed Stock	237	0	0
£500 London, Brighton & South Coast Railway Ordinary Stock	515	0	0
£500 Great Eastern Railway 4 per cent. Debenture Stock	475	0	0
Balance at Bank	98	14	0
Ditto on deposit	600	0	0
Publications (Treasurer's Estimate)	180	0	0
	£3,936	19	0

Liabilities.

	£	s.	d.
Four Cheques	27	7	9
Life Compositions	2,089	10	0
Balance General Fund	1,820	1	3
	£3,936	19	0

WILLIAM DUDDELL, *Honorary Treasurer.*

Audited and found correct,

HARRY M. ELDER.
THOMAS H. BLAKESLEY.

LIFE COMPOSITION FUND.

	£	s.	d.
174 Fellows paid £10	1,740	0	0
3 Fellows paid £15	45	0	0
4 Fellows paid £21	84	0	0
7 Fellows paid £31. 10s.	220	10	0
	£2,089	10	0

NOTE.—Four Fellows who paid £10 deceased during year 1913.

Audited and found correct,

WILLIAM DUDDELL, *Honorary Treasurer.*

HARRY M. ELDER.
THOMAS H. BLAKESLEY.

The Election of Officers and Council then took place, the new Council being constituted as follows :—

President.—Prof. Sir J. J. THOMSON, O.M., D.Sc., F.R.S.

Vice-Presidents, who have filled the Office of President.—Prof. G. C. FOSTER, D.Sc., LL.D., F.R.S. ; Prof. W. G. ADAMS, M.A., F.R.S. ; Prof. R. B. CLIFTON, M.A., F.R.S. ; Prof. A. W. REINOLD, C.B., M.A., F.R.S. ; Prof. Sir ARTHUR W. RÜCKER, M.A., D.Sc., F.R.S. ; Sir W. DE W. ABNEY, R.E., K.C.B., D.C.L., F.R.S. ; Prin. Sir OLIVER J. LODGE, D.Sc., LL.D., F.R.S. ; Prof. SILVANUS P. THOMPSON, D.Sc., F.R.S. ; R. T. GLAZEBROOK, C.B., D.Sc., F.R.S. ; Prof. J. PERRY, D.Sc., F.R.S. ; C. CHREE, Sc.D., LL.D., F.R.S. ; Prof. H. L. CALLENDAR, M.A., LL.D., F.R.S. ; Prof. A. SCHUSTER, Ph.D., Sc.D., F.R.S.

Vice-Presidents.—Prof. T. MATHER, F.R.S. ; A. RUSSELL, M.A., D.Sc. ; F. E. SMITH, R. S. WHIPPLE.

Secretaries.—W. R. COOPER, M.A. ; S. W. J. SMITH, M.A., D.Sc., F.R.S.

Foreign Secretary.—R. T. GLAZEBROOK, C.B., D.Sc., F.R.S.

Treasurer.—W. DUDDELL, F.R.S.

Librarian.—S. W. J. SMITH, M.A., D.Sc., F.R.S.

Other Members of Council.—W. H. ECCLES, D.Sc. ; Sir R. A. HADFIELD, F.R.S. ; Prof. G. W. O. HOWE, M.Sc. ; Prof. J. W. NICHOLSON, M.A., D.Sc. ; Major W. A. J. O'MEARA, C.M.G. ; C. C. PATERSON ; Prof. O. W. RICHARDSON, M.A., D.Sc., F.R.S. ; Prof. the Hon. R. J. STRUTT, F.R.S. ; W. E. SUMPNER, D.Sc. ; R. S. WILLOWS, M.A., D.Sc.

The following Papers were read :—

1. On the Moving Coil Ballistic Galvanometer. By Mr. R. LL. JONES.

2. On Vacuum-tight Lead-Seals for Leading-in Wires in Vitreous Silica and other Glasses. By Dr. H. J. S. SAND.

February 27th, 1914.

Meeting held at the Imperial College of Science.

Prof. Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

Prof. G. CAREY-FOSTER gave a short biography of Prof. Frederick Guthrie, to whom the Physical Society of London owed

its initiation, as an introduction to the first GUTHRIE LECTURE, which was then delivered by Prof. R. W. WOOD, of Johns Hopkins University, Baltimore, who lectured on "THE RADIATION OF GAS MOLECULES EXCITED BY LIGHT."

March 13th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Time Measurements of Magnetic Disturbances and their Interpretation. By Dr. C. CHREE.

2. On the Ratio of the Specific Heats of Air, Hydrogen, Carbon Dioxide and Nitrous Oxide. By Mr. H. N. MERCER.

3. The Asymmetric Distribution of the Secondary Electronic Radiation produced by X-Radiation. By Mr. A. J. PHILPOT.

March 27th, 1914.

Meeting held at the Imperial College of Science.

Prof. Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

The following Papers were read :—

1. A New Type of Thermogalvanometer. By Mr. F. W. JORDAN.

2. An Instrument for Recording Pressure Variations due to Explosions in Tubes. By Mr. J. D. MORGAN.

3. The Direct Measurement of the Napierian Base. By Mr. R. APPLEYARD.

May 8th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Some Gyrostatic Devices for the Control of Moving Bodies. By Dr. J. G. GRAY.

2. A Graphic Treatment of Cusped Wave-fronts and of the Rainbow. By Mr. W. R. BOWER.

May 22nd, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Volatility of Thorium Active Deposit. By Messrs. T. BARRATT and A. B. WOOD.

2. The Passage of α-Particles through Photographic Films. By Messrs. H. P. WALMSLEY and W. MAKOWER.

3. On a Null Method of Testing Vibration Galvanometers. By Mr. S. BUTTERWORTH.

4. Experiments with an Incandescent Lamp. By Dr. S. W. J. SMITH.

June 12th, 1914.

Meeting held at the Imperial College of Science.

Prof. T. MATHER, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. Note on the Connection between the Method of Least Squares and the Fourier Method of Calculating the Coefficients of a Trigonometrical Series to represent a given Function or Series of Observations. By Prof. C. H. LEES.

2. A Magnetograph for Measuring Variations in the Horizontal Intensity of the Earth's Magnetic Field. By Mr. F. E. SMITH.

3. The Atomic Weight of Copper by Electrolysis. By Mr. A. G. SHRIMPTON.

4. Note on an Improvement in the Einthoven String Galvanometer. By Mr. W. A. APTHORPE.

June 20th, 1914.

Members of the Society, with a number of friends, visited the works of the Cambridge Scientific Instrument Co., after which they were the guests of the Management and Directors at luncheon in the Hall of St. John's College, Cambridge.

In the afternoon a meeting of the Society was held in the Cavendish Laboratory, Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

The following Papers were read :—

1. Production of Very Soft Röntgen Radiation by the Impact of Positive and Slow Cathode Rays. By Sir J. J. THOMSON.

2. On the Homogeneity of Atmospheric Neon. By Mr. F. W. ASTON.

Those present were then entertained to tea by Sir Joseph and Lady Thomson.

June 26th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves round the Earth's Surface. By Prof. J. A. FLEMING.

2. Atmospheric Electricity Observations made at Kew Observatory. By Mr. GORDON DOBSON.

3. Thermal and Electrical Conductivities of Some of the Rarer Metals and Alloys. By Mr. T. BARRATT.

4. Some Investigations of the Arc as a Source of High-frequency Oscillations. By Mr. F. MERCER.

I. *The Ice Calorimeter. With Remarks on the Constancy of the Density of Ice.* By EZER GRIFFITHS, *B.Sc., Fellow of the University of Wales.*

RECEIVED JULY 17, 1913.

I. INTRODUCTION.

THE primary object of the work recorded in this Paper was the redetermination by an electrical method of the constant of Bunsen's ice calorimeter—*i.e.*, the mass of mercury drawn into the instrument by the addition of one mean calorie of heat.

Apart from the intrinsic value of this constant in calorimetry, its redetermination by an independent method is desirable on account of the marked discrepancy between Dieterici's value of J, determined by the ice calorimeter, and that found by other observers using entirely different methods. Also, Bunsen's calculated value of the latent heat of fusion of ice is 0·3 per cent. higher than that obtained in direct determinations.

Table I. is, I believe, a complete summary of the values hitherto published for the calorimeter constant.

TABLE I.

Authority.	K in mgms.	Reference.
Bunsen	15·41	"Phil. Mag.," XLI., 1871.
Schuller and Wartha	15·44	"Wied. Ann.," 2, 1877.
Than	15·42	"Chem. Ber.," 10, 1877.
Velten	15·47	"Wied. Ann.," 21, 1884.
Zakrzevski	15·57	"Bull." l'Acad. Cracovie, 1891.
Staub	15·26	Inaug. Diss. Zürich, 1890.
Dieterici	15·49	"Ann. Phys.," 16, 1905.

Reference to the original memoirs, where available, shows that the values are based on observations of the heat imparted to the calorimeter by a small quantity of water contained in an envelope whose thermal capacity is comparable with that of the contained water.

Moreover, in the majority of cases no attempt has been made to vary the conditions, and thus detect systematic errors. For example, Bunsen's value is the mean of two experiments repeated under precisely similar conditions; the glass envelope weighed 0·2 gm., contained 0·3 gm. water, and a platinum sinker (weight 0·5 gm.) was also attached.

Of the determinations quoted in the above table Dieterici's would appear to carry the greatest weight as the quantity of water was varied from 0·6 gm. to 2 gms. His value, however, differs from the mean of the others by 0·4 per cent. Dieterici has also performed experiments in which the heat was supplied electrically, with the object of determining J in terms of the mean calorie. Assuming J as 4·184,* we have from his first series,† the value 15·27 for the calorimeter constant, and from the second series ‡ the value 15·46. The second series, however, is the more reliable, as the electrical quantities (E and C) were determined with greater accuracy by potentiometer measurements.

On account of the slow rate of heat supply the correction for "drift" § was considerable, amounting to as much as 1 per cent. in some of his experiments; and, moreover, it appears to have been very erratic, being positive in some experiments and negative in others.

II. Outline of the Method used in this Investigation.

Although all electrical methods in calorimetry are dependent on the accuracy of the determinations of the electrical standards and of J, in this particular application they are free from two sources of error inseparable from the direct determination just discussed; loss in transfer and correction for the thermal capacity of the envelope.

Moreover, if the value assumed for J is stated and the electrical units are expressed in terms of international standards the value of the calorimeter constant can at any time be corrected with ease and certainty.

In the experiments described herein the results are based on determinations of electromotive force and resistance; but neither of these quantities had to be measured during the actual course of an experiment.

The current was so adjusted that the difference of potential at the ends of the heating coil was exactly equal to the E.M.F. of a number of standard cadmium cells in series, the equality of the balance being indicated by the non-deflection of a sensitive galvanometer in the standard-cell circuit.

* *See* p. 4, §VI.
† "Wied. Ann.," 33, 1888.
‡ "Ann. Phys.," 16, 1905.
§ The term "*drift*" is used to denote the progressive freezing or thawing of the ice mantle independently of the measured energy supply.

All adjustments were made on a duplicate coil some time before the commencement of an experiment; the change-over being effected by a mercury key which also recorded on a chronograph tape.

During an experiment the equality of the potential balance could be maintained with great exactness by means of a high resistance rheostat forming a shunt on an adjustable low resistance in the main circuit.

The conditions were varied as much as possible, E.M.F. due to 3, 4, 5, 6, 7 and 8 standard cells being balanced.

Thus the rate of energy supply in the fastest experiments was more than seven times that in the slowest.

Considerable precautions were taken to reduce the magnitude of the "drift." In some experiments the correction was negligible, and on the average it only amounted to 1 part in 500.

In order to understand the arrangement of the apparatus it is necessary to consider in some detail the sources of error peculiar to ice calorimeters and the precautions necessary for accurate work with these instruments.

III. Sources of Error.

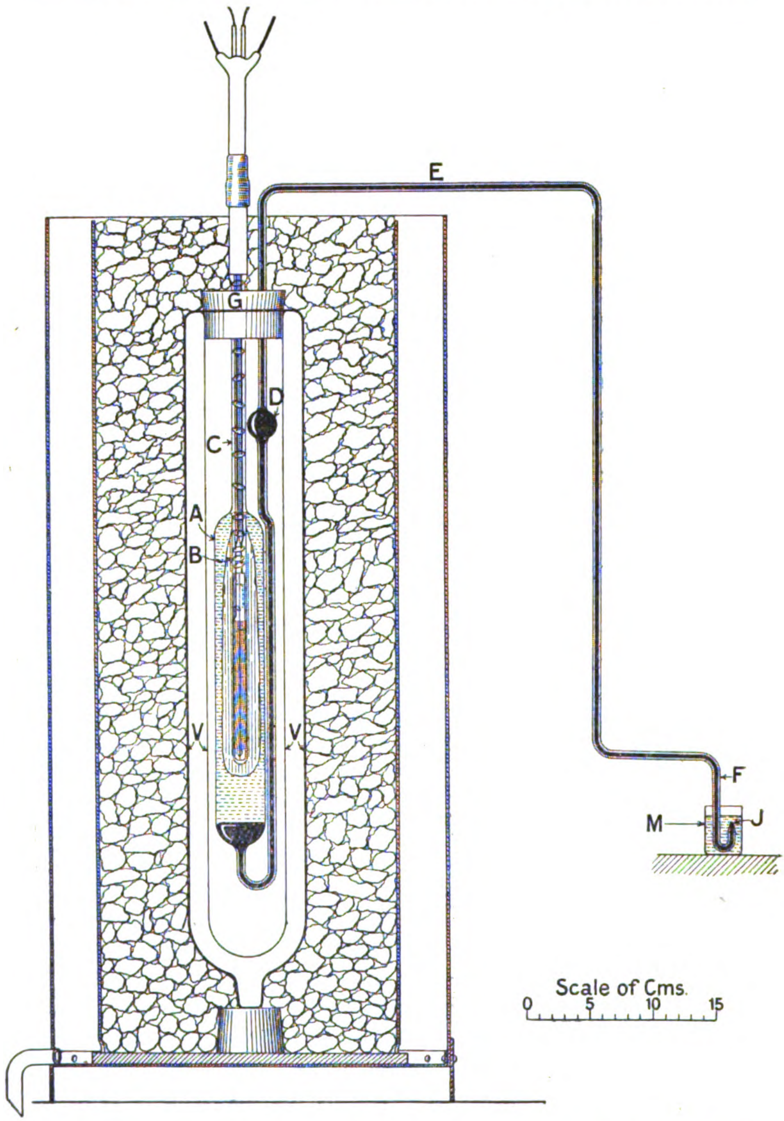
"*Drift.*"—Bunsen, in his classical Paper on the ice calorimeter, describes this effect, and attributes it to the slight difference—caused by impurities—in the melting point of the snow outside and the mantle inside the instrument. He devised the method of correction which has since been universally adopted. Unfortunately, however, the rate of "drift" is not a uniform one, so the correction is only an approximation.

Prof. Boys showed that the rate of drift could be reduced to about one-tenth its value by fixing the calorimeter in an empty vessel surrounded by ice, the effect being to reduce the rate of interchange of heat between the instrument and the surrounding ice or snow. Prof. Callendar further improved this device by using a second glass bulb fused around that of the calorimeter, and exhausting the outer bulb.

This arrangement is undoubtedly very satisfactory when the vacuum is good, but the difficulties of construction and filling are considerable.

The plan adopted in this work is shown in Fig. 1. The calorimeter is suspended within a transparent cylindrical vacuum vessel, the mouth of which is closed by a rubber cork fixed to the calorimeter.

The heat insulating property of the vacuum vessel, as also its transparency, were invaluable in this investigation, for it was



desirable to watch and control the formation of the ice mantle. The cylindrical form of the vessel rendered its withdrawal

and replacement in the packing of powdered ice a simple matter provided the packing had been well rammed down.

Pressure Lowering the Melting Point.—In the customary form of ice calorimeter there is a difference in level between the surface of the mercury in the bulb and that in the capillary tube, sometimes amounting to 50 cm. In the first design of this calorimeter the capillary terminated at the level E in Fig. 1. After the instrument had been set up and the condition allowed to become steady, observations of the position of the mercury meniscus in the capillary were repeated at intervals for several hours. It was invariably found that a minute drift was taking place in the direction indicating a thawing of the ice mantle.

This effect was ascribed to the external head of mercury, causing a depression of the melting point of the mantle below that of the surrounding ice.

Since the pressure amounted to three-quarters of an atmosphere the melting point would be lowered by $\frac{1}{200}$°C.

This pressure effect was eliminated by extending the capillary and making the level approximately the same inside and out.

No drift was perceptible over long periods when experiments were not performed, and the slight drift which took place during, and after, an experiment was undoubtedly due to slow conduction down to the calorimeter of the heat generated in the leads.

Density of Ice said to be Variable.—This is one of the objections most frequently quoted against the use of the ice calorimeter in work of precision.

If well grounded it seriously limits the possibilities of the instrument, and the evidence upon which it is based may be summarised as follows:—

Nichols ("Phy. Rev.," VIII., 1899) reviews the work of previous investigators on the density of ice and describes his own experiments. He concludes that the density of ice mantles, determined by weighing in petroleum, is $0{\cdot}91615 \pm 0{\cdot}00009$. This result agrees with the mean value deduced from different methods by Plücker and Geissler, Kopp and Bunsen (for a similar variety of ice) to four places of decimals.

His experiments on the causes of the variations in density of artificial ice were not completed. The method was to freeze the ice mantle around the innner tube of a calorimeter by

pouring in a mixture of CO_2 and ether. The unfrozen water was shaken out as completely as possible, and the adhering water frozen, the remaining space being then completely filled with mercury. The weight of the mercury, together with that of the ice, gave the data for the computation of the density of the ice mantle.

Although the results were consistent among themselves the absolute value was subsequently found to be erroneous on account of the deformation of the glass vessel under the weight of the contained mercury.

Nichols at first thought the discrepancy—amounting to 1 per cent.—between his value and Bunsen's was due to the much lower temperature at which the mantle was formed in his own experiments. He, therefore, made some determinations with mantles frozen by means of alcohol at -5 deg. to -10 deg. as refrigerant in the manner devised by Bunsen. The measurements appeared to indicate that the mantles formed by the use of alcohol at -5 deg. to -10 deg. were less dense than those formed by means of CO_2 and ether at -70 deg. by at least 1 part in 1,000, and further that one of the latter mantles decreased in density by nearly this amount after standing 24 hours in an ice bath.

The use of CO_2 and ether resulted in a very rapid formation of ice and the mantle, when a certain size was reached, invariably became filled with a network of fine cracks.

Vincent * later took up the subject, and also investigated the coefficient of cubical expansion of ice.

He prepared the ice by means of a freezing mixture, and appears to have obtained a different density for each sample prepared. Table II. summarises all his results :—

TABLE II.

Experiment.	Density ice at 0 deg.	Weight assigned to the experiment.
1	0·916335	3
2	0·915460	2
3	0·916180	2
4	{ 0·915540	1
	0·916060	2
Weighted mean .... 0·9160		

His value for the coefficient of expansion of the above samples are consistent, and no connection between variation

* "Phys. Rev.," XV., 1902.

in density and expansion can be traced. Vincent's mean value for the density is 1 part in 5,000 less than the mean of the results of Plücker and Geissler, Bunsen and Nichols.

Hence, we may sum up the evidence as follows :—

1. Observers, using entirely different methods, agree within 2 parts in 10,000 in assigning the value 0·9161 to the density of artificial ice.

2. That no other physical property, such as melting point or coefficient of expansion, appears to vary with the density.

3. That it has not been possible to obtain two or more definite and reproducible values for the density of artificial ice.

It would appear, therefore, that an explanation of the small variations in the density is to be sought for in the presence of occluded water * in the samples of ice examined.

Calculation shows that the presence of 1½ per cent. of water would be more than sufficient to account for the discrepancies observed by both Nichols and Vincent.

And in this connection we have the very significant fact that the value of the latent heat of fusion of ice obtained by calculation from the constants of the ice calorimeter exceeds the value obtained by direct determinations using ice in bulk by about 0·7 per cent.

The latent heat of fusion is the one physical property which would be seriously influenced by the presence of comparatively small amounts of occluded water.

On consideration it will be seen that the presence of occluded water in the ice mantle of a Bunsen's ice calorimeter would have no influence on the results obtained with this instrument.

"*Stiction.*"—A method which has been considerably used on account of its convenience is that of observing the position of the end of the mercury column in a graduated capillary tube.

Since, however, the meniscus is a receding one the readings are liable to considerable error on account of the phenomenon usually termed "stiction." Also it is necessary to calibrate the tube and to provide means of adjusting the position of the column in this tube.

The method of directly weighing the mercury drawn in is free from these objections, but the form of the orifice requires consideration, since the ingress of air bubbles into the tube must be prevented.

* The term "occluded water" includes any amorphous modification binding the crystalline mass together.

The difficulty appears to be due to a film of air which is trapped between the mercury in the orifice and that in the measuring cup when the latter is inserted in position; this film breaks the column and forms an air bubble in the tube.

It was found that experiments so affected would be consistent among themselves, but the absolute value would be as much as 1 per cent. too low compared with experiments when free from air bubbles. After many trials the form of jet * shown at J in Fig. 1, was developed.

A short piece of 2 mm. bore tubing was fused on at F. The end of this tube had been ground off to a sharp even edge; the bore being left parallel and unrestricted.

When the mercury cup M was inserted beneath and raised into position this form of orifice showed no tendency to trap air; the meniscus maintained a well-defined convex form; and any slight errors in the weight due to variations in the form of the meniscus would mean out in a series of experiments.

IV. Description of the Apparatus.

The bulb A of the calorimeter was 27 cm. long by 4 cm. diameter; the inner tube B was 20 cm. long by 9 mm. diameter. The stem C (26 cm. long) was enlarged at its upper end to closely fit the head of the heating coil, a short piece of rubber effecting the joint.

The capillary † DEF was in one continuous length with a small bulb blown at D.

The usual method of attaching the capillary to the calorimeter by means of a cork, was avoided owing to the possibility of creep and the trapping of air bubbles.

The bulb D was inserted to prevent air bubbles being drawn into the main bulb when the calorimeter was allowed to warm up without a mercury cup at the orifice. The rubber cork G supported the instrument within a vacuum vessel. This vessel V was 52 cm. long by 6 cm. internal diameter.

The assembled apparatus was placed in a double walled can containing finely powdered ice, the water from the melting ice being allowed to drain away freely at K.

* The form of jet devised by Schuller and Wartha was tried, but was unsatisfactory on account of the tendency of the mercury to drop completely out of the little bulb forming the orifice with slight vibration, caused by changing the mercury cups. This may possibly have been due to the opening being too large.

† The bore of this tube was 2 mm. as far as the bulb D, and from D on to F 1 mm.

Heating Coil.—The resistance in which the heat was developed was of bare manganin wire, of about 20-ohm resistance, wound on a mica rack of X form. The diameter of the coil was 7 mm., and as the internal diameter of the central tube B was but 9 mm., the transference of heat to the ice mantle was rapid. The windings extended a distance of 11 cm. along the rack, and two straight pieces of manganin connected the coil to the junctions of current and potential leads.

Both potential and current leads were of manganin to eliminate the thermoelectric forces; the former leads were of thin wire, while each of the latter consisted of two thick wires in parallel to diminish the heating effect of the current in the calorimeter stem. The resistance of the coil alone without the leads could be determined "in situ." A small correction had to be applied to the value so determined on account of the change in resistance by the heating effect of the current in the experiments.

This correction factor had been investigated in the case of a duplicate coil and a table drawn up.

The small temperature coefficient of manganin made the correction of small magnitude; the factor in the heaviest rate of supply only amounted to 1·0003. The coil was immersed in paraffin oil up to the junctions with current and potential leads.

V. Method of Filling the Calorimeter and of Forming the Mantle.

Filling.—The calorimeter was fixed in an inverted position so that the capillary tube was at the highest point; a funnel containing freshly boiled distilled water being attached to the orifice end of the capillary.

By repeatedly heating and cooling both bulb and tube could be completely filled with water, and finally the water was boiled a few times in the calorimeter. The desired quantity of mercury was introduced by heating and cooling with the orifice under mercury.

Freezing the Ice Mantle.—The preliminary cooling to 0°C. was effected by immersing the vacuum vessel and its contents in the ice packing for some time. For freezing a quill tube containing ether (cooled in liquid air) was inserted into the oil contained in the central tube of the calorimeter, the operation being repeated a considerable number of times.

Unless some ice was present in the bulb (from a previous freezing) considerable undercooling would take place, and then

superfusion would set in suddenly, the entire space between the inner tube and the bulb becoming filled with a loose mass of ice plates and needles mixed with water. This fibrous mass was allowed to thaw until only a few nuclei remained, and then the cooling was recommenced.

The formation now was a well-defined mantle of ice growing around the tube B, and by varying the depth of immersion of the cooled ether tube the region of growth of the mantle could be varied at will.

Lowering a small Geissler thermometer into the oil in E, immediately on the withdrawal of the cooling tube, indicated that the interior was but very little below 0°C. in temperature.

Very rapid formation of the ice mantle is not desirable, as, for example, when liquid air was poured directly into the tube B the mantle produced was filled with large cracks, and serious strains set up in the ice and glass.

On completion of the freezing the oil was removed by suction, the interior washed a few times with fresh oil, and then the requisite amount of paraffin run in.

Three or four hours had to elapse before the conditions were sufficiently settled to proceed with an experiment.

After switching off the heating current the mercury cup was allowed to remain under the orifice for periods of 3 to 10 minutes before being removed for weighing.

Observations of the change in weight of the second mercury cup supplied the data for the computation of the drift during an experiment.

It was found that the rate of drift depended to a certain extent on the time which had elapsed since the previous experiment, and it appeared that the heat generated in the leads would continue to pass into the calorimeter for a considerable time after the current supply was stopped. In consequence more than two experiments could rarely be performed in one day.

VI. The Absolute Value of the Quantities Involved in the Measurements.

Value of J.—The direct determinations of the calorimeter constant given in Table I. are expressed in terms of the mean calorie. Hence for conversion of the electrical measurements to the same unit we require the value of J expressed in ergs per mean calorie. Two independent determinations of J in terms of the mean calorie have been made.

1. The direct determination by Reynolds and Moorby * gave the value of J as $4 \cdot 183 \times 10^7$ ergs. The range in Reynolds' and Moorby's experiments, however, was not exactly 0° to 100°, but on the average 1·2° to 1·4° to 100°. Since the variation of the specific heat of water with temperature near 0° is considerable a small correction † has to be appplied to the above number.

After such correction J in terms of the mean calorie becomes $4 \cdot 1836 \times 10^7$ ergs.

2. The indirect determination of Callendar and Barnes, based on measurements of electromotive force and resistance, gave the value 4·185 (Barnes, 1909).

Hence we may assume the value $4 \cdot 184 \times 10^7$ for J with the probability that the error does not exceed 1 part in 4,000.

Time.—An electrically driven seconds pendulum of Invar steel recorded seconds on a chronograph tape. The error of the pendulum was determined by comparison with a rated chronometer.

The change-over key (controlling the heating current) was so designed that any time-lag between the marking of the tape and the actual switching on was compensated for during the operation of switching off. The tape could be read to a twentieth of a second, so the probable error in the timing should not exceed 1 part in 5,000.

Mass.—The set of weights had been calibrated and the absolute value of the 100 grms. determined at the National Physical Laboratory.

The change in weight could be determined to 0·1 mgm.

Electromotive Force.—The set of standard cells were frequently compared by means of a Thomson-Varley potentiometer with two National Physical Laboratory standards BC—1 and 2. The E.M.F. of these cells at 20°C. was certified as 1·01830 international volts.

Resistance.—The resistance of the coil had been determined in international ohms at the National Physical Laboratory about nine months previous to this work. Its value had since increased by 1 part in 4,000, the change being probably due to slight oxidation and rough usage in the preliminary experiments.

* "Phil. Trans.," A., 1897.

† "Thermal Measurements," E. H. Griffiths, p. 107.

VII. Experimental Results.

Column 1.—Number of standard cells balanced at the ends of the heating coil.
,, 2.—Change in weight of mercury cup corrected for drift.
,, 3.—Time in seconds of the electrical supply.
,, 4.—Calorimeter constant in mgms. reduced to vacuo.
,, 5.—Difference of individual experiments from mean.

Table III.

No. Cd. Cells.	Weight of Hg.	Time (sec.).	Constant.	Difference.
3	2·4605	1,431·60	15·467	+0·019
3	2·1670	1,265·50	15·525	−0·039
4	4·5200	1,481·40	15·504	−0·018
4	3·4485	1,132·05	15·479	+0·007
4	3·1520	1,034·75	15·479	+0·007
4	2·5380	832·65	15·489	−0·003
5	3·2041	673·40	15·475	+0·011
5	5·1630	1,084·75	15·480	+0·006
6	2·3328	339·95	15·499	−0·013
7	4·2530	456·70	15·456	+0·030
7	3·3853	362·60	15·493	−0·007
8	9·6725	793·50	15·488	−0·002
		Mean	15·486	

Hence, the value of the calorimeter constant is 15·486, *with a probable error of* 0·08 *per cent.*

VIII. Calculation of the Latent Heat of Fusion of Ice.

It is of considerable interest to compare the value of the latent heat determined directly with that obtained from the ice calorimeter, assuming the density of mercury, water and ice at 0°.

Calorimeter constant	15·486 mgms.
Density of mercury at 0°	13·5956 gms. per c.c.
,, water ,, ...	0·99988 ,,
,, ice ,, ...	0·9161* ,,

Hence, calculated latent heat of fusion of ice

=80·30 *mean calories.*

Bunsen's value for the latent heat is 80·02 (assuming his own value for the density of ice and the calorimeter constant).

The most recent direct determination by Mr. A. W. Smith,†

* The uncertainty of 1 part in 5,000 in the density of ice would have an effect of less than 1 part in 500 in the calculated value of the latent heat of fusion.

† "Phys. Rev.," 1903.

using artificial ice prepared from pure water, gives the value of the latent heat as 79·77, assuming J=4·184 and E.M.F. of Clark cell at 15° as 1·433 volts.

The difference of 0·7 per cent. between Smith's direct determination by an electrical method and the value calculated above, is difficult to explain on any other grounds than that of the presence of occluded water in the samples used for the direct determination.

If such is the case it illustrates the difficulty of obtaining ice entirely free from water, as Smith prepared the ice with considerable care.

SUMMARY OF RESULTS.

1. The evidence, adduced by various observers, in favour of ice having variable density cannot be regarded as conclusive.

2. The constant of the ice calorimeter is 15·486 mgms mercury per mean calorie assuming J=4·184 and E.M.F. of standard cadmium cell at 20° as 1·01830. This value agrees with Dieterici's determination, using water in quartz bulbs, to within 1 part in 3,000. Hence the discrepancy between Dieterici's value of J and that of other observers is not due to errors in his determination of the calorimeter constant, but is either due to inaccuracy in the absolute value of his electrical standards or systematic error in his experiments.

3. The calculated value of the latent heat of fusion of ice exceeds that obtained by measurements with ice in bulk by more than the probable experimental errors.

In conclusion, my thanks are due to Principal Griffiths, at whose suggestion the work was undertaken, and to my brother, Edgar A. Griffiths, for his assistance.

Research Laboratory, University College, Cardiff,

July 7, 1913.

ABSTRACT.

The primary object of the work was the re-determination, by an electrical method, of the constant of Bunsen's ice-calorimeter. The heat was supplied by a manganin coil wound on a mica rack which fitted the interior tube of the calorimeter, and the results are based on determinations of E.M.F. and resistance. The current was adjusted so that the difference of potential at the ends of the heating coil was exactly equal to the E.M.F. of a number of standard cadmium cells n series.

The conditions were varied as much as possible. Thus the rate of energy supply in the fastest experiments was more than seven times that in the slowest.

Errors due to progressive freezing or thawing of the ice mantle were greatly diminished by suspending the calorimeter within a transparent vacuum vessel of cylindrical form, the stem and capillary of the calorimeter projecting through a rubber stopper, and the vacuum vessel being completely embedded in powdered ice.

The mean value of the calorimeter constant was found to be 15·486 milligrammes of mercury per mean calorie.

The Constancy of the Density of Ice.—Various observers have advanced evidence tending to show that the density of ice at 0°C. is not a definite constant. A consideration of their work leads to the conclusion that the small variations of density found for different samples might be simply due to the presence of occluded water or an amorphous modification cementing the ice crystals together.

The value (80·30) of the latent heat of fusion of ice, calculated from the ice calorimeter, supports this view, as it is higher by about 0·7 per cent. than the value obtained by direct determinations with ice in bulk.

DISCUSSION.

Prof. H. L. Callendar noted with pleasure that the author had very closely verified Dieterici's value, using an entirely different method. He had had occasion to look into Dieterici's determinations in connection with the specific heat of water, and thought them very reliable. They agreed very closely indeed with the results obtained by Dr. Barnes and himself by the continuous electric method. The most uncertain element in Dieterici's work was the thermal capacity of his silica bulb, which amounted to about 25 per cent. of the whole. The extreme uncertainty of the calculation of latent heat from the volume changes on melting was not, perhaps, sufficiently realised. The calculation depended on the *difference* of the specific volumes of ice and water, so that any error in the value assumed for the density of ice seriously affected the value obtained for L. Thus the author, taking Vincent's result for the density of ice (=0·9161 gramme per cubic centimetre), had calculated L=80·3 *mean calories*, whereas, if 0·9167, the density as given by Barnes, had been assumed 79·5 (approx.) would have been obtained for the latent heat.

Dr. J. A. Harker mentioned a possible error due to the suction of mercury at atmospheric temperature, through a capillary of varying temperature, into the body of the calorimeter. Was the author's method of getting rid of "drift" superior to the method of putting one calorimeter inside another and larger one, and, by varying the pressure on the ice in the second one, adjusting its temperature until the "drift" was zero? He had at one time tried glacial acetic acid instead of ice. It had the advantage that when heat was supplied mercury was pushed out instead of sucked in. Also the transition temperature was between 16°C. and 17°C., and the constant was about 30.

Mr. F. E. Smith referred to the question of the leads to the coil. If they were too thick too much heat was conducted along them; if too thin too much heat was produced in them by the current. If the cross-section was assumed to be 2 sq. mm. then the resistance of a pair of manganin leads, each of which was 25 cm. long, would be 0·1 ohm. Since the heating coil was of 20 ohms resistance it followed that the heat produced in the leads was 0·005 times that produced in the coil. This heat was not accounted for in the calculations, and he would be glad if Mr. Griffiths would state what became of it. In his opinion much of it passed into the oil in the calorimeter. The experiments made by the author did not appear to him to indicate the

magnitude of the effect, since the heat passing from the leads to the oil would be proportional to the time of an experiment and also to the square of the current used.

Prof. SILVANUS THOMPSON asked whether there was any evidence of an allotropic or amorphous form of ice. According to the theory of crystal formation by arrangements of densest packing of molecules, any amorphous form should be less dense (as is the case of fused silica as compared with quartz crystal) and its specific heat per unit mass should be greater. Was the author sure that the water used was perfectly air-free?

Prof. C. H. LEES said the author had raised his opinion of the ice-calorimeter. He would like to know where it now ranked in the author's own estimation among other instruments used for similar purposes.

The AUTHOR agreed that the calculated value of the latent heat must be accepted with reserve, as it was difficult to predict in what direction the assumed value of the density of ice might be in error. The presence of occluded water in the specimens used would make the value too high, while minute air bubbles, due to dissolved air separating out, would make it too low. He had not yet attempted the substitution of other substances for ice. The use of glacial acetic acid would obviate the trouble at the orifice, since mercury would be ejected instead of sucked in. The heating coil was so designed that the heat generated in the leads was small. Moreover, only a fraction of this would pass into the bulb of the instrument, since the coil proper extended nearly the entire length of the inner tube. The heat generated in the leads outside the bulb would be partially absorbed by the glass stem and the air surrounding the leads. The slight conduction by the glass stem and leads down to the bulb would cause a minute "drift" lasting for a considerable time after switching off the heating current. This was corrected in the manner indicated in the Paper. He regarded the variation of the density of ice to be due to occluded water and not to allotropic crystallisation. He considered the water to be present as an amorphous cement binding the crystals together, In reply to Prof. Lees, he considered electrical methods, as, for example, the continuous electric method of Prof. Callendar, superior to the ice calorimeter. One advantage of the latter was, however, that there was no radiation correction.

II. *An Electrostatic Oscillograph. By* H. Ho, *Prof. of Electrical Engineering, Imperial University, Tokio, and* S. Koto, *Asst. Prof. of Electrical Engineering, Imperial University of Kyushu.*

Communicated by Mr. R. S. Whipple.

Received September 30, 1913.

Introductory.—As far as we know, the first attempt to record oscillations by an electrostatic force was made by E. T. Jones* in 1907. His instrument was connected in idiostatic fashion, however, and vibrated with a frequency twice as high as that of the impressed voltage, consequently it cannot be said to have much value as an oscillograph. In the discussion on E. A. Watson's Paper,† in 1910, J. T. Irwin showed some curves which were taken on an electrostatic oscillograph, but as no description either of the instrument itself or of the results obtained has been published since that time, we are not in a position to criticise his invention or compare it with our own. The present device, which was brought to a practical form two years ago in the laboratory of the Engineering College, Tokio Imperial University, is the outcome of an endeavour to supply the need for a suitable oscillograph for recording high voltages, which we felt keenly in the investigation of the electrical properties of insulating materials, and we give here a rough description of its general principles and construction in the hope of interesting some persons engaged upon the same line of research.‡

The present practice of recording very high voltages with an ordinary electromagnetic oscillograph with high non-inductive resistance connected in series with it, possesses many disadvantages, which are enumerated as follows :—

(*a*) The large volume and the high cost of the non-inductive resistance.

(*b*) The considerable loss of energy in the resistance, which makes the method totally unsuitable for cases in which the source of the energy supply is limited, as, for example, when

* "A Short Period Electrometer," "Phil. Mag.," 1907.

† "Losses of Transmission Lines due to Brush Discharge," "Journal" of I.E.E., 1910.

‡ The instrument is now constructed by the Cambridge Scientific Instrument Co., Ltd., England.

using an influence machine, or when the disturbance due to the current taken by the oscillograph cannot be neglected.

(c) The effect of the electrostatic capacity of the resistance, which is more and more perceptible as the size of the resistance is increased and the voltage becomes higher, the consequence being that the current flowing into it through the oscillograph may not be in phase with the voltage to be recorded.

We wish to draw particular attention to this last point. H. L. Curtis and F. W. Grover * have shown that the effective inductance of so-called non-inductive resistances on the market reached such a high value as—100,000 micro-henrys in a coil of 10,000 ohms. Needless to say, this would cause a considerable phase advancement of the current if such a resistance be used with an oscillograph, and the adoption of some special method of winding for minimising the capacity becomes imperative, which means very costly apparatus. In our electrostatic oscillograph the reduction of the voltage to be recorded to a value suitable for the vibrator is effected by means of condensers which are quite inexpensive and occupy a very small space, the power consumption being practically zero. The electrostatic capacity of the vibrator proper is exceedingly small, but oil-condensers in multiple with it are required to adjust its potential and a charging current is taken by them. It is, however, incomparably less than the current consumed by an ordinary oscillograph; hence, in most cases it causes no appreciable disturbance of the external potentials. The majority of the disadvantages can be thus eliminated by using an electrostatic oscillograph for high voltages, say, above 1,000 volts, there being no upper limit whatever, provided suitable condensers for series connection are used.

Description.—The general arrangement is shown diagrammatically in Fig. 1, in which the P.D. between a_0 b_0 is required to be recorded. C is an oil-condenser to be used, if necessary, for reducing the said voltage to a suitable value at the terminals a b of the oscillograph. Two bronze strips, s_1 and s_2, are stretched between two parallel metallic plates, F_1 and F_2, called "field plates"; these strips are parallel to one another and to the plates, and carry a small mirror m in the centre, the tension being adjustable in the manner usually adopted. Contrary to the method used in an ordinary oscillograph, the strips are insulated from each other by a silk thread, t, which connects

* "Bulletin" of the Bureau of Standards, Vol. VIII., No. 3.

them and passes over an ivory pulley, p, thus forming two electrically independent conductors in the electric field of the parallel—plate condenser F_1, F_2. The plate $\mathbf{F_1}$ has an opening w_1, for the passage of the light ray to and from the mirror, and an exactly similar one is cut in the other plate, F_2, to maintain the symmetry of the electric field. The plates and the strips, mounted on an ebonite frame as shown in Fig. 2, complete the vibrator, which is immersed in an oil bath, provided with the necessary number of terminals with suitable insulating bushings for the pressure to be employed. Two

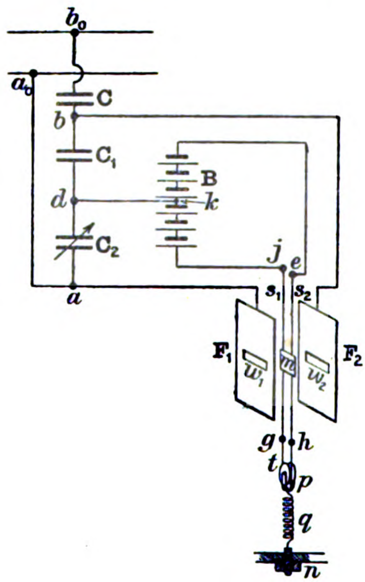


FIG. 1.

oil-condensers, C_1 and C_2, in series are connected to the points $a\,b$, in multiple with the field plates, as in Fig. 1, one of which condensers must be adjustable.

B is a water-battery, or a number of small dry cells, the latter being decidedly preferable, connected in series and mounted on an insulating stand, the electrical centre k of this battery being connected to the point d between C_1 and C_2. Its terminals are connected to the two strips s_1, s_2 in order to charge them for the same purpose as the needle of a quadrant electrometer is charged. The width of the field plates is 9 mm., the

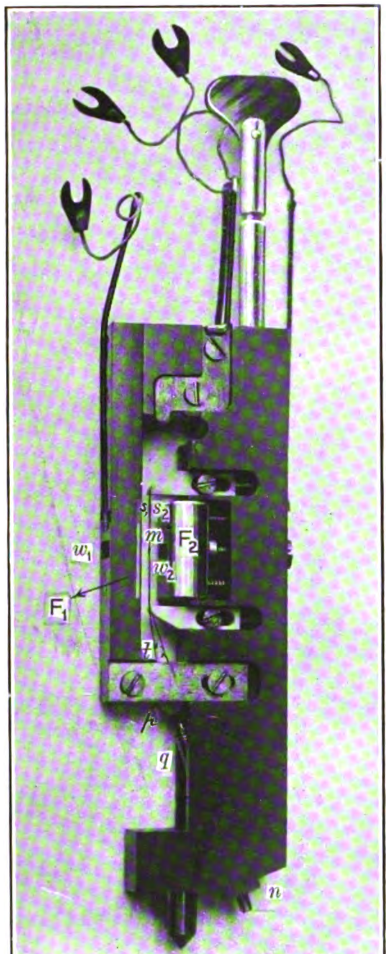


FIG. 2.

To face p. 18.

distance apart 5 mm., and the vibrator has approximately the same dimensions as the latest Duddell vibrator. These two types of vibrators can thus be placed conveniently side by side in the same oil bath, for which purpose one of the field plates F_2 is made of soft iron to serve as a part of the magnetic circuit.

Theory and Method of Use.—Fig. 3 represents two insulated strips, s_1, s_2, placed between two insulated parallel metallic plates, F_1, F_2. First, suppose equal and opposite charges $+q$ and $-q$ be now given to s_1 and s_2, a distribution of the charge is produced on the conductors F_1 and F_2—let us call it distribution No. 1—but their potentials remain at zero as before,

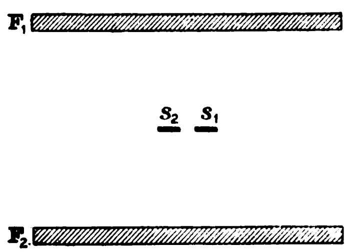


FIG. 3.

while the strips acquire equal and opposite potentials proportional to q, which may be denoted by

$$\left.\begin{array}{r} v_{s_1}=kq \\ v_{s_2}=-kq \\ v_{f_1}=v_{f_2}=0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Next, suppose no charge be given to the strips, but that a P.D. $(v'_{f_1}-v'_{f_2})$ be applied between the plates F_1 and F_2. The Faraday-tubes thereby produced do not differ appreciably from those obtained when no strips are present, as they are placed at right angles to the flow of the tubes and the thickness is very small compared with the distance between the plates. Since the strips are in the exact centre between the plates their potentials must then be

$$v'_{s_1}=v'_{s_2}=\tfrac{1}{2}(v'_{f_1}-v'_{f_2}). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let the distribution of electricity produced on F_1, F_2 in this case be called distribution No. 2; the total charge on each of the strips is, of course, zero.

Let the battery B on an insulated stand and the two condensers C_1 and C_2 now be connected up as in Fig. 4, the middle

point of the E.M.F. of the battery being connected with the point d between the condensers, and suppose the P.D. $V_a - V_b$ be applied at $a\ b$, then evidently

$$V_{f_1} - V_{f_2} = V_a - V_b \quad \ldots \ldots \quad (3)$$

$$V_{s_1} = \tfrac{1}{2}(V_a - V_b) + \tfrac{1}{2}E_B \quad \ldots \ldots \quad (4)$$

$$V_{s_2} = \tfrac{1}{2}(V_a - V_b) - \tfrac{1}{2}E_B, \quad \ldots \ldots \quad (5)$$

where E_B is the total E.M.F. of the battery. We wish to determine the distribution of electricity under this condition. In

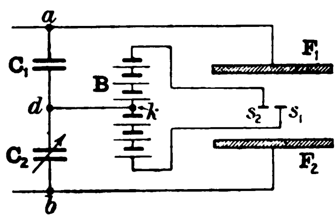


FIG. 4.

the case which we have first supposed, let the equal and opposite charges given to s_1, s_2 be in accordance with the equations

$$\left.\begin{aligned} q_{s_1} &= \frac{1}{k}\frac{E_B}{2} \\ q_{s_2} &= -\frac{1}{k}\frac{E_B}{2} \end{aligned}\right\} \quad \ldots \ldots \quad (6)$$

Then, by (1)

$$\left.\begin{aligned} v_{s_1} &= \frac{E_B}{2} \\ v_{s_2} &= -\frac{E_B}{2} \end{aligned}\right\} \quad \ldots \ldots \quad (7)$$

The distribution No. 1 is produced on F_1F_2, and their potentials (v_{f_1} and v_{f_2}) are both zero.

Next, in the case we have secondly supposed, let

$$v'_{f1} = V_a,$$

$$v'_{f2} = V_b,$$

then the distribution No. 2 is produced on F_1F_2, and the strips are at the potential

$$v'_{s1} = v'_{s2} = \tfrac{1}{2}(V_a - V_b).$$

Suppose these two distributions to be superimposed, the result

must then produce another state of equilibrium, and the potentials must be as follows :—

$$(v_{f1}+v'_{f1})-(v_{f2}+v'_{f2})=V_a-V_b=V_{f1}-V_{f2},$$
$$v_{s_1}+v'_{s_1}=\tfrac{1}{2}(V_a-V_b)+\tfrac{1}{2}E_B=V_{s_1},$$
$$v_{s_2}+v'_{s_2}=\tfrac{1}{2}(V_a-V_b)-\tfrac{1}{2}E\ =V_{s_2}.$$

But, as may be seen, they are the same as (3), (4), (5), hence we may conclude that the electric distribution, corresponding to the condition expressed by (3), (4) and (5) is the superimposing of two such distributions as we have just described. That is to say, on the strips are superimposed :—

(α) Equal and opposite charges proportional to E_B.

(β) Induced charge which is zero in total, on each strip.

On the plates are superimposed.

(γ) the distribution No. 1 corresponding to (α),

(δ) the distribution No. 2 proportional to (V_a-V_b).

Between (β) and (γ), between (β) and (δ) and between (α) and (γ) no force can be produced that gives rise to the deflection of

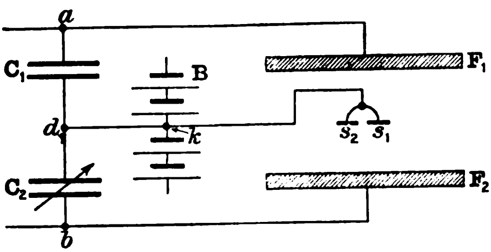


FIG. 5.

the mirror, that between (α) and (β) being only influential in producing a turning moment, the momentary value of which is proportional to the product of the momentary values of E_B and (V_a-V_b). Hence, if E_B be constant, the deflection of the mirror will follow the variation of (V_a-V_b) accurately, provided proper damping is obtained. This gives a rough idea of the principle of the action of our electrostatic oscillograph.

In the above we have supposed the pair of the strips to be in the geometrically exact centre between the field plates, but this is difficult to attain in practice. Consequently, it is found in practical use that it is not sufficient to make C_1 equal to C_2, but the following adjustment is always necessary. Disconnect the strips from the terminals of the battery, join the strips

together and connect to the point k, as in Fig. 5, and apply an alternating E.M.F. at ab, which will cause the light spot on the screen to vibrate with a frequency twice as high as that of the E.M.F., as may be observed by the revolving mirror. This vibration is produced by the electrostatic force acting on the pair of strips, caused by a lack of symmetry and the small inevitable inequality of tension between the strips. Adjust one of the condensers, C_1C_2, till this vibration totally disappears; this ensures that the potential of the strips will remain equal to that of the same position when there were no strips at all. The adjustment of the condenser being thus made, connect up as shown in Fig. 4, and the oscillograph will then be ready for use. If we impress an alternating E.M.F., the imperfect adjustment of the said condenser will manifest itself by the dissimilarity of the forms of the positive and negative waves, and the incorrect choice of the point k is discerned at once by the shifting of the wave from the zero line.

It is advisable to connect the metallic oil-bath to the point d, and when the vibrator is used with another electrostatic vibrator or with an electromagnetic vibrator in the same bath it must be shielded on both sides by tinfoil suitably insulated with mica plates, and kept at the potential of the point d in its own circuit. It is perhaps more convenient to use a separate oil-bath for each vibrator, the optical system being suitably modified.

Oil plays the most important part in our oscillograph; not only does it serve as the damping agent and as the insulator, but its high specific inductive capacity as compared with air does much in increasing the sensitiveness, and but for its well-known property of causing very little phase difference between the electric intensity applied and the consequent displacement, the electrostatic oscillograph would have been an absolute impossibility.

In cases where the voltage is low but the source of energy is extremely limited, so that a sufficient current cannot be taken to actuate the ordinary oscillograph, we may employ the electrostatic vibrator and apply the voltage in question to the strips s_1s_2, while the terminals of a high-tension battery or of an influence machine, as in Fig. 8, are connected to F_1F_2 instead of to the strips.

Sensitiveness.—Those who are not familiar with the electrostatic phenomena and are apt to undervalue the force electrostatically produced, may have an incorrect idea of the sensi-

tiveness of our oscillograph, and assume that it must have a very long natural period to get an appropriate deflection. To contradict this impression it will suffice to state that our vibrator, when adjusted for the natural period of $\frac{1}{3300}$ second, as determined experimentally by the resonance method and at the same time by calculation in accordance with the formulæ for the natural period of a bifilar oscillograph, gave a wave 2 cm. in amplitude at the distance of 70 cm. by $(V_a - V_b) = 2{,}000$ volts effective, the total E.M.F. of the battery being 300 volts. The dimensions of the field plates were 15 mm. by 9 mm., and the distance between them 5 mm.

The sensitiveness can be easily adjusted by changing the number of cells B. The proper range of the voltage at the terminals of the plates, when the connection of Fig. 1 is adopted, would appear to be 1,000—6,000 volts effective, although voltages up to 9,000 have been measured.

Parallel Working with an Ordinary Oscillograph.—Fig. 6 shows the curves obtained with a falling plate camera by using the electrostatic vibrator in parallel with an ordinary

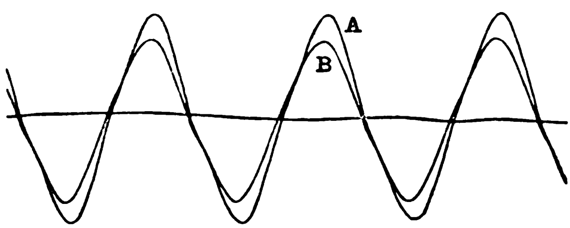


FIG. 6.

electromagnetic vibrator for a voltage of 2,000 volts effective, 60 ~; the curve A was produced by the electrostatic and B by the electromagnetic vibrator. In series with the latter vibrator we connected a "non-inductive" resistance of the type supplied with a wattmeter, as we had no suitable resistance specially made for oscillograph work, and the result is somewhat startling, showing in a most pronounced manner the effect of the capacity of the resistance by the advance of phase thereby produced, thus confirming the statement of Curtis and Grover in the Paper already referred to, and incidentally emphasising the superiority of the electrostatic oscillograph for high-voltage work. We have reason to believe that the high resistances supplied by oscillograph makers at present will give results not much different from this.

Fig. 7 shows the same experiment with a generator which gives a particularly pointed E.M.F. wave—the frequency being again 60 ∾.

Application to Recording Very Small Currents.—The recording of very small currents, lower than $\frac{1}{1000}$ ampere, say, is impossible with an ordinary oscillograph, unless a current

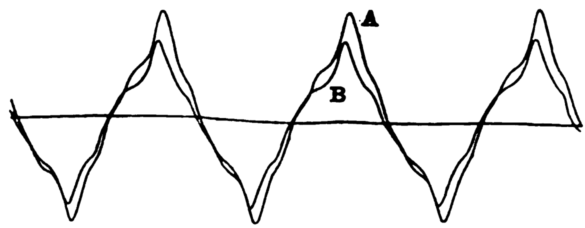


FIG. 7.

transformer is used. But in some kinds of investigations the use of a current-transformer is not desirable, and the electrostatic oscillograph may be called upon again for this kind of service with advantage. The connection for this purpose is shown in Fig. 8, in which R_1 and R_2 are two exactly equal

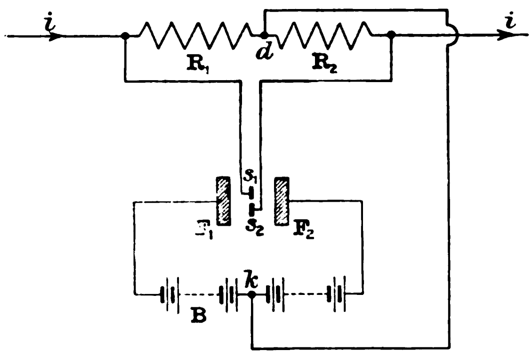


FIG. 8.

resistances with as little inductance and capacity as possible, connected in series in the circuit of the current to be recorded, the middle point *d* being connected to *k* the centre point of the E.M.F. of the high-tension battery B. It is less expensive to use an influence machine with two equal condensers joined in series, as shown in Fig. 9, in place of the battery.

Fig. 10 is the result of an attempt to record the charging current while testing the dielectric strength of a glass plate 0·7 cm. thick between tinfoil electrodes 21 cm. by 15 cm. in air, with a pressure of 9,000 volts effective, the edges of the electrodes showing active brush discharges. A is the current curve taken in the said manner and B the voltage curve taken with

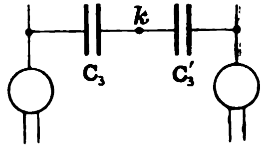


FIG. 9.

another electrostatic vibrator. By passing a known current, the current curve was found to indicate a maximum height of about 0·0005 ampere ; the P.D. between F_1 and F_2 was kept at 3,500 volts with a Wimshurst machine. On account of the lack

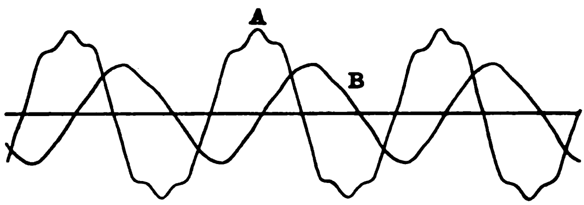


FIG. 10.

of a suitable number of condensers and resistances, the adjustment was not as satisfactory as could be desired, and the result is rather qualitative than quantitative, but it demonstrates the utility of the instrument for investigations in which small currents have to be recorded at very high voltages, and the loss of voltage in R_1 and R_2 can be neglected in comparison with the applied voltage.

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ABSTRACT.

The Paper describes an electrostatic oscillograph suitable for recording very high voltages. Two vertical bronze strips pass symmetrically between two parallel metallic plates called "field plates." They are connected at their lower ends by a silk fibre which passes under an ivory pulley. An extremely small mirror is

fixed to the strips. This arrangement constitutes the vibrator, which, mounted on an ebonite frame, is immersed in an oil bath. To the upper extremities of the strips are connected the terminals of a direct-current voltage of about 300. The alternating voltage to be recorded is connected to the "field plates," in parallel with which there are two oil condensers in series. The electrical midpoint of the direct-current battery is connected to a point between the condensers.

The turning moment on the strips is proportional to the product of the momentary values of the alternating-current voltage and the direct-current voltage, so that if the latter is constant, the deflection of the mirror accurately follows the variation of the former.

Oil plays an important part, not only acting as a damping agent and insulator, but increasing the sensitiveness on account of its high dielectric constant.

In cases where voltage is low, but the source of energy is so limited that a sufficient current cannot be taken to actuate the ordinary oscillograph, the electrostatic vibrator may be used by applying the voltage in question to the strips, while the terminals of a high-tension battery, or influence machine, are connected to the "field plates." The instrument may also be used for recording very small currents by replacing the oil condensers by two exactly equal resistances, which are traversed by the current.

DISCUSSION.

Mr. A. CAMPBELL thought it was a considerable advantage to be able to do without high resistances. It had been suggested to him that the instrument might be of great use in work with X-rays and in other cases where the charge was being taken from an induction coil.

Mr. R. S. WHIPPLE emphasised the commercial advantage of the instrument. High resistances were very expensive. Einthoven was the first to propose an electrostatic oscillograph, and a Belgian inventor had constructed an instrument similar in principle.

Prof. G. W. O. HOWE stated that in the two curves given the difference was assumed to be due to a "lead" in the electromagnetic instrument. Was there any actual evidence that it was not due to "lag" in the other? If the oil did produce a phase-difference, it would introduce a "lag" and give the observed effect.

Mr. E. H. RAYNER (communicated): The oscillograph described by the authors and other similar instruments are intended to be instruments for depicting curves of current intensity. By the use of a series resistance the ordinary oscillograph gives a record of the voltage of a circuit. A watt-meter oscillograph would often give valuable information, as it is the power dissipated in a circuit which is often of primary importance, as in testing insulation, for instance. The authors' instrument appears to be usable as a watt oscillograph by substituting in Fig. 8 the secondary of a transformer for the polarising battery, the middle point of the winding being connected to the junction of R_1 and R_2 just as the battery is in the diagram. The primary of the transformer would be supplied from the alternator, which is connected to the circuit through R_1 R_2. There would only be a slight error due to the small difference from 180 deg. between primary and secondary of this transformer. If the alternator were a two-phase one the "idle" component of the current could be at once depicted by supplying the transformer from the other phase.

Mr. J. T. IRWIN said he was already aware of the device suggested by Mr. Rayner, having described it in 1907. The principal difficulty with an electrostatic oscillograph was that a very high polarising voltage is re-

quired if the zero is to be stable. The authors had reversed the process—using a small polarising voltage compared with the alternating-current voltage. Hence it is only when the alternating-current voltage is applied that a change of zero is produced. In most cases this cannot be compensated beforehand, as we do not know the value of the potential to be applied. Castor oil was not suitable for use in oscillographs, as it became acid and conducting under high voltages. Well-filtered paraffin had not this defect.

Mr. D. Owen regretted that the Paper contained so little in the way of actual work accomplished with the instrument. If one took the values stated in the Paper for the inductance of the high resistances used with E.M. oscillographs, and assumed a frequency of 100∼ per second, calculation seemed to show that the phase-differences introduced by the use of such resistances were much too small to account for the discrepancies in the experimental curves.

Mr. G. L. Addenbrooke said that if castor oil free from dust and moisture were used he did not think oscillograms with the instrument would be affected perceptibly by action in the oil at 40 periods or higher. With a dry oil much lower periodicities might be used. The influence of moisture is greater with mineral oils than with castor. The dielectric strength varies almost directly with the percentage of moisture present.

Dr. G. W. C. Kaye (communicated): One of the features that interests me in the Paper is the possibility of applying the very neat and ingenious apparatus of the authors to the measurement of the potential curve of the current from an induction coil. Such a curve exhibits rapid and marked discontinuities and abruptness, and the difficulty of satisfactorily recording the detail by ordinary oscillograph methods has proved to be almost impossible. This is more especially the case when the coil is working an X-ray bulb, which ordinarily passes a current of the same order as that through the oscillograph itself. The various peaklets of the potential wave on an X-ray tube doubtless have their analogies in the cathode ray velocities; and it would be interesting to correlate the shape of the potential loop with the magnetic spectrum of the cathode rays in a coil-driven discharge tube.

Prof. Koto, in reply, said that the frequency of the voltage used for the tests was 60∼, the records being taken on a falling-plate camera. He saw no objection whatever to the use of the oscillograph as a wattmeter, as suggested by Mr. Rayner and carried out by Mr. Irwin. He stated that the oil used was that supplied by the Cambridge Scientific Instrument Co., and that from some rough measurements he had made he found it to have a specific inductive capacity of about 2. The deviation of the strips from the centre was a cause of the change of zero, as Mr. Irwin suggested, if a large polarising voltage be applied. To suppress this effect the authors made the distance between the two field plates, consequently the distance between the strip and the plates, sufficiently great to make this inevitable slight deviation from the centre a very small percentage of the distance between the strip and the plates, and consequently this deviation from symmetry would cause no appreciable "out of balance" of the forces due to the polarising voltage acting upon the strips and effecting this change of zero. Referring to the effect of the oil on the phase relationship of the curves, Prof. Koto stated that he had no doubt that the curves obtained on the electrostatic oscillograph and reproduced in the Paper were correct, but he intended to make a series of experiments on different oils to see whether any serious phase differences are caused by the oils having different dielectric hysteresis constants.

III. *On the Thermal Conductivity of Mercury by the Impressed Velocity Method.** *By* H. Redmayne Nettleton, *B.Sc., Assistant Lecturer in Physics at Birkbeck College.*

Received July 17, 1913.

1. Introduction.

For some time the author has been interested in temperature gradients modified by the movement of the conductor itself, and has already, with the aid of ordinary thermometers, obtained the general form of temperature curve down a column of mercury heated at the top, maintained cold at the bottom, and moving uniformly upwards from the lower to the higher source. Assuming the isothermals in such a moving column to be horizontal, he has obtained an expression for the thermal conductivity of mercury†; but, owing to the nature of the assumption and the necessity to resort to graphic interpolation, no accuracy other than that of the right order was claimed for the value obtained.

The present communication describes an apparatus so designed that with a single thermo-junction—carried by a cathetometer capable of rotation—the temperature may be found at any desired point within a wide vertical vacuum-jacketed tube containing mercury. As a result of experiments with flows of different magnitudes the conclusions have been arrived at :—

1. That the isothermals in a vertical column of mercury moving upwards from cold to hot are remarkably horizontal even when the disturbance of temperature gradient by flow is very large.

2. That, owing to this fact and the great accuracy with which Newton's law holds in a vacuum-jacketed vessel for the range of temperature necessary, the impressed velocity method is very suitable for determining the thermal conductivity of mercury at the temperature of the enclosure; the method, in fact, possessing all the advantages of continuous flow calorimetry.

* The expenses of this research—other than the cost of the mercury—were defrayed by a Government grant received through the Royal Society.

† "Proc." Phys. Soc, London, Vol. XXII., 1910. Also "Phil. Mag.," April, 1910.

2. Statement of Theory Employed.

The differential equation pertaining to the distribution of temperature down a moving conductor has already been discussed by the author, and it suffices here merely to state the solution in the form most suitable for the present research.

Consider a column of mercury heated by steam at the top and maintained cold by ice at the bottom, and let the mercury be flowing uniformly upwards, m grammes per second crossing a horizontal section. Assume the isothermals horizontal, and let all temperatures be measured from the temperature of the enclosure taken as zero, and all distances, reckoned upwards as positive, be measured from the isothermal within the tube, which is at the same temperature as the enclosure, and which for brevity is hereafter referred to as the zero isothermal.

Let K denote the thermal conductivity of mercury,
s=the specific heat of mercury,
A=the cross-section of the tube employed,
p=the perimeter of the tube,
E=the Newtonian coefficient of emissivity.

Then the temperature θ at any distance, x, from the zero isothermal is given by the equation

$$\theta=\mathrm{M}e^{\frac{ms}{2\mathrm{KA}}\cdot x}\sinh \lambda x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\lambda=\sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2+\frac{\mathrm{E}p}{\mathrm{KA}}}$, and is thus constant for any one flow, and M is a constant depending on the temperatures and positions of the fixed sources.

Since $\sinh \lambda(-x)=-\sinh \lambda x$, it follows at once that if θ_1 and θ_2 be the respective temperatures at distances L/2 above and below the zero isothermal, then

$$\frac{\theta_1}{-\theta_2}=e^{\frac{ms\mathrm{L}}{2\mathrm{KA}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the ratio of the two temperatures being thus independent of the heat lost by radiation.

If the mercury is at rest equation (1) becomes

$$\theta=\mathrm{M}\sinh \mu x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\mu=\sqrt{\frac{\mathrm{E}p}{\mathrm{KA}}}$, and can easily be evaluated—as is well known—from observations of temperature at three equidistant points.

Though equation (2) is generally the most convenient for calculating the value of K, there is another very simple method of treatment showing the essence of the method—especially its relation to continuous flow calorimetry—and bringing out most clearly what method of procedure would be necessary if the experimental tube could not be regarded as uniform in cross-section.

Let g_1 be the value of the temperature gradient at a place where θ_1 is the temperature and A_1 the cross-section, and let g_2, A_2, θ_2 similarly refer to a lower position. We have then in the steady state

$$KA_1g_1 - KA_2g_2 = ms(\theta_1 - \theta_2) + R. \quad . \quad . \quad . \quad (4)$$

where R is the heat lost by radiation between θ_1 and θ_2.

Now, $R = \int_{\theta_2}^{\theta_1} Ep\theta \, . \, dx$, and can be made approximately zero other than by taking θ_1 and θ_2 near together—namely, by choosing θ_1 and θ_2—so that the heat lost between θ_1 and the zero isothermal equals that gained between the zero isothermal and θ_2, a selection which resolves itself into choosing two areas approximately equal. If, then, by this means R be made negligible and the cross-section is sufficiently uniform, we have

$$K = \frac{ms}{A}\left(\frac{\theta_1 - \theta_2}{g_1 - g_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The maximum error made possible by the conductivity of the glass vessel containing the mercury and of the carrier within holding the thermo elements is easily found by considering the horizontal isothermals to extend into the material of the glass. If a be the area of the cross-section of the glass and k its thermal conductivity, the term $ka(g_1 - g_2)$ should strictly be added to the left-hand side of equation (4). It follows that the values of K obtained either from equations (5) or (2), both of which can be deduced from the fundamental differential equation of the method, are too high, owing to the effect of the glass, but the error is necessarily less than 100 ka/KA per cent.

3. Description of the Apparatus.

The experimental glass tube T containing the mercury is about 4·5 cm. in diameter and 60 cm. long, the upper portion being within the annular heater A; the middle portion of the tube is vacuum-jacketed for some 28 cm., and the lower

portion dips into the large reservoir R of mercury which constitutes the lower temperature source. The object of the high vacuum is to secure constant conditions of emissivity without resort to cotton wool, it being a great advantage to see through the vessel prior to filling it with mercury at the commencement of each experiment. The iron reservoir R was made from a bottle of mercury by cutting off the top and cleaning and enamelling the inside ; it rests on a tripod in the ice vessel V, which stands on the strong bridge B. This bridge also supports the annular heater carrying the experimental tube, which latter thus dips like a barometer or siphon into the mercury in R.

The thermo-junction of iron and constantan was prepared with skill by Messrs. Cossor, Limited. The thin insulated wires contained within the vertical glass tube t of about 6 mm. diameter were fused through the glass at a (Fig. 1) with

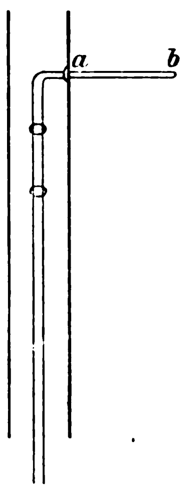


FIG. 1.

inappreciable external distortion, the wires from a to the junction b being close together and in the same horizontal plane. The tube t was held at Cl by the steel carrier, as seen in Fig. 2, being bent and brought up along the carrier side, as shown, the wires emerging from the tube at Fe, Cn. The length ab was such that when the tube t was resting against the inner wall of the experimental tube T the point b would just reach the centre of the latter.

The cathetometer K, which carries the thermo-junction, was fitted with translational and rotational adjustments, and stood on the sector S, which itself could be rotated about a vertical axis, X, over the smooth glass track W laid out on the horizontal

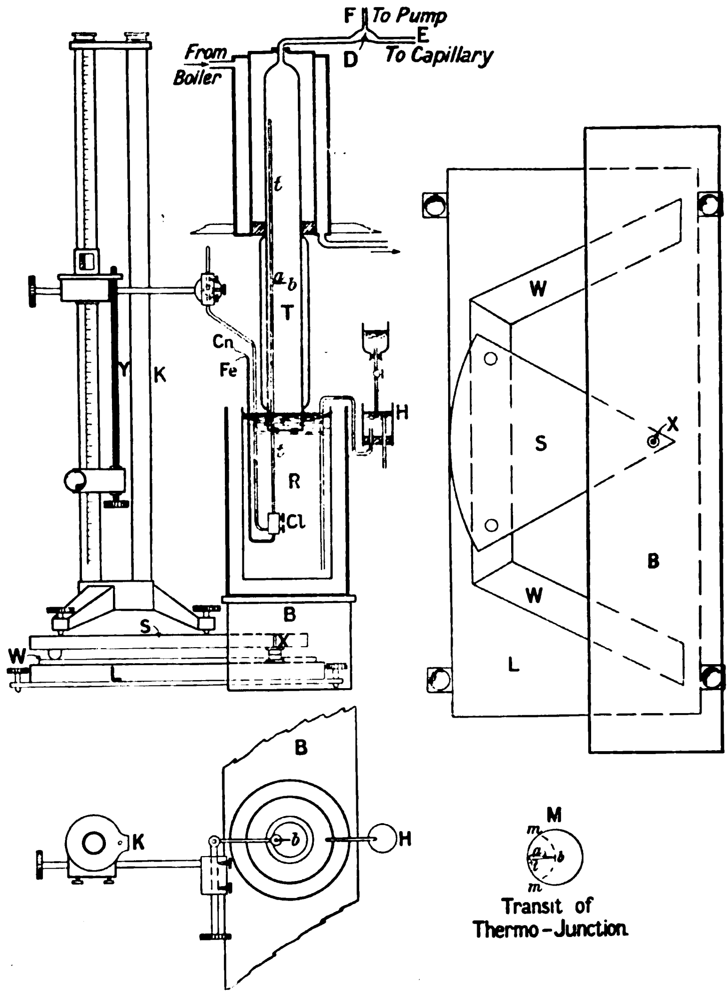


Fig. 2.

table L. Thus the straight portion t of the tube containing the thermo elements could be brought accurately into the vertical and into the same straight line as the axis of rotation.

By turning the screw Y the tube *t* could then be raised or lowered vertically alongside the inner wall of the experimental tube T level, and situated as shown, while by rotating the sector S the tube *t* would rotate about its axis without any lateral displacement, the thermo-junction describing an arc, as seen in plan at M, the limiting position being when the actual junction touches the wall at *m*. In order that the longitudinal movement up or down of the tube *t* should in no way modify the distribution of temperature down the mercury column, it is necessary that, whatever be the position of *t*, its extremities should be well within the respective temperature sources A and R, a condition requiring that these constant sources and *t* itself should be of sufficient length.

The other junction of the thermo elements at the temperature of the enclosure was contained in a glass tube lying within a vessel of water in the proximity of the apparatus, while the joins of the iron wire to the galvanometer leads were likewise in glass tubes immersed side by side in water. The galvanometer was a quartz fibre instrument of the Broca type, and with the thermo-couple employed and the range of temperature to be measured deflections on the distant scale were accurately proportional to temperature changes.

The top of the experimental tube was narrowed down to capillary bore and attached by pressure tubing to the bulb tube D. At E was attached pressure tubing leading to a drawn-out capillary with its extremity well below the constant head H, this capillary constituting the resistance whereby a constant flow of mercury could be maintained up the tube T. Pressure tubing attached at F led to a Fleuss pump, enabling the whole experimental tube and capillary to be completely filled with mercury drawn up from the reservoir R. Convection currents around the annular heater were allayed by the free use of cotton wool. The Author would here record his thanks to Mr. J. M. Scholer, B.Sc., for his skill in adapting the cathetometer to permit the necessary movements and for the actual construction of the rotating table and its accessories.

4. Method of Experiment.

Before each experiment the table L was levelled to the horizontal, and the cathetometer and tube *t* to the vertical, *t* being also adjusted to be coaxial with X. The vacuum tube T was then let down over *t*, levelled and brought up so that *t* was just on the point of touching the walls. Mercury was then

drawn up to the level of the thermo-junction, which, on rotating the sector S, was seen to sweep out the necessary arc without appreciable lateral displacement. The positions of the sector when the junction was touching the walls at *m* were carefully marked. All levelling adjustments being satisfactory, the apparatus was completely filled, and the experiment started, usually about eight o'clock at night. Steam was passed through A and ice placed in V, while a pinch cock was released and the height of the capillary adjusted to secure a suitable uniform flow of mercury. In two to four hours—according to the magnitude of the flow and the state of gradient when it was started—the steady state was attained, a shorter time being required than when the column was stationary.

The measurement of relative temperatures at definite places by the movement of the carrier tube *t* was now proceeded with and found to be satisfactory, for on moving to a definite cathetometer reading the junction registered its final temperature almost immediately. Moreover, on returning to the original position, the original temperature was recorded to an accuracy of the same order as the accuracy of re-setting. Starting with the thermo-junction in its highest position—the resistance of the circuit being adjusted to obtain a maximum deflection—it was lowered by carefully rotating the screw, readings being taken when the arrow head of the vernier lay on each whole and half centimetre division. The temperature curve was then retraced, *t* being raised from the lowest position to the starting point. The relative temperatures given in the results are obtained by adding the upward and downward readings. This process was tedious, and in the earlier experiments lasted an hour and a half; in later experiments many readings in the neighbourhood of the zero isothermal were omitted.

The flows were measured by the weight of mercury—ranging from 871 grammes to 1,422 grammes—outflowing in 15 minutes. They were weighed to the nearest gramme, and were at least constant to one part in five hundred. A flow of 1,000 grammes per hour is equivalent at 20°C. to a linear velocity of 17·84 cm. per hour. As, however, the density of mercury is not required, and varies slightly with the temperature, whereas the product of the density, the velocity and the area of the cross-section is everywhere constant, it is natural and convenient to measure the flow by weight.

The mean internal cross-section of the experimental tube T for the 12 cm. over which practically all measurements of

temperature were made was found accurately after the research, with the tube t in situ, with the aid of distilled water and a cathetometer and needle. It was found to be 16·55 sq. cm., which agreed with the value found when t was removed and its cross-section found with a gauge and allowed for by subtraction. The experimental tube was found to be slightly conical, its cross-section varying by 1 per cent. in 11 cm. but as the extreme distances between readings used in calculating the thermal conductivity range from 6 cm. to 10 cm. the error is very small.

The effect of the upward flow is to make the actual range of temperature over the vacuum-jacketed portion of the tube T very much smaller than when the mercury is at rest. Calibration showed that for the 12 cm. explored the range varied from 20°C. to 25°C., according to the magnitude of the flow. In order, therefore, as a secondary experiment to find the value of Ep/KA from the temperature gradient when the mercury column is at rest, ether (B.P. 34·96 deg.) was circulated through the heater instead of steam. The temperature range over the investigated portion was then of the same order as in the dynamical experiments.

The specific heat of mercury is taken as 0·0333, which is Barnes'* value at 15°C.

5. Results of Experiments.

(*a*) *Horizontal Nature of Isothermals.*—On rotating the thermo-junction over a horizontal section there was no appreciable alteration of galvanometer deflection even when the junction was on the point of touching the walls and lying along the side of an inscribed hexagon. Many such rotations have been tried at different parts of the vacuum tube with flows of different magnitude, but all with precisely the same order of result; whereas on careful rotation of the junction the spot of light might move 1 mm. on the scale in either direction, on turning the screw to lower the junction vertically through 0·1 mm. a definite and larger effect would be produced. It is hence to be concluded that the isothermals in a vacuum-jacketed column of mercury are remarkably horizontal even when the temperature gradient is considerably modified by flow. It should here be pointed out that it is not perhaps surprising that even when the flow is swift no increased effect can be noticed, for the longitudinal result of a very swift flow

* Barnes, Brit. Assoc. "Report," p. 530, 1902.

over the explored portion of the tube is to render the range of temperature itself very small. The steep portion of the gradient under fast flows lies within a very small length within, or only just outside, the annular heater. The author is confident that the thermo-junction was in no way short-circuited, a second carrier tube and junction, registering temperatures somewhat lower down the vacuum tube, giving similar results.

(*b*) *Relative Temperatures with Flows of Varying Magnitude.* Table I. shows the chief results of the research—viz., the relative temperatures at definite positions of the cathetometer carrier.

(*c*) *Method of Calculating* K.—One typical example suffices to show the method of calculating K in each experiment. Usually five different values of $\log_{10}\frac{\theta_1}{-\theta_2}/L$ are found, where L ranges from 6 cm. to 10 cm. It is clear that L should not be taken too small, for an error in observation of θ_1 or θ_2 would affect the ratio seriously.

TABLE II.—*Calculation of* K *from Readings of Experiment XI. Magnitude of Flow* 1,099 *grammes per* 15 *minutes.*

L in cm.	Cathetometer readings	and deflections.	θ_1.	θ_2.	$\log_{10}\frac{\theta_1}{-\theta_2}/L$.
	76·0	41·15			
10	71·0	− 7·45	48·60	−26·45	0·02642
	66·0	−33·90			
	75·5	34·77			
9	71·0	− 7·45	42·22	−24·36	0·02654
	66·5	−31·81			
	75·0	28·79			
8	71·0	− 7·45	36·24	−22·27	0·02643
	67·0	−29·72			
	74·5	23·22			
7	71·0	− 7·45	30·67	−19·99	0·02656
	67·5	−27·44			
	74·0	17·98			
6	71·0	− 7·45	25·43	−17·61	0·02660
	68·0	−25·06			
			Mean value	....	0·02651

From formula (2) of section 2 we now obtain directly K=0·0201.

It will be seen that θ_1 and θ_2 are reckoned from −7·45 as zero, the reading corresponding to the position 71·0 on the cathetometer. The true zero was at −6·0, a little—but never more than 2·5 mm. in the most extreme case—above the

TABLE I.

Number and date of experiment	I. Dec. 31 and Jan. 1.	II. Jan. 14 and 15, 1913.
Magnitude of flow in grammes per 15 min.	1,296	1,159
Temperature of enclosure in deg: C.	14·0	15·0
Reading corresponding to "zero temperature"	−7·0	−11·9
Cathetometer readings :—		
76·5	...	...
76·0	48·35	...
75·5	40·54	47·50
75·0	33·43	40·45
74·5	26·83	33·77
74·0	20·83	27·50
73·5	15·10	21·60
73·0	9·80	16·23
72·5	4·95	11·21
72·0	0·31	...
71·5	− 3·93	2·15
71·0	− 7·75	...
70·5	−11·43	− 5·88
70·0	−14·99	...
69·5	−18·11	−12·82
69·0	−21·09	−16·02
68·5	−23·74	...
68·0	−26·26	−21·73
67·5	−28·60	...
67·0	−30·84	−26·80
66·5	−33·00	...
66·0	−34·96	−31·36
65·5	−36·84	−33·47
65·0	...	−35·50
64·5	...	−37·42
64·0	...	−39·15
63·5	...	−40·88
63·0	...	−42·53

To face p. 36.

position 71·0, and could, if necessary, be found by interpolation. As, however, the ratio $\log_{10} \frac{\theta_1}{-\theta_2} / \mathrm{L}$ would be constant everywhere if emissivity were negligible, the error is inappreciable, and interpolation is avoided by taking as the zero the nearest reading to it.

(*d*) *Calculation of* E*p*/KA *and Verification of Theoretical Formulæ.*—The value of E*p*/KA is not required in calculating the thermal conductivity, but additional support is given to the method by determining it, and thus finding the values of λ and μ in equations (1) and (3) of section 2. These equations can then be verified.

Applying to the results of experiment VII., in which the mercury was stationary and heated at the top by ether vapour. the well-known formula :—

$$\mu \frac{\mathrm{L}}{2} = \log\ (n + \sqrt{n^2 - 1}),$$

where $2n$ is the ratio $\frac{\theta_1 + \theta_3}{\theta_2}$, θ_1, θ_2 and θ_3 being successive temperatures separated by the distance L/2, we obtain a mean value

$$\sqrt{\mathrm{E}p/\mathrm{KA}} = \mu = 0{\cdot}0598(7) = 0{\cdot}06 \text{ very approximately.}$$

The readings of experiment VII. will now be found to agree to a high accuracy with those derived from the formula

$$\theta = 80{\cdot}1 \sinh 0{\cdot}06x.$$

Similarly, the equation

$$\theta = \mathrm{M}e^{\frac{ms}{2\mathrm{KA}}x} \sinh x \sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2 + \frac{\mathrm{E}p}{\mathrm{KA}}},$$

which holds for flows, can easily be tested ; for

$$\frac{ms}{2\mathrm{KA}} = \log_{10} \frac{\theta_1}{-\theta_2} / \mathrm{L} \times 2{\cdot}3026.$$

Applying this to experiment XI., already taken as a typical case, we obtain

$$\frac{ms}{2\mathrm{KA}} = 0{\cdot}06104,$$

$$\lambda = \sqrt{\left(\frac{ms}{2\mathrm{KA}}\right)^2 + \frac{\mathrm{E}p}{\mathrm{KA}}} = 0{\cdot}0856;$$

whence $\theta = \mathrm{M}e^{x \times 0{\cdot}061} \sinh (x \times 0{\cdot}0856),$

which, if M be found and taken as 81·4, agrees very well with

the distribution of temperature as found by experiment. The value of Ep/KA as found from a statical experiment is thus satisfactory when applied to determine λ in dynamical experiments.

(*e*) *Summary of Results of 10 Consecutive Experiments under Flow.*—Table III. is a summary of the deductions made from the sequence of temperatures already given in Table I.

TABLE III.

Magnitude of flow in grammes per 15 mins.	Temperature in degrees Centigrade.	$\text{Log}_{10}\frac{\theta_1}{\theta_2}/L.$	$\frac{ms}{2KA}$	λ.	K.
1,296	14·0	0·03139	0·07228	0·094	0·02004
1,159	15·0	0·02778	0·06397	0·088	0·02025
1,199	15·0	0·02898	0·06673	0·090	0·02009
1,003	15·0	0·02449	0·05639	0·082	0·01988
1,422	15·5	0·03420	0·07875	0·099	0·02019
1,079	15·8	0·02592	0·05968	0·085	0·02021
1,099	17·5	0·02651	0·06104	0·086	0·02013
871	17·3	0·02098	0·04831	0·077	0·02015
1,301	15·5	0·03177	0·07315	0·095	0·01988
1,361	15·5	0·03283	0·07559	0·097	0·02013
			Mean....................		0·020095

Thus the result of all experiments is to obtain at 15·5°C. a value for the thermal conductivity of mercury 0·0201 c.g.s. unit.

6. DISCUSSION OF THE METHOD AND THE RESULTS.

The horizontal nature of the isothermals, the constancy of the value of K as deduced from flows of widely different magnitude and the verification of the theoretical formulæ have given the author some confidence in the method of this Paper, though the value he obtains is higher than that usually accepted. How widely the values obtained by other experimenters on this problem differ is seen from the following table :—

TABLE IV.

Experimenter.	Method.	Temperature.	Value of K.
H. F. Weber, 1880	Flat plate..................	17°C.	0·0162
Angström, 1864 ...	Periodic heat flow	50°C.	0·0177
Berget, 1888.........	Guard ring	0°-100°C.	0·02015
R. Weber, 1903 ...	Electrical measurement of heat....................	0°-34°C.	0·0197

The author's results thus differ widely from those of H. F. Weber and Angström, but are in close agreement with those of Berget and R. Weber. It should be pointed out that objections have been raised to Berget's method (*a*) on account of the uncertainty of the area from which the heat is collected, (*b*) on account of supposed convection currents. The first objection may be serious, but a study of the thermal effect of flows has convinced the author that the second objection is of little moment. For, in the first place, in Berget's inner tube, protected by a guard ring, the isothermals are almost sure to be horizontal, in which case the algebraic effect of convection currents as heat transmitters can easily be shown to be zero; in the second place, it can be shown that in the case of a column of fluid heated at the top natural convection currents will aid rather than hinder the attainment of horizontal isothermals.

Owing to the desire to rotate the thermo-electric carrier tube the lower temperature source was possessed of considerable width; a large quantity of mercury was thus needed for the research. The temperatures of the two constant heat sources—viz., 0°C. and 100°C.—were well suited, owing to the effect of the upward flow, for determining the thermal conductivity over a small range of temperature in the neighbourhood of 15°C. For higher temperatures a thermostatic enclosure would be necessary as well as a higher temperature source.

The essential feature of the method is the simplicity of the calorimetry depending on the well-known value of the specific heat of mercury. Emissivity loss is satisfactorily dealt with, and the range of temperature explored is small. Moreover, only relative, not absolute, temperature differences need be measured.

An interesting problem would be an investigation of the nature of the flow taking into account viscosity and density differences due to temperature. It has, however, been shown elsewhere by the author that as long as the isothermals are horizontal the nature of the flow does not affect the results of these experiments.

This research was carried out in the physical laboratories at Birkbeck College, and the author has been stimulated by the interest shown throughout its development by Dr. A. Griffiths, head of the department.

ABSTRACT.

The Paper gave an account of the determination of the thermal conductivity of mercury at the ordinary temperature of the room by the impressed velocity method first described by the author in the "Proceedings" of this Society, Vol. XXII., 1910. The mercury is contained within a vacuum-jacketed syphon tube about 17 sq. cm. in cross-section, which is heated at the top and maintained ice-cold at its lower extremity. The distribution of temperature down this tube is determined with the aid of a single thermo-junction, while the mercury is at the same time flowing uniformly up the tube at speeds ranging from 870 to 1,420 grammes per 15 minutes. The calorimetry is thus essentially continuous flow calorimetry within the temperature gradient, the quantity of heat passing down the mercury being controlled and measured by the flow of liquid. The wires forming the thermo-elements—viz., iron and constantan—were contained within a vertical tube of 6 mm. external diameter, through the glass of which the wires were fused; this carrier tube lay within the vacuum vessel. The thermo-junction could be raised or lowered to the desired extent by a cathetometer, or could be rotated in a horizontal plane. The isothermals were found to be remarkably horizontal.

If θ_1 is the temperature at a distance L/2 above the isothermal surface at a temperature equal to that of the enclosure, and θ_2 the temperature at an equal distance below, then $\log_e \theta_1/-\theta_2 = ms\text{L}/2\text{KA}$ where s is the specific heat of mercury, K its thermal conductivity, A the cross-section of the tube and m the mass of mercury crossing a section per second. Using this relationship a mean value of 0·0201 c.g.s. units at 15·5°C. is obtained for K.

The advantages of the method lie in the simplicity of the calorimetry and in the fact that temperature ratios rather than temperature differences are required.

DISCUSSION.

Dr. C. Chree said there might be some doubt as to the exactitude with which the mathematical formulæ represented the physical facts, but the method seemed promising for determining the change with temperature of the thermal conductivity, and he inquired whether investigations had been directed towards that end. As the cross-section of the vessel appeared in the formulæ, it would give increased confidence if vessels of different section were tried and found to give accordant results.

Dr. J. A. Harker asked if the author was satisfied as to the distribution of the flow at different parts of the cross-section, and wished to know how the results compared with those of other observers.

Mr. F. E. Smith said that it appeared to him that the practice of Mr. Nettleton did not exactly tally with the conditions imposed by theory. Why did Mr. Nettleton assume that the temperature of some water in a glass tube in the proximity of the apparatus was the same as that of the outer wall of the vacuum jacket? It appeared from the data that it was not, and this being so an error was introduced, inasmuch as the isothermal taken as the "zero isothermal" was not so. If the assumed "zero isothermal" was appreciably higher than the true one the deduced thermal conductivity would be too high. Again, theory imposed a condition on the outer wall of the vacuum jacket; it must be of uniform temperature. Was this so in practice? One end of the jacket was

practically at 0°C. and the other at 100°C.; there must be a temperature gradient. He would suggest that the outer wall of the vacuum jacket be cooled with circulating water in which the second thermo-junction was immersed. The beauty of Mr. Nettleton's method was apparent to all, and he trusted he would continue his experiments.

Dr. A. RUSSELL regretted that the author, following the usual custom, called the law for the cooling of a body by radiation Newton's law, and the coefficient the Newtonian coefficient of emissivity. Newton considered the case of a block of iron being cooled by a current of air, so that the heat lost by radiation was very small compared with that lost by convection. In this case he found that the heat lost by the iron was very approximately proportional to the difference of temperature between the air and the iron. This law had been verified recently up to differences of temperature between the cooling body and the air as high as 200°C. and 300°C. In the author's experiment, however, the tube had a vacuum jacket, so that the heat lost by convection was quite negligible. As the difference of temperature was, comparatively speaking, small, we saw by Stefan's law that the author's assumption was justifiable. In the speaker's opinion the real "Newton's law of cooling" was of great importance in practical work, and he thought that teachers and the writers of text-books ought to lay greater stress on it.

Dr. W. H. ECCLES called attention to the fact that at first sight the method would be thought to be incapable of high accuracy, because of the form of the expression for K, the conductivity. This expression had as its denominator what was practically the difference between two nearly equal magnitudes, and these magnitudes were themselves differences. That was to say, the denominator was of the nature of a second differential coefficient of an experimental curve. That such consistent results were obtained indicated remarkable precision of experiment.

Prof. LEES asked how small a difference of temperature throughout the cross-section would have been detected by the thermo-couple.

Dr. GRIFFITHS, in reply, stated that, as shown in the earlier Paper, the distribution of the flow was not important, so long as the isothermals were plane.

The AUTHOR communicated the following: In reply to the chairman, Prof. Lees, it should be stated that a difference of temperature of 0·05°C. over a horizontal surface would easily have been detected in any position of the cathetometer. In the neighbourhood of the "zero isothermal" all resistances, except that of the galvanometer and wires, could be removed and a more delicate test made. It is safe to say that in this neighbourhood a difference of 0·02°C. would have been detected easily. Thus the isothermals are remarkably horizontal, the result far exceeding the author's expectations. Mr. F. E. Smith has raised an important point in his remarks about the "cold junction" at the temperature of the enclosure. The cold junction could not be inside the vacuum, as iron and constantan cannot be fused through glass and a vacuum of high standard maintained. An attempt to surround an earlier vacuum vessel with a glass water jacket (it is necessary to be able to see through the vacuum when adjusting the carrier-tube) resulted in its collapse, and no risks were taken with the present vessel, which is valuable, as at least 10 attempts at making a third have failed. The "cold junction" was maintained in a tube under water just outside the vacuum vessel to protect it from air draughts, but on holding it in the air just outside the vacuum vessel no serious temperature difference was recorded. The vacuum, of course, was of "thermos" or X-ray standard. When the "warm junction" was in such a position as to produce no electric current the zero-isothermal was located. The author cannot doubt that the temperature of the cold junction was rightly that of the enclosure; for it stood the

double test of Newton's law holding with respect to it in a statical experiment, as well as the constancy of the ratio $\log \frac{\theta_1}{-\theta_2}/L$ in many dynamical experiments. The height of the zero-isothermal varies in every experiment, being higher the faster the flow, as can be seen in the actual temperature curves given in the Paper previously alluded to. Under no flow the zero was very low, being near the lower extremity of the vacuum-jacketed portion of the experimental tube—too low, in fact, to allow of the test $\theta_1/-\theta_2=1$ over any considerable range. The range of temperature between the extremities of the vacuum would be of the order 30°C.—35°C

IV. *Polarisation and Energy Loss in Dielectrics.* By A. W. Ashton, *D.Sc., M.I.E.E., Battersea Polytechnic, Battersea, S.W.*

Received in Revised Form October 9, 1913.

§ 1. *Introduction.*—The object of this Paper is to discuss the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D.

Pellat* showed that (at least qualitatively) the displacement in a dielectric can be represented by

$$D_t = KE_0 + \varepsilon KE_0(1 - e^{-at}),$$

where D_t=displacement at time t since P.D. E_0 was applied, a, K and ε are constants, K being the instantaneous capacity of the condenser and a the reciprocal of the time constant of the viscous displacement, while ε is a coefficient such that displacement after infinite time of charge is equal to $KE_0(1+\varepsilon)$.

Pellat recognised that it would be necessary in some cases to use several terms having different values for a in order to represent quantitatively the results obtained from dielectrics.

It was pointed out by Schweidler† that by modifying Pellat's equation into the form

$$D_t = KE_0 + KE_0 \Sigma\varepsilon(1 - e^{-at})$$

the charging current for constant voltage becomes for $t > 0$

$$i_t = KE_0 \Sigma a\varepsilon e^{-at},$$

and in the series of exponential terms a and ε can be so chosen as to make

$$\Sigma a\varepsilon e^{-at} = \beta t^{-n}.$$

This is necessitated by the results of experiments in which condensers have been charged for a considerable time at a constant voltage. The charging current for $t > 30$ seconds has been shown in a large number of cases to be of the form

$$i_t = i_a + \beta KE_0 t^{-n},$$

where $n < 1$, and it has generally been considered that i_a is a true conduction current whilst the recoverable energy is due to the term $\beta KE_0 t^{-n}$.

* "Annales de Chimie et de Physique," Ser. 7. Vol. XVIII., p. 150, 1899.

† "Annalen der Physik," Vol. XXIV., p. 711, 1907.

§2. *Relations between a and ε for any given Value of the Exponent n in the Equation for Polarisation Current.*—If we consider the above equation for polarisation current it can be shown that, in order that the current may be proportional to t^{-n}, the variations of a and ε must be such that the product $a\varepsilon$ is proportional to a^n.

In a physical sense $K\varepsilon$ may be considered to be the viscous displacement due to unit P.D. (*i.e.*, the viscous capacity) corresponding to all molecules of the dielectric having the same displacement time constant.

Let $$C_1 = \Sigma a\varepsilon e^{-at_1} = \beta t_1^{-n}$$
and let $$C_2 = \Sigma a\varepsilon e^{-at_2} = \beta t_2^{-n}.$$

In order to make $C_1 = kC_2$, where k is any positive number, we must have $t_1^{-n} = kt_2^{-n}$, *i.e.*, $t_1 = k^{-1/n}t_2$.

Although a varies continuously we can choose the values of a in the series so that the values of $a\varepsilon$ are in geometrical progression and write

$$C_1 = a_1\varepsilon_1 e^{-a_1t_1} + a_2\varepsilon_2 e^{-a_2t_1} + \ldots\ldots\ldots\ldots + a_r\varepsilon_r e^{-a_rt_1},$$

where $a_2\varepsilon_2 = ka_1\varepsilon_1$, $a_3\varepsilon_3 = ka_2\varepsilon_2$, &c.

In this case $k\varepsilon_m$ is the total capacity of that group of molecules whose time constants have a mean value $1/a_m$, and which fall within certain limits as determined by the values of a in the terms on either side.

Now $$C_2 = \frac{1}{k}\{a_2\varepsilon_2 e^{-a_1t_2} + a_3\varepsilon_3 e^{-a_2t_2} + \ldots + a_{r+1}\varepsilon_{r+1} e^{-a_rt_2}\},$$

and since, when $1/a$ is either very small or very large compared with t, $a\varepsilon e^{-at}$ becomes negligible, we can write $C_2 = \frac{C_1}{k}$, provided $a_2t_1 = a_1t_2$, $a_3t_1 = a_2t_2$, $a_4t_1 = a_3t_2$, &c., &c., and the first and last terms of the series are negligibly small.

Since $t_1 = k^{-1/n}t_2$, in order to make $a_2t_1 = a_1t_2$, $a_3t_1 = a_2t_2$, &c., we must have $a_2 = k^{1/n}a_1$, $a_3 = k^{1/n}a_2$, &c., and therefore $a_2^n = ka_1^n$, $a_3^n = ka_2^n$, &c., from which it follows that since $a_2\varepsilon_2 = ka_1\varepsilon_1$, $a_3\varepsilon_3 = ka_2\varepsilon_2$, &c., therefore the values of a and ε must be so chosen that $a\varepsilon$ shall be proportional to a^n.

From this it follows further that ε is proportional to $(1/a)^{1-n}$ and we must have the following law :—

Where the polarisation current of a dielectric is proportional to t^{-n} the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$th power of the time constant.

§ 3. *Energy Loss and Variation of Capacity with an Alternating E.M.F. as determined from the Equations of Hopkinson and Wilson.*—Equations for the displacement in a viscous dielectric under the action of an alternating E.M.F. have been given by Hopkinson and Wilson* on the assumption that the charging current is proportional to t^{-n}, where $n<1$. From these equations the apparent capacity is given by

$$C=K+p^{n-1}\beta K\Gamma(1-n)\cos(1-n)\frac{\pi}{2}$$

and the alternating-current conductivity by

$$S=S_0+p^n\beta K\Gamma(1-n)\sin(1-n)\frac{\pi}{2},$$

where S_0 is the true conductivity under a steady E.M.F. and $p=2\pi f$, where f=frequency of the applied E.M.F.

For all frequencies greater than about one per second S_0 is negligible compared with S, the energy loss being therefore proportional to f^n and the viscous capacity proportional to f^{n-1}.

Now the apparent conductivity for a steady E.M.F. is

$$S''=\beta K t^{-n}$$

and the ratio of the alternating-current conductivity to the apparent conductivity under a steady E.M.F. is

$$r=p^n t^n \sin(1-n)\frac{\pi}{2}\Gamma(1-n)$$

$$=\frac{p^n t^n \pi}{2\Gamma(n)\cos(1-n)\frac{\pi}{2}}.$$

This ratio is of considerable importance in connection with cable testing in which it has been the practice to measure the apparent insulation resistance after one minute's electrification in order to detect faults in manufacture.

The above expression for alternating-current conductivity becomes negative when $n>2$ and this is also the case with regard to the viscous capacity when $n>1$. The variations of energy loss and capacity with frequency can be shown to be true within the above limits for n by investigating the series obtained on the assumption that the value of ε and a are such that $a\varepsilon$ is proportional to a^n.

* "Phil. Trans.," Vol. 189A., 1897.

§ 4. *Variation of Energy Loss with Frequency as determined from Pellat's Theory.*—The total alternating-current conductivity of the dielectric due to viscous polarisation is

$$S=K\Sigma\frac{p^2\varepsilon a}{p^2+a^2}.$$

If we consider two terms of the series, a_1, ε_1 and a_2. ε_2 such that $a_2=ka_1$, then $a_2\varepsilon_2=k^n a_1\varepsilon_1$ and the conductivity at frequency $p_2/2\pi$ for the group a_2, ε_2 is

$$S'_2=\frac{Kp_2^2a_2\varepsilon_2}{p_2^2+a_2^2}.$$

Now, if S'_1=conductivity at frequency $\frac{p_1}{2\pi}$ such that $p_2=kp_1$ for the group a_1, ε_1, then it follows that

$$S'_2=\frac{Kp_1^2a_1\varepsilon_1k^{n+2}}{k^2(p_1^2+a_1^2)}=k^nS'_1.$$

Therefore in the series for the total conductivity of the dielectric at frequency $p_2/2\pi$, the terms are each k^n times the corresponding terms in the series for conductivity at k^{-1} times the frequency, and provided the terms of the two series vanish in the limit when a is indefinitely increased or diminished we can write S proportional to p^n.

Now it can be shown that S′ is a maximum for a given frequency in the case of that term of the series for which

$$a=p\sqrt{\frac{n}{2-n}}.$$

For values of n lying between 0·5 and 1·5 the maximum energy loss occurs in those groups for which a does not greatly differ from p, while for values of a much greater than this $S'=\frac{Kp^2\varepsilon}{a}$. Further, since $\frac{\varepsilon}{a}$ is proportional to a^{n-2} the terms in the series must vanish in the limit as a increases provided n is <2. For values of a much smaller than p, $S'=K\varepsilon a$, where εa is proportional to a^n and the terms in the series vanish in the limit as a is diminished provided n is positive. It appears therefore that for all positive values of n <2 the alternating-current conductivity is proportional to the nth power of the frequency.

§5. *Variation of Viscous Capacity with Frequency as determined from Pellat's Theory.*—The capacity under the application of an alternating E.M.F. is given by $C=K\left(1+\Sigma\frac{a^2\varepsilon}{p^2+a^2}\right)$. If, as before, a_1, ε_1 and a_2, ε_2 are values of a and ε such that $a_2=ka_1$, then it follows that $a_2\varepsilon_2=k^n a_1\varepsilon_1$. The viscous capacity for the group a_2, ε_2 at a frequency $p_2/2\pi$ is

$$\begin{aligned} C'_2 &= \frac{Ka_2^2\varepsilon_2}{p_2^2+a_2^2} \\ &= \frac{Kk^{n+1}a_1^2\varepsilon_1}{k^2(p_1^2+a_1^2)} \\ &= k^{n-1}C'_1, \end{aligned}$$

where C'_1 is the viscous capacity for the group a_1, ε_1, at frequency $\frac{p_1}{2\pi}=\frac{p_2}{2\pi k}$. Provided the terms of the two series vanish in the limit as a either increases or decreases the viscous capacity will be proportional to the $(n-1)$th power of the frequency.

Now it can be shown that $\frac{a^2\varepsilon}{p^2+a^2}$ is a maximum for $a=p\sqrt{\frac{1+n}{1-n}}$, and, provided n is not approaching unity, there is a well-defined maximum value for which a is in the neighbourhood of p.

As a diminishes so that a^2 becomes negligible compared with p^2 the capacity of each group$=\frac{Ka^2\varepsilon}{p^2}$, which is proportional to a^{n+1}, and the terms of the series vanish in the limit for all positive values of n. On the other hand, as a increases so that p^2 is negligible compared with a^2 the capcity of each group becomes $=K\varepsilon$, which is proportional to a^{n-1}, and the terms of the series only vanish in the limit as a increases provided $n<1$. The viscous capacity will therefore be proportional to f^{n-1} provided $n<1$.

It will be noticed that for a given frequency the groups of molecules having the greatest influence on the capacity and conductivity may have very different time constants. Thus the ratio of the value of a for maximum conductivity to that for maximum viscous capacity is given by $d=\sqrt{\frac{n(1-n)}{(2-n)(1+n)}}$. The value of d is approximately one-third when n lies between 0·1 and 0·9 but increases rapidly as n approaches unity.

§ 6. *Variation in Capacity when n is Unity.*—The variation in capacity with change of frequency can easily be determined for the case in which $n=1$. In this case ε is constant and a alone varies. The terms in the series for capacity are all equal until a becomes comparable with p. From this point the terms in the series diminish to zero, and this part of the series is the same for all values of p. The number of equal terms becomes greater as p diminishes. If a_1, a_2 be values of a and p_1, p_2 values of p such that $\frac{a_1}{p_1}=\frac{a_2}{p_2}$, then the viscous capacity C_1 for frequency $p_1/2\pi$ differs from the capacity C_2 for frequency $p_2/2\pi$ by $K\Sigma_{\alpha_1}^{\alpha_2}\varepsilon$.

We may therefore write, if $p_2>p_1$,

$$C_1=C_2+C_0\log_{10}\frac{p_2}{p_1},$$

where

$$C_0=K\Sigma_a^{10a}\varepsilon.$$

From this it follows that for frequencies increasing in geometrical progression the capacity decreases in arithmetical progression.

A similar formula can be shown to represent the variation of capacity with time under the action of a steady E.M.F.

The above properties of the dielectrics should be true whether n is a constant over the whole range or not, since, except in cases where n is approaching its limiting value for the particular property considered (such as $n=1$ for capacity variation), the range of values of a having most effect in any given case will be restricted within limits closely corresponding to the frequency or the time for which the variation is being investigated.

§ 7. *Experimental Evidence of Theory.*—A large amount of valuable experimental research on dielectrics has been recently carried out by various investigators and it is proposed to examine some of the results obtained in order to test the above conclusions.

Addenbrooke* has investigated the energy loss in gutta-percha for comparatively slow cycles and has expressed his results for each frequency as a multiple of the apparent loss with a steady E.M.F. after two minutes' application. Except for frequencies of less than 2 cycles per second the ratio of losses can be shown to be fairly accurately represented by

* *See* "The Electrician," Vol. LXX., p. 673.

$r=93f^{0.79}$, where f=periods per second. Table I. gives observed and calculated values of r.

TABLE I.—*Ratio of Loss in Gutta Percha under an Alternating E.M.F. to that under a Steady E.M.F. as given by Addenbrooke.*

f.	r (observed).	r (calculated).	Error %.
0·2	20	25·5	+27·0
0·4	38	44·0	+16·0
1·0	89	93·0	+ 4·5
2·0	160	160·0	0
4·0	276	281·0	+ 1·8
8·0	480	482·4	+ 0·5
16·0	830	829·0	− 0·1
32·0	1,440	1,440·0	0
42·0	1,830	1,785·0	− 2·5

A large number of dielectrics have been tested by Fleming and Dyke* over a wide range of temperature for alternating-current conductivity and capacity at telephonic frequencies. From these experiments they conclude that for most dielectrics the alternating-current conductivity can be represented by a formula $S=A+Bf$, where f is frequency and A and B are constants, of which, however, A is very small for most good insulators. Their results can also be shown to be closely represented by an equation of the form $S=Qf^n$, where Q is a constant, the case in which $A=0$ corresponding to $n=1$. The value of n varies from 0·4 to 1·4 but is nearly unity for a considerable number of the dielectrics tested. Table II. shows the values of n for celluloid at temperatures varying from −184°C. to 80°C. taken from the same Paper, and, except at 15°C. and 60°C., the calculated values do not differ more than 2 per cent. from the observed values.

TABLE II.—*Values of n determined from Fleming and Dyke's Tests on Celluloid.*

n.	°C.	n.	°C.	n.	°C.
0·85	−184°	1·22	−11°	0·56	60°
0·90	−140°	1·15	2·5°	0·49	68°
1·02	−120°	1·05	15°	0·44	80°
0·99	− 75°	0·71	30°	...	...
1·20	− 38°	0·59	51°	...	...

Curtis† in connection with a series of tests for dielectric loss and capacity variation in mica condensers, has discussed the question as to the agreement between his results and Schweidler's

* *See* "Journal" Inst. Elec. Eng., Vol. XLIX., p. 323,1912.
† *See* "Bulletin" of Washington Bureau of Standards, Vol. VI., p. 431.

modification of Pellat's theory and appears to have com to the conclusion that the agreement is not satisfactory. In at least one case, however, agreement between his results and the theory can be shown to be remarkably close.

Columns 1, 2 and 5, Table III., show the results Curtis obtained where $\varphi=90-\theta$ for power factor $\cos\theta$. ΔC is variation in capacity (parts in 100,000). S is the conductivity for alternating currents for a condenser of 1 microfarad capacity. The fourth column gives the conductivity calculated from $S=5{\cdot}334\times 10^{-9}p^{n}$, where $n=0{\cdot}654$, and the sixth column the viscous capacity calculated from $\Delta C=853{\cdot}3p^{n-1}$.

TABLE III.—*Conductivity and Capacity of Mica Condenser (Curtis).*

f.	ϕ.	$S\times 10^{6}$ (observed).	$S\times 10^{6}$ (calculated).	ΔC (parts in 100,000 observed).	ΔC (parts in 100,000 calculated).
1,200	50″	1·827	1·8317	7	38·86
100	2′ 0″	0·3654	0·3606	60	91·83
50	2′ 30″	0·2284	0·2291	85	116·7

The increase in capacity from $f=1{,}200$ to $f=100$ is 53 observed and 52·97 calculated, while from $f=100$ to $f=50$ the corresponding values are 25 and 24·87.

The agreement is even more remarkable with regard to the values of βK calculated from the conductivity and capacity measurements. For the former we have

$$S=\frac{\beta K\pi p^{n}}{2\Gamma(n)\cos(1-n)\frac{\pi}{2}}=5{\cdot}334\times 10^{-9}p^{n},$$

where $n=0{\cdot}654$.

From this $\beta K=4{\cdot}005\times 10^{-9}$.

From the capacity variation

$$\Delta C=8{\cdot}533\times 10^{-9}p^{n-1}$$

in practical electromagnetic units, and from this $\beta K=3{\cdot}87\times 10^{-9}$, which agrees with the former value within $3\frac{1}{2}$ per cent.

H. A. Wilson* has carried out an investigation of the change in capacity due to absorption under the action of a steady E.M.F., and has shown that the capacity after T minutes bears a ratio $1+B\log(1+p/T)$ to the capacity after 1 minute's charge.

* *See* "Proc." Roy Soc., Vol. 82A., p. 409, 1909.

In the case of the ebonite condenser it can be shown that the capacity at 30°C. is very closely represented by

$$C=0{\cdot}4545+0{\cdot}3618\,T^{0{\cdot}098},$$

the maximum deviation of the calculated values from the values observed being 0·15 per cent. From the above equation $n=0{\cdot}902$, which is in agreement with values of 0·9 to 1·0 given by Fleming and Dyke's results on ebonite.*

It is interesting to note that equations for conductivity and capacity variation under an alternating E.M.F. can be readily obtained from the above equation.

For t in seconds we have

$$C=0{\cdot}4545\times10^{-6}+0{\cdot}2421\times10^{-6}\,t^{0{\cdot}098}$$

in electromagnetic units, from which it follows that

$$\beta K=2{\cdot}373\times10^{-8}.$$

The alternating-current conductivity is given by

$$S=3{\cdot}534\times10^{-8}\,p^{0.902}.$$

The capacity under alternating E.M.F. is given by

$$C=0{\cdot}4545\times10^{-6}+2{\cdot}278\times10^{-7}\,p^{-0{\cdot}098}.$$

The above discussion of results obtained on various dielectrics shows the possibility of correlating the various absorption phenomena of a dielectric. Before coming to any very definite conclusion, however, it is proposed to carry out a series of tests on various dielectrics over a wide range of frequency, as well as with steady E.M.F.s, in order to determine to what extent n is constant for a given dielectric at a definite temperature.

Pellat's Theory.†

The following brief résumé of Pellat's theory of "true polarisation" differs mainly from the original treatment in that the condenser is assumed to have only one layer of dielectric between the plates instead of also having two layers of air.

Let λ=specific inductive capacity of the dielectric,

φ=electric intensity,

σ=surface density on the condenser plates.

Then $$\varphi=\frac{4\pi}{\lambda}(\sigma-\jmath), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\jmath$ is called by Pellat the true polarisation. At the end of

* *See* "Journal" Inst. Elec. Eng., *loc. cit.*

† *See* Pellat, "Annales de Chemie et de Physique," Vol. XVIII., p. 150. Also Pellat and Beaulard, "Comptes Rendus," Vol. CXXX., p. 1457.

infinite time j takes a final value, J. It is assumed that the rate of variation of j is proportional to the difference between its present and final values—that is,

$$\frac{dj}{dt}=a(J-j) \qquad (2)$$

From which it follows that

$$j=J(1-e^{-at}). \qquad (3)$$

Pellat justifies the assumption made in equation (2) by proving from it Curie's Principle of Superposition.

The final polarisation J is a function of the final electric intensity Φ, and we may write $J=h\Phi$, where h is a constant for weak fields but a function of Φ for strong fields.

If a steady E.M.F., E_0, be applied to the dielectric we have

$$J=h\frac{E_0}{c}, \qquad (4)$$

where c=thickness of dielectric, and from (1) and (3)

$$\sigma=\frac{\lambda E_0}{4\pi c}+cJ(1-e^{-at}). \qquad (5)$$

Put $K=\frac{\lambda A}{4\pi c}$, where A=area of each condenser plate, then the displacement (D_t) at time t is given by

$$D_t=KE_0+AhE_0(1-e^{-at}) \qquad (6)$$

and $$D=KE_0+\varepsilon KE_0(1-e^{-at}); \qquad (7)$$

where $$\varepsilon=\frac{Ah}{K}.$$

The current is therefore given by

$$i=KE_0\varepsilon ae^{-at}. \qquad (8)$$

The capacity and the energy loss under the application of an alternating E.M.F. were deduced by Pellat and Beaulard on the assumption that h is a constant.

Let $e=E_m \sin pt$ be the applied E.M.F. From equation (2)

$$\frac{dj}{dt}+aj=\frac{ah}{c}E_m \sin pt. \qquad (9)$$

The solution of equation (9) is $j=B\sin(pt+\theta)$, where

$$B=\frac{E_m ha}{c\sqrt{p^2+a^2}}$$

and $$\cos\theta=\frac{a}{\sqrt{p^2+a^2}}, \quad \sin\theta=\frac{-p}{\sqrt{p^2+a^2}}. \qquad (10)$$

From (1), (4) and (10) we have

$$\sigma=\frac{\lambda E_m \sin pt}{4\pi c}+B\sin(pt+\theta).$$

Therefore $i=A\frac{d\sigma}{dt}=pKE_m\cos pt+\frac{pKE\varepsilon_m a}{\sqrt{p^2+a^2}}\cos(pt+\theta)$;

that is, $i=pKE_m\left\{1+\frac{\varepsilon a^2}{p^2+a^2}\right\}\cos pt+\frac{p^2KE_m\varepsilon a}{p^2+a^2}\sin pt.$

The capacity under an alternating E.M.F. is

$$C=K\left(1+\frac{\varepsilon a^2}{p^2+a^2}\right)$$

and the rate at which energy is lost is given by

$$w=\frac{p^2KE_m{}^2\varepsilon a}{2(p^2+a^2)}.$$

ABSTRACT.

The Paper discusses the relations which should exist between the coefficients in Pellat's equation (as modified by Schweidler) giving the displacement in a viscous dielectric as a function of the time of charge and the P.D. It is proved that where the polarisation current of a dielectric under the action of a steady E.M.F. is proportional to t^{-n} (t=time), the viscous capacity of any group of molecules having the same time constant of displacement is proportional to the $(1-n)$th power of the time constant.

Starting with this hypothesis it is proved that under an alternating E.M.F. the energy loss varies as the nth power of the frequency, and the viscous capacity varies as the $(n-1)$th power. An equation is also obtained giving the variation in capacity for the particular case in which $n=1$.

Experimental results published by various investigators are given, and are shown to be not inconsistent with Pellat's modified theory. A brief résumé of Pellat's Theory is also given.

DISCUSSION.

Dr. RUSSELL said that the author's method was ingenious and that the formula deduced for the dielectric current agreed well with many experimental results. He thought, however, that Pellat's theory was not sound and that von Schweidler's modification of it was too elaborate to be of much practical use. He pointed out that Dr. Grover had proved that, from the mathematical point of view, Hopkinson's and von Schweidler's theories were identical. He considered that Fleming and Dyke's experimental results could not be ignored, and that none of the theories advanced up to the present explained them satisfactorily. Very different formulæ gave results which were practically identical over wide ranges, and so they were of doubtful help in enabling us to discover the causes of the phenomena. He considered that the nomenclature of dielectric theory stood badly in need of standardising. For instance, what the

author called the displacement was practically the same as Maxwell's electric displacement, but Sir J. J. Thomson and several other writers called it the polarization.

Dr. ECCLES stated that the Paper compelled remark on the artificial nature of Pellat's hypothesis. This hypothesis led to an expression for the displacement current as a sum of exponential terms. Experiment required it to be a power of t, or the sum of several power terms. This required that the exponential terms of Pellat's hypothesis should be infinite in number—which deprived the hypothesis of physical meaning. It would be more logical, therefore, to begin with the power terms as fundamental, and to discard the false start given by Pellat.

Mr. ADDENBROOKE stated that in order to get at the proper theory of experiments the best dielectrics should not be used, as the effects were so minute that the chances of error were great. With poorer dielectrics, the actions were on a greater scale and easier to observe. Using an ellipsoid suspended at 45 deg. to the electric field, he had been able to carry his experiments down to a frequency of 1 cycle in two hours, taking, of course, time readings of current and voltage. Dr. Ashton had not mentioned a Paper by Wagner, which seemed to cover much the same ground.

Prof. C. H. LEES mentioned a later Paper on the subject by Curtis, who had found that Schweidler's equations held if several terms were taken.

The AUTHOR, in reply, agreed with Dr. Russell as to the need for a standard nomenclature with regard to dielectrics, and called attention to the numerous methods of expressing the quantity known as the electric force. With regard to Dr. Russell's objection that the exponent n has not been shown to be constant, except over a limited period of time, some of the author's published results show that n has remained constant for over 24 hours, but it appeared impossible to carry out satisfactory experiments over long periods without adopting stringent precautions for keeping the temperature and the applied voltage absolutely constant. The author did not wish to give the impression that Fleming's and Dyke's results were not of the highest accuracy, but he believed their results were equally consistent with the equations deduced from Pellat's theory as with the equation of Fleming and Dyke. With regard to the criticism of Dr. Eccles and Prof. Lees that a theory involving the representation of results by an infinite series was of little use, the author contended that the theory did not necessarily involve an infinite series. The upper limit of a is fixed by the point at which the capacity ceases to be affected by an increase in frequency. The lower limit is fixed by the time required to fully polarise the dielectric by a steady E.M.F. An infinite series would lead to the displacement being infinite for t=infinity. The great advantage of the theory is the fact that it correlates all the so-called "absorption" phenomena, and if established by experiment it could be used to extend our knowledge of the properties of dielectrics. For instance, it can be shown that if the theory is true the hysteresis loop must be elliptical, and that a third harmonic cannot be introduced by dielectric hysteresis as it can be by magnetic hysteresis. With regard to Mr. Addenbrooke's work on slow cycles, it should be pointed out that at frequencies of the order of one per second the rate of absorption of energy becomes comparable with the rate of dissipation in the dielectric, and according to Pellat's theory the former will depend on the particular point in the cycle at which the E.M.F. was first applied to the condenser.

V. *A Lecture Experiment to Illustrate Ionisation by Collision and to show Thermoluminescence. By* F. J. Harlow, *A.R.C.S., B.Sc., Assistant Lecturer in Physics, Sir John Cass Technical Institute.*

Read November 14th. Received November 10th.

An experiment to test the effect of providing the electrodeless discharge with a continuous supply of ions from incandescent solids proved to be a suitable one for demonstrating to an audience several points in connection with the production of ionisation by collision, and was thought, therefore to be worth recording.

Experimental Arrangement.

A spherical discharge bulb was constructed, with four leads passing into it carrying two small coils of platinum wire coated with aluminium phosphate and lime respectively. These substances yield when heated abundant supplies of ions, positive from the phosphate and negative from the lime. The platinum wires were placed near the surface of the glass, and in the plane of the coil wound round the bulb for the purpose of producing the electrodeless discharge. The arrangement used to set up the necessary electrical oscillations in this coil was that involving the use of two condensers, described by Sir J. J. Thomson in "Recent Researches." The alternative method of exciting the discharge, which involves the use of one condenser only, was found to be unsuitable on account of the large electrostatic effect which is produced, the ionisation from this masking somewhat the effect to be described.

Effect of Incandescent Lime and Aluminium Phosphate.

The intensity of the electrodeless discharge produced by the above arrangement depends on both the gas pressure and the induced E.M.F. For any given E.M.F. the discharge can be excited over a limited range of pressure only, and within this range, as the pressure is diminished, its intensity increases to a maximum, after which it decreases slightly, then becomes visibly intermittent and finally disappears altogether. By heating up either the lime or the aluminium phosphate to

incandescence two noticeable effects are to be observed. First, the intensity of the discharge, provided this is not too great initially, is considerably increased; and, secondly, the lower limit of the pressure range over which the discharge can be produced is considerably extended.

The first of these effects shows in a striking way the increased ionisation in the gas caused by the collision of ions from the incandescent solid with the molecules of the gas. This increase is not so very noticeable at the higher pressures, but becomes very marked as the pressure is reduced. The colour, too, of the discharge often undergoes a distinct change, similar in character to that which occurs when the pressure is reduced or the E.M.F. increased, which points rather to an increased velocity at collision. The effect of the negative ions from the lime is generally greater than that of the positive ions from the phosphate, unless the latter is heated very strongly, when, according to Garrett,* negative ions are given off in addition.

The second effect—viz., that of increasing the range of the discharge—is, perhaps, the more striking, as, when the pressure is sufficiently low that no discharge can be obtained in the ordinary way, a vivid discharge ensues as soon as either the lime or the phosphate is heated. This is an illustration of the fact that the sparking potential is considerably reduced by the presence of initial ionisation, since the discharge being intermittent is practically a succession of sparks. The extension of the range of the discharge in this way is greater with the negative ions than with the positive, which shows that the latter are less effective than the former in producing ionisation by collision. The precise limits of the range of pressure over which the discharge can be excited depends largely upon the induced E.M.F. The values obtained with one particular arrangement will show the order of the effect of the ionisation from the heated solids. The discharge produced without the assistance of the lime or phosphate commenced at 0·3 mm. pressure, and disappeared at 0·05 mm. When the lime or phosphate was heated no appreciable difference in the pressure at which the discharge started could be detected; but the lower limit with the phosphate was extended to 0·02 mm., and with the lime to a pressure lower than could be recorded by a McLeod gauge reading to 1/1000 mm.

* "Proc." Phys. Soc., November, 1910.

An interesting point demonstrated by the experiment is the well-known spreading of the discharge as the pressure is diminished on account of the increased diffusion of the ionisation at the lower pressures. The discharge at the very low pressures completely fills the bulb, whereas at the higher pressures it is just a ring near the surface of the glass. As the pressure is reduced, however, the discharge gets feebler and feebler as the number of molecules in the bulb, and therefore the number of collisions, is gradually reduced.

To make the experiment most effective, the ordinary discharge should be made quite feeble. This can be done most conveniently within certain limits by an adjustment of the spark-length ; but it may be necessary to adjust the number of turns in the oscillation coil. The best effect with the lime is given after it has been heated for some considerable time, for, as Willows and Picton have shown,* the electrical activity of lime is increased enormously by this process. If, however, it be exposed again to air at atmospheric pressure, the activity returns to a very small value. On the other hand, Garrett has, shown† that the activity of aluminium phosphate is decreased by continued heating, assuming eventually a final steady value, so that this is more effective when first set up.

Thermoluminescence.

When the experiment described above was set up it was found that the lime possessed the property of displaying thermoluminescence. A discharge intensified by the phosphate alone was maintained for a few minutes, the lime being kept cold. When this was stopped, and the lime gently warmed by a current much less than that required to raise it to incandescence, a golden yellow luminosity was emitted which persisted for some considerable time, unless the lime was heated above a certain temperature, when the luminosity instantly ceased. The lime had then to be exposed afresh to the discharge before the phenomenon recurred. The experiment can be shown most effectively by switching on a current of sufficient strength to make the lime incandescent. As the lime is warming up it first flashes out intensely the thermoluminescence, then ceases to be luminous, and finally becomes incandescent. The phenomenon of thermoluminescence, which

* "Proc." Phys. Soc., June, 1911.
† Loc. cit.

usually occurs with substances containing certain others as impurity, is thought to be due to a soft Röntgen radiation* produced in the discharge at the moment of collision of the ions with the gas molecules. This radiation, called "Entladungstrahlen," by Wiedemann, is considered to produce a chemical compound stable at ordinary temperatures, but which breaks up, however, when warmed, emitting a characteristic luminosity as it does so. The method used for coating the platinum wire with lime was that of putting on a layer of sealing wax and raising to incandescence. The white residue consists largely of lime, and adheres well to the platinum, but contains barium sulphate and other substances as impurities. The thermoluminescence was thought at first to be due to these impurities, but nominally pure lime was found to produce the phenomenon equally well. This case of thermoluminescence does not appear to have been observed before.

Tests made with the aluminium phosphate showed that it also gave a feeble blue thermoluminescence which, however, could not be obtained after the phosphate had been heated for some time.

I am indebted to Dr. R. S. Willows for his kind interest and helpful suggestions in connection with this experiment.

Note added November 28th.

Since the above Paper was first sent in for publication the peculiarity of lime and aluminium phosphate in exhibiting thermoluminescence has received further attention. As mentioned in the Paper, the phenomenon does not usually occur with pure substances, so that it seemed curious that it should be observed with lime and aluminium phosphate, both of which were thought to be pure. A test was made to see whether or not the lime, in which the phenomenon was most marked, gave any thermoluminescence apart from the platinum. A piece of platinum foil coated with lime was first held near the vivid electric sparks passing across a spark-gap, and then gently warmed in the hot air rising from a small non-luminous gas flame. When the requisite temperature was reached the characteristic luminosity appeared. The experiment was now repeated with lime on a watch glass, but no thermoluminescence could be observed at all. It is thus the combination of the lime and platinum which causes the phe-

* J. J. Thomson "Conduction in Gases," pp. 602, 624.

nomenon. This observation suggests that there may be a close connection between thermoluminescence and electrical activity. This suggestion receives considerable support from an observation of the effect in the electrodeless discharge bulb. As already mentioned, Willows and Picton have observed that, with continued heating, the electrical activity of lime is considerably increased. Now this was found to be the case with the thermoluminescence. After keeping the lime incandescent for several hours, both its efficiency in the experiment described and the thermoluminescence increased enormously. Experiments on the effects of other forms of radiation have revealed the fact that both ultra-violet light and Röntgen radiation from a bulb are capable of exciting the phenomenon. Other metals besides platinum have also been tried. Lime on nickel, which is also electrically active,* gives thermoluminescence of the same colour; lime on aluminium gives a blue luminosity similar to that given by aluminium phosphate on platinum, but lime on copper gives no effect at all.

It thus seems probable that thermoluminescence may be closely connected with the electrical activity of salts and oxides upon metals; experiments are now in progress in connection with these phenomena, which it is thought will reveal their origin.

ABSTRACT.

A method of demonstrating to an audience both ionisation by collision and the reduction of the sparking potential by the presence of initial ionisation is described in the Paper.

A spherical bulb, in which an electrodeless discharge is excited, contains two coils of platinum wire, coated with lime and aluminium phosphate respectively, which can be heated by means of a current. Within the range of pressure for which the electrodeless discharge can be excited, provided the discharge is not too intense, both the lime and aluminium phosphate, on being raised to incandescence, give a considerably increased effect, showing that the ions given off by these substances are effective under the action of the induced E.M.F. in producing an enormous number of others by collision.

The lower limit of the range of pressure over which the discharge can be excited with a given induced E.M.F. is considerably extended by heating either the lime or the aluminium phosphate, the former, however, being much more efficient than the latter in this respect. This extension of range is an illustration of the fact that the sparking potential is diminished by the presence of ions, and by negative ions more than by positive.

If the lime and aluminium phosphate are subjected while cold to an

* Willows and Picton, loc. cit.

intense discharge for some time they exhibit the phenomenon of thermoluminescence, the lime on being warmed gently giving out a golden yellow luminosity and the aluminium phosphate a blue. The effect with lime apparently increases with an increase of electrical activity caused by continued heating, which suggests that the thermo-luminescence and electrical activity are closely associated. This suggestion receives support from the fact that lime alone does not exhibit thermo-luminescence, and that both lime on platinum and lime on nickel, which are electrically active, do. Further experments are being made on this point, which the author thinks will throw light on the origin of the anomalous electrical activity of lime on platinum.

A demonstration of these experiments was given.

VI. *An Experimental Method for the Production of Vibrations on Strings illustrating the Properties of Loaded or Unloaded Telephone Cables.* *By* J. A. FLEMING, *M.A., D.Sc., F.R.S.*

RECEIVED NOVEMBER 28TH, 1913.

THE problem of determining the possible vibrations of a loaded string—that is, a flexible string having small masses attached to it at equal distances—is one to which many mathematicians have given attention. It suffices to mention the well-known discussion of the problem by Lagrange,* the full treatment of it by Lord Rayleigh,† and an interesting Paper by Mr. C. Godfrey‡ published in 1898.

Of late years the matter has acquired an electrical interest from the close analogy existing between the transmission of mechanical vibrations along a loaded string and the propagation of alternating electric currents along a telephone cable having inductance coils inserted in it at equidistant intervals. In seeking for experimental methods of illustrating the properties of such loaded cables it was natural to turn for assistance to the visible oscillations produced on strings loaded or unloaded.

Almost the only method hitherto used for creating sustained vibrations on strings has been the elegant device of F. E. Melde, in which a string attached to the prong of a tuning fork is set into stationary sympathetic vibrations when the length and tension of the string is adjusted so that the vibrations travel along it in a time which has some integer ratio to the periodic time of the fork.

The objections to this method are (1) that we cannot put more than a certain tension on the string without stopping the fork even if the latter is electrically driven, (2) that when long loaded strings are employed we need very large forks to provide a sufficiently low frequency in the impulses, (3) that we

* "Mécanique Analytique," Vol. I., part 2, sec. VI., § III.
† "Theory of Sound," Vol. I., Chap. VI., 2nd edition.
‡ "On Discontinuities Connected with the Propagation of Wave Motion along a Periodically Loaded String," "Phil. Mag.," April, 1898, p. 356.

cannot alter the frequency of vibration except by changing the fork, (4) that the vibrations of the string take place only in one plane and are not very visible unless viewed from a certain standpoint.

In thinking over these difficulties it occurred to me that the fork could with great advantage be replaced by a small continuous-current electric motor, the speed of which can be easily controlled and measured and made low or high enough to study the vibrations on a string of any length, loaded or not loaded, and with any reasonable applied tension.

The best arrangements were found to be as follows :—

On the shaft of a small continuous-current motor, say, $\frac{1}{8}$ H.P. or $\frac{1}{4}$ H.P., is attached a pulley or disc which has a crank pin (K) inserted on its outer face at a distance of about 0·5 in. from

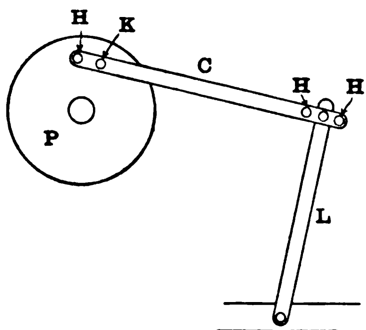


Fig. 1.

the centre (*see* Fig. 1). To this pin is attached a light crank shaft (C) which is connected at the other end to a rocking lever (L). A small hook (H) is attached to the crank shaft, quite close to the crank pin, and another at the far end, close to the junction with the top end of the rocking lever.

On the other end of the motor shaft is an endless screw gearing with a toothed wheel, the latter carrying a pin on its face which strikes once in every revolution against a small piece of steel clock-spring. This mechanism serves to count the speed of the motor, as we can take the time of 20 or 1,000 revolutions of the wheel by a stopwatch and, knowing the gear ratio, obtain the speed of the motor. The hook on the crank shaft near the crank pin describes, therefore, a circular path in space without rotation and the hook at the far end of the

crank, shaft moves to and fro with an approximately simple harmonic motion.

If, then, we fasten a long stretched flexible cord to either hook we have the means of impressing upon the end of the cord simple harmonic motions either (1) in the direction of its length, or (2) transverse to its length, or (3) a circular motion equivalent to the combination of two simple harmonic motions differing 90 deg. in phase, each taking place in a direction at right angles to the direction of the stretched cord.

The remote end of the cord is attached to a pin which is screwed into a nut travelling on a long screw similar to the slide rest of a lathe (*see* Fig. 2). We can by this means put any desired tension on the cord, and also measure the tension by a spring balance inserted between the end of the cord and the terminal pin. If, then, we attach one end of the cord to the hook nearest the crank pin of the motor with the string stretched parallel to the axis of the motor and apply a certain tension at the other end of the string, and set the motor in rotation, we shall have vibrations propagated along the cord such that each part of the cord describes a circular motion. By properly adjusting the tension and the speed of the motor stationary oscillations may be set up on the cord which are the resultant of two simple transverse harmonic motions at right angles and differing 90 deg. in phase. The cord, therefore, when vibrating appears spindle shaped in virtue of the persistence of vision, and if the tension is adjusted we can make the cord vibrate in a single loop or in 1, 2, 3, &c., loops up to a dozen or more segments corresponding to the various harmonics of the fundamental mode of vibration of the cord (*see* Fig. 2).

By the slide-rest arrangement the tension can be accurately adjusted so as to produce any required number of sections and keep them steady. By sending the beam from an arc lamp lantern along the cord these stationary vibrations can be rendered visible to large audiences. We can easily measure the half wave-length or distance from node to node by a beam compass and having also the means of measuring the speed of the motor we can calculate the velocity with which a wave motion of any wave-length producible travels along the cord.

Arrangements can also easily be made to apply a tension to the cord by means of a weight attached to the end in the usual manner by passing the end of the cord over a fixed pulley.

The cord can be loaded by fixing on it at equal intervals

glass beads or cork or wooden balls, and the possible modes of vibration on this loaded cord studied. By loading half of the cord or a middle portion, we can examine the reflection effects taking place when a vibration passes from one medium to another of different mean density. The following is the simple analytical treatment of the problem.

Let us assume that the mass of each load on the cord is m, in which we must include the mass of the length of cord intervening between two loads. Let d be the distance between the masses. Suppose also that in any shape into which the cord is thrown in its vibrations the angle made by any section of the cord with the straight line joining the extremities of the cord is so small that we may take the sine of this angle as equal to its tangent. Also let T be the tension of the cord, and let y_n be the transverse displacement of the nth mass. Then, following a method used by Prof. Tait,* we have for the equation of motion of that mass

$$m\frac{d^2y}{dt^2}=\mathrm{T}\left(\frac{y_{n+1}-y_n}{d}\right)-\mathrm{T}\left(\frac{y_n-y_{n-1}}{d}\right), \quad . \quad . \quad . \quad (1)$$

which expresses the fact that the difference of the components of the tensions taken perpendicular to the undisturbed cord on either side of any mass is the moving force on it. If we put $y_n=\mathrm{Y}\cos(pt-qx)$, where $p=2\pi/\tau$ and $q=2\pi/\lambda$, τ and λ being the periodic time and wave-length of the motion respectively, then on substitution in (1) we have

$$-mp^2\cos(pt-qx)=\frac{\mathrm{T}}{d}\{\cos(pt-q(x+d))-2\cos(pt-qx)$$
$$+\cos(pt-q(x-d))\} \quad (2)$$

or

$$p^2=\frac{4\mathrm{T}}{md}\sin^2\tfrac{1}{2}qd. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Writing n for $p/2\pi$ and V for $n\lambda$ we have

$$\mathrm{V}=n\lambda=\sqrt{\frac{\mathrm{T}}{m/d}}\cdot\frac{\sin\pi d/\lambda}{\pi d/\lambda}. \quad . \quad . \quad . \quad . \quad (4)$$

The above is a correct analysis if the vibrations are in one plane, but as regards the case under consideration, viz., when each portion of the string is moving in a circle having its plane

* *See* article "Wave," "Encyclopædia Britannica," 9th edition.

perpendicular to the line of the undisturbed cord, we must arrive at the equation of motion in another way.

Since each load then describes a circle of constant radius y, the centrifugal force on a mass m moving with angular velocity p is mp^2y; and if we consider the load at y it is $mp^2y \cos(pt - qx)$. This force, however, is balanced by the sum or difference of the resolved parts of the tensions on either side of it, taken perpendicular to the axis of revolution. Hence we have

$$mp^2y_n = \mathrm{T}\left(\frac{y_n - y_{n-1}}{d}\right) - \mathrm{T}\left(\frac{y_{n+1} - y_n}{d}\right)$$

and by substitution for y_n of $\mathrm{Y}\cos(pt-qx)$ gives us the same equation (2) and hence (4) follows, as above.

If d/λ is very small, that is, if the loads are very close together, then $m/d = \rho$ is the average density per unit of length, and the formula reduces to $\mathrm{V}_0 = \sqrt{\mathrm{T}/\rho}$, which is the known expression for the velocity of propagation of a wave along a cord of mass per unit length ρ and longitudinal tension T. Hence, if we call this latter the wave velocity corresponding to the uniformly distributed or smoothed out masses we have finally

$$\mathrm{V} = \mathrm{V}_0 \frac{\sin \pi d/\lambda}{\pi d/\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Also (5) can be written

$$n = \frac{1}{\pi d}\sqrt{\frac{\mathrm{T}}{m/d}} \sin \pi d/\lambda \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From which it is seen that no frequency greater than $\sqrt{\mathrm{T}}/\sqrt{\pi^2 md}$ can be propagated along the loaded cord. This corresponds to about three loads per wave, or when $d = \lambda/2$.

The values of $(\sin \pi d/\lambda)/(\pi d/\lambda)$ in terms of λ/d or the number of loads per wave are given in the Table I. below :—

TABLE I.

λ/d.	$(\mathrm{Sin}\ \pi d/\lambda)/(\pi d/\lambda)$.	λ/d.	$(\mathrm{Sin}\ \pi d/\lambda)/(\pi d/\lambda)$.
1	0	6	0·955
2	0·637	7	0·967
3	0·827	8	0·974
4	0·900	9	0·980
5	0·935	10	0·984

It is obvious that this ratio runs rapidly up towards unity and that even when λ/d is as small as 10 the discontinuous loading is equivalent in practical effect to continuous loading, as if the added mass was uniformly distributed along the string.

These deductions from theory can be easily illustrated and confirmed by the use of the vibrating string apparatus above described.

The best cord to use with it is a light inelastic cord called cotton cord, and a useful diameter is about 1 mm. or 2 mm. The sample I have used has a weight of 0·00839 gramme per centimetre.

A length of 2 or 4 metres was employed and one end attached to the motor, the other being led over a light aluminium pulley well pivoted and terminated in a scale pan in which weights were placed. Using appropriate tensions and frequencies the cord was thrown into stationary vibrations giving 1, 2, 3, 4, &c., half-wave lengths or loops. The frequency N (per second) and wave-length λ were measured and also the tension T of the cord in grammes weight, reduced to dynes by multiplication by 981. The product $N\lambda$ was then compared with the quotient $\sqrt{T}/\sqrt{\rho}$, where $\rho=0{\cdot}00839$. The following Table II. gives the results :—

TABLE II.—*Transverse Vibrations of an Unloaded String.*

ρ = weight per cm. 0·00839 gramme. Length 2 metres.

Number of half waves or loops.	Tension of string T.		Frequency of oscillations per sec. N	Wave-length in cms. λ.	Velocity of propagation.	
	Grammes.	Dynes.			$N\lambda$.	$\sqrt{T}/\sqrt{\rho}$.
1	1,000	981,000	27·9	390	10,900	10,850
2	335	329,000	31·8	200	6,360	6,280
3	150	147,000	32·9	130	4,280	4,200
4	87	85,400	32·9	98	3,220	3,220
5	50	49,100	30·7	79	2,320	2,420
With a cord 4 metres in length vibrations up to the 10th harmonic could be obtained giving from 10 to 19 loops or half-waves.						
10	55	54,000	31·2	80	2,496	2,540
19	15	14,710	32·2	40	1,288	1,320

It will be seen that the wave velocity given by product $N\lambda$ is approximately equal to the quotient $\sqrt{T}/\sqrt{\rho}$. There is a certain difficulty in measuring the wave-length as the nodes are not very sharply defined, but the agreement is fairly good and proves that the velocity of propagation of the wave varies as the square root of the tension and inversely as the square root of the density of the cord.

A second set of measurements was made employing a constant tension of 110 grammes on the cord and variable speeds of

the motor to produce different wave-lengths, and the results are embodied in Table III.:—

TABLE III.—*Transverse Vibrations of an Unloaded String.*
ρ = weight per cm. = 0·00839 gramme. Length = 2 metres.

Number of half waves or loops.	Tension of string = T.		Frequency of oscillations per sec. = N	Wave-length in cms. λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
1	110	108,000	8·5	390	3,310	3,580
2	110	108,000	17·8	201	3,580	3,580
3	110	108,000	26·1	134	3,500	3,580
4	110	108,000	36·1	98	3,540	3,580
		Length of cord 4 metres.				
2	110	108,000	8·7	404	3,520	3,580
4	110	108,000	17·8	201	3,580	3,580
6	110	108,000	26·9	134	3,600	3,580
8	110	108,000	36·7	98	3,600	3,580

These last results similarly prove that within the limits of wave-length employed the velocity of propagation is independent of the frequency and equal to the quotient $\sqrt{T}/\sqrt{\rho}$.

On the cord 4 metres in length there was no difficulty in producing as many as 20 half-waves quite stationary, each having a length of 20 cm.

The next step was to prepare a loaded cord by placing on the same cotton cord glass beads each weighing 0·208 gramme at distances of 20 cm. The mean density or weight per centimetre length of cord was thereby raised to 0·0188 gramme. The same experiments were then repeated with this loaded cord using variable tensions (Table IV.), and constant tension (Table V) with the results given.

TABLE IV.—*Transverse Vibrations of a Loaded String.*
Loads of 0·208 gramme at intervals of 20 cm. Mean weight per cm. ρ = 0·0188 gramme. Length of cord = 4 metres. Variable tension.

Number of half waves or loops.	Tension of string = T.		Frequency of oscillations per sec. = N	Wave-length in cms. λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
3	810	795,000	25·0	266	6,650	6,510
4	550	540,000	26·8	200	5,360	5,360
5	380	373,000	28·0	158	4,420	4,460
6	265	260,000	29·1	130	3,790	3,720
7	205	201,000	29·1	110	3,210	3,280
8	180	177,000	31·2	97	3,020	3,060

TABLE V.

Same loaded string as in Table IV., but with constant tension.

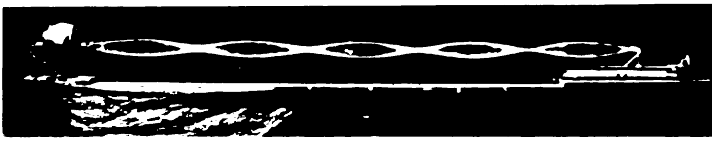
Number of half waves or loops.	Tension of string = T.		Frequency of oscillations per sec. = N	Wave-length in cms. = λ.	Velocity of propagation.	
	Grammes.	Dynes.			Nλ.	$\sqrt{T}/\sqrt{\rho}$.
2	210	206,000	8·1	400	3,240	3,310
3	210	206,000	12·8	266	3,400	3,310
4	210	206,000	16·6	200	3,320	3,310
5	210	206,000	19·9	160	3,200	3,310
6	210	206,000	24·6	132	3,240	3,310
7	210	206,000	28·6	110	3,160	3,310

It will be seen that here also the wave velocity measured by the product Nλ is approximately equal to the quotient $\sqrt{T}/\sqrt{\rho}$ and that the wave velocity is independent of the frequency.

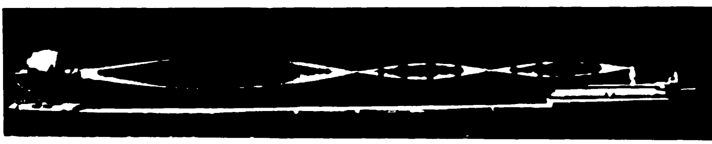
The loaded cord, however, differs from the unloaded in this respect, that whereas there was no difficulty in producing on the unloaded string as many as 19 or 20 stationary half-waves or loops, each about 20 cm. in length, it was found quite impossible to produce waves as short as this on the loaded string with loads 20 cm. apart. The latter refuses to oscillate sufficiently quickly and will not propagate waves of a higher frequency than corresponds to a wave-length of rather more than double the distance between the loads. On putting on a higher frequency at one end of the cord the vibrations or loops on it are seen to die away quickly, but sustained stationary oscillations on the whole length of the loaded string cannot be set up unless the frequency is less than the critical value.

This method of exciting the oscillations on a string can also be employed to exhibit in a beautiful manner the wave reflection that takes place at the junction between two strings of equal tension but unequal mass per unit of length or at the junction between an unloaded and a loaded string.

If we place glass beads at regular intervals on one-half of a string 4 metres long and attach the extremity of the unloaded half to the motor wheel and apply a tension at the remote end, then standing waves will be produced on both parts when the tension is adjusted (*see* II., Fig. 2). These standing waves have, however, different wave-lengths on the two parts which are inversely as the square roots of the mean cord densities. Thus in one experiment tried the standing waves had a wave-length of 100 cm. on the unloaded half of the string and a wave-length of 66 cm. on the loaded half, These two parts of the

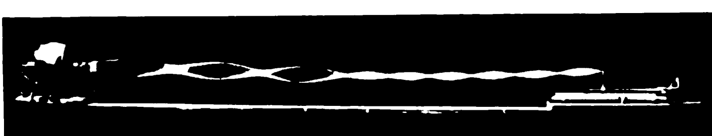


I.—Standing Waves on Unloaded String.

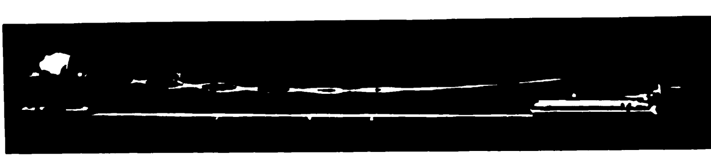


II.—Standing Waves on String Half Loaded, Half Unloaded.

III.—Reflection of Wave by Load at Centre of String.



IV.—Standing Waves on String Half Single, Half Fourfold.



V.—Standing Waves on Loaded String, Wave-length Covers about Three Loads.

Fig. 2.

To face p. 68.

string had mean densities of 0·00839 and 0·0188 gramme per centimetre respectively. The ratio of the square roots of these last numbers is as 29 is to 43·3 and $29 \times 100 = 2{,}900$ whilst $43{\cdot}3 \times 66 = 2{,}858$. Hence the wave-lengths are nearly inversely as the square roots of the mean cord densities. The amplitude of the waves on the unloaded half of the cord was, however, much greater than that of the waves on the loaded cord, thus showing the loss of amplitude on reflection. The same thing can be better shown by employing a cord one half of which is single and the other half made of four similar cords in parallel and tied together at intervals. When this is vibrating we have waves on the four-fold part which are half the wave length of those on the single part (*see* IV., Fig. 2).

Again, if the tension was decreased slightly so as to lower the velocity and decrease the length of the waves a point was soon reached at which whilst standing waves were produced on the unloaded half of the cord, no waves were produced on the loaded half. This happened when the length of the waves on the unloaded half was about 60 cm. and therefore those on the unloaded half should have been about 40 cm., or equal in length to twice the distance between the loads; so that one wave would cover three loads or a little more.

This experiment proves clearly that there is an inferior limit to the possible wave-length and a superior limit to the possible frequency of the vibrations which can be produced on the loaded string exactly as predicted by theory. The same reflection of the waves and loss of amplitude on passing a junction between two uniform cords of different mass per unit of length can be shown by joining two cords of equal length, one a plain cotton cord and the other a piece of whipcord or heavy gilt thread, and attaching one end of the length to the crank of the motor and applying a tension to the other end. By adjusting this tension it is easy to show the loss in amplitude of the wave in passing such a junction, or even the total reflection of the wave (*see* III., Fig. 2). This phenomenon has its electrical analogue in the loss in amplitude or reflection of the electrical waves at the junction between two conductors, say, an unloaded land line and a submarine cable, or loaded line, the two having different capacities and inductances per mile when telephonic currents are transmitted through such a circuit.

Again, we can show by this vibration apparatus the advantage of tapering off the loading in the case of a junction between

a loaded and an unloaded line owing to the diminution of the reflection losses. When a wave passes from one medium to another of different refractive index, and if this index changes somewhat suddenly at the junction plane we know that wave reflection takes place. If, however, the index changes gradually the reflection losses are much reduced or else annulled. Hence, if a loaded telephone cable is connected to an unloaded cable partial reflection of the wave takes place at the junction. If, however, the loading is tapered off, these reflection losses are much reduced. This can be visibly illustrated by the vibrating string apparatus very easily. If we load one-half of a string with glass beads but taper off the loading by using towards the middle point beads or knots of gradually decreasing mass, and then attach one end of the unloaded half to the motor crank and apply a tension to the other end of the whole cord we shall see that the reflection losses at the junction are much reduced.

The partial reflection and consequent loss of amplitude in the transmitted wave can even be shown by placing a single bead or load of appropriate mass in the centre of an otherwise unloaded string. If one end of the string is vibrated and a suitable tension applied we can produce stationary waves of large amplitude on that part of the string between the motor and the load, but the amplitude of the vibrations produced on the far side of the load is much less, thus showing the partial loss of amplitude in the transmitted vibration owing to the reflection losses.

In conclusion, a short method may be given for arriving at an expression for the velocity of propagation of an electric wave along a coil-loaded telephone cable, which was first given by Prof. I. Pupin in his important memoirs on this subject,* but which may be obtained more simply in the following manner :—

Let us assume we have a two-wire cable having capacity C and leakance S due to the dielectric between the wires, both reckoned per unit length of the circuit. Let us suppose that loading coils, each having inductance L_1 and resistance R_1, are inserted at intervals d along the cable. Also let the cable itself have a conductor resistance r and inductance l per unit

* *See* I. Pupin, "Transactions of the American Institute of Electrical Engineers," Vol. XVI., p. 93, 1899, and Vol. XVII., p. 445, 1900. Also *see* J. A. Fleming, "The Propagation of Electric Currents in Telephone and Telegraph Conductors," 2nd edition, p. 110.

of length. Let us denote the currents through three successive loading coils by i_{n-1}, i_n and i_{n+1}, and the P.D. between the wires by v_1 at a point half-way between the $(n-1)$th and nth coil, and by v_2 half-way between the nth and $(n+1)$th coil. Furthermore, let the currents and potentials be simple harmonic functions of the time and vary as the real part of ε^{jpt}, where $j=\sqrt{-1}$ and $p=2\pi$ times the frequency. Hence $d/dt=jp$ and $d^2/dt^2=-p^2$.

Consider, then, the section of the cable between the $(n-1)$th and nth loading coil. The average gradient of the current in that section of the cable is $(i_{n-1}-i_n)/d$, and if this gradient line is nearly straight the gradients must be equal to $(S+jpC)v_1$.

Also in the same manner for the next section we shall have $(i_n-i_{n+1})/d$ equal to $(S+jpC)v_2$.

Therefore, taking the difference we have

$$\frac{(i_{n-1}-i_n)-(i_n-i_{n+1})}{d}=(S+jpC)(v_1-v_2). \quad . \quad . \quad (7)$$

But if the impedance of each coil is large compared with that of the line between the coils, we can say that,

$$v_1-v_2=(R_1+jpL_1)i_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If, however, we include the resistance r and inductance l of the cable itself apart from those of the loading coil we must write this last equation in the form

$$v_1-v_2=i_n\left\{\left(r+\frac{R_1}{d}\right)+jp\left(l+\frac{L_1}{d}\right)\right\}d. \quad . \quad . \quad . \quad (9)$$

In actual practice the inductance of the length d of the cable between the loading coils is negligible compared with that of the loading coil, and the resistance of the loading coil is small compared with that of the length of cable between the loading coils. Hence, it is sufficient to write the above equation in the form

$$v_1-v_2=i_n(R+jpL)d, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where R and L are the average resistance and inductance of the loaded cable per unit of length of the circuit. Hence, from (7) and (10) we have

$$i_{n-1}-2i_n+i_{n+1}=i_n(S+jpC)(R+jpL)d^2. \quad . \quad . \quad (11)$$

If the current is a simple periodic current then i_n may be taken as proportional to $\cos(pt-qx)$, where $p=2\pi/\tau$, and

$q=2\pi/\lambda$ and $x=nd$. The equation (11) then becomes, by substitution,

$$\cos(pt-q(x-d))-2\cos(pt-qx)+\cos(pt-q(x+d))$$
$$=\cos(pt-qx)\{RS-p^2LC+jp(CR+LS)\}d^2 \quad . . \quad (12)$$

But since $-p^2=(jp)^2$ equation (12) can be written

$$(jp)^2+2\left(\frac{R}{2L}+\frac{S}{2C}\right)jp=-\left(\frac{4}{LCd^2}\sin^2\tfrac{1}{2}qd+\frac{RS}{LC}\right) \quad . . \quad (13)$$

Solving this quadratic we have

$$jp=-\left(\frac{R}{2L}+\frac{S}{2C}\right)\pm j\sqrt{\frac{4}{LCd^2}\sin^2\pi d/\lambda-\left(\frac{R}{2L}-\frac{S}{2C}\right)^2} \quad (14)$$

Hence the frequency of the oscillations is given by

$$p/2\pi=f=\frac{1}{\pi d}\sqrt{\frac{1}{LC}\sin^2\pi d/\lambda-\left(\frac{Rd}{4L}-\frac{Sd}{4C}\right)^2} \quad . \quad (15)$$

Hence the velocity $W=f\lambda$ with which the wave travels is given by

$$W=\sqrt{\frac{1}{LC}\frac{\sin^2\pi d/\lambda}{(\pi d/\lambda)^2}-\frac{\lambda^2}{4\pi^2}\left(\frac{R}{2L}-\frac{S}{2C}\right)^2} \quad . \quad . \quad . \quad (16)$$

If, then, the primary constants R, L, S and C are so related that $R/L=S/C$, that is, if the cable is sufficiently loaded to be distortionless, we have

$$W=\frac{1}{\sqrt{LC}}\frac{\sin\pi d/\lambda}{\pi d/\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Accordingly the cable will not transmit a wave if the frequency f is increased to a point at which f is greater than $1/\pi d\sqrt{LC}$, that is, if the periodic time T is less than $\pi\sqrt{Ld \cdot Cd}$. If the wave-length is made less than the coil or load interval d, then, as shown by Mr. Godfrey (*loc. cit.*), there are curious discontinuities as the wave-length is progressively decreased. We are presented with a phenomenon analogous to the formation of absorption lines in spectra, in which a medium is transparent or more or less opaque in accordance with the wave-length of the incident light. If, in the formula (15) above, we put $S=0$, it becomes identical with the formula given by Pupin. If we put both R and S equal to zero, and if d/λ is small, then the wave velocity becomes $1/\sqrt{LC}$, and is the same as if the added inductance is uniformly distributed along the cable.

ABSTRACT.

Prof. Fleming exhibited an apparatus for the production of vibrations on strings loaded or unloaded. This method is an improvement on that of Melde, in which transverse vibrations are created on a string attached to the prong of a tuning fork.

Melde's method has the disadvantage that it is not possible to put great tension on the string without stopping the fork, or to use very heavy or loaded strings without using forks of great size. Moreover, the frequency cannot be varied except by changing the fork.

In the Author's method the vibrations are produced on a string by attaching one end to the shaft of a small continuous-current motor of about $\frac{1}{8}$ H.P. The motor has on one end of its shaft a mechanism for counting revolutions, and on the other end a disc which has a crank pin inserted in its outer face, and to this pin is attached a light crankshaft connected at its outer extremity with a rocking lever. The string is fastened to a hook on the crankshaft near to the crank pin.

The other end of the string is attached to a fixed point which can be moved by means of a screw, in some cases a spring balance being interposed to measure the tension.

When the motor is started the string has a circular motion given to its end which is equivalent to two simple harmonic motions at right angles to each other.

If the tension is rightly adjusted the string then vibrates in sections, and the number of sections can be adjusted by altering the tension. The distance from node to node can then easily be measured and the frequency determined from the speed of the motor. In this way the velocity of the wave is measured, and can be compared with the velocity determined by taking the square root of the quotient of the tension by the linear density of the string.

A number of tables were exhibited showing the velocity of the wave motion on strings of various kinds confirming the known laws of string vibrations.

This method is particularly useful in studying the properties of loaded strings. Strings made of flexible cotton cord can be loaded with glass or wooden beads and set in vibration. In this manner it can be shown experimentally that when the wave length on the string extends over a distance of more than 8 or 10 loads, the string vibrates as if the loading matter were uniformly distributed, but the string cannot propagate vibrations when the half wave length approaches equality to the distance between two loads.

It is also possible to show by this apparatus very prettily the reflection of a wave at a load placed at any point on the string, and also that this reflection is reduced by tapering off the loading.

We can thus imitate with this loaded vibrating string all the phenomena of inductive loading in telephone cables on the Pupin system.

The theory of the vibrations of a loaded string has been studied by many mathematicians—for example, by Lagrange, Lord Rayleigh, Prof. P. G. Tait, and a very interesting Paper was published on the subject by Mr. Charles Godfrey in the "Philosophical Magazine" for April, 1898.

This method of causing a string to vibrate by a motor affords a simple method of demonstrating all the laws of vibrating strings to a class

and also can be employed to show experimentally results of loading telephone cables in various ways, and the reflections that occur when cables with different constants are connected in series.

DISCUSSION.

Dr. Russell complimented Prof. Fleming on his short and easy proof of Pupin's law.

Mr. Campbell remarked that Prof. Tait, of Edinburgh, used to have a hand machine with which he could produce eight or nine loops on a long vibrating string. But the success of his demonstrations depended largely upon the skill of the operator.

VII. *On the Moving Coil Ballistic Galvanometer.* By

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Received November 3, 1913.

1. In the theory of the moving coil ballistic galvanometer as usually presented, the motion of the coil is assumed to be oscillatory. But when a search coil of low resistance, such as is employed to explore magnetic fields, is connected to the galvanometer, the motion may become aperiodic. The theory as usually given in text-books is not directly applicable to this case. To evade this difficulty the suggestion is sometimes made to insert sufficient resistance in the circuit to make the motion oscillatory. This is unnecessary, for, as the following considerations show, the throw of the galvanometer is proportional to the change in flux, even when the motion is dead-beat, provided the change is quickly effected. As it is far quicker and much more convenient to work with a galvanometer when its motion is aperiodic than when it is oscillatory, the matter is of some practical importance, and the treatment given below may not be without interest.

In the Grassot fluxmeter, with its very weak torsional control, the throw is proportional to the change in flux, and is, within limits, independent of the time taken to effect the change. On the other hand, in an ordinary galvanometer, with its stronger control, the throw will be very small when the rate of change is the slowest permissible with the fluxmeter, and will have a much greater (its maximum) value when the change is instantaneous. Further, we would naturally expect, even when the change is instantaneous, the throw to be greater when the torsional control is weak than when it is strong. This inference is confirmed by the investigation given below.

In this Paper the aperiodic motion of the moving coil galvanometer is first considered; then an account of some observations with a galvanometer is given which confirm some deductions from the theory; and finally the results obtained are applied to determine the exact relation between its throw and the change in the flux through the search coil which produced it.

2. The coil of the galvanometer turns about a vertical axis in the plane of the coil, passing through its centre of gravity and perpendicular to the field. In the position of

rest the normal to the plane of the coil is at right angles to the lines of force, and the number of lines passing through it is zero. The deflection θ of the coil is conveniently measured by the angle this normal makes with its direction in the position of rest. Let A be the effective area of the coil and H the strength of the field. Then N, the number of lines passing through the coil in the position θ, is given by

$$N=AH\sin\theta.$$

The E.M.F. induced in the galvanometer coil in this position by its motion is $-A\,.\,H\cos\theta\,.\,\dot{\theta}$. This is equal to $-AH\,.\,\dot{\theta}$, since θ is always small in practice. Let i be the current in the coil at this instant. Then

$$i=-\frac{AH}{R}\,.\,\dot{\theta}-\frac{L}{R}\,.\,\frac{di}{dt},$$

where R is the total resistance of the galvanometer and the search coil in series with it. Since the movement is slow, we can neglect the second term and thus obtain

$$i=-\frac{AH}{R}\,.\,\dot{\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The couple acting on the moving system tending to increase θ, due to this current, is $-\frac{A^2H^2}{R}\,.\,\dot{\theta}$, where again we assume θ small and write $\cos\theta=1$.

The equation of motion of the coil is

$$\ddot{\theta}+\frac{A^2H^2}{IR}\,.\,\dot{\theta}+\frac{c}{I}\,.\,\theta=0,$$

where I is the moment of inertia of the coil about its axis of rotation and c is the torsional control of the suspending strip.

This equation may be written

$$\ddot{\theta}+2\lambda\dot{\theta}+n^2\theta=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Its solution is

$$\theta=e^{-\lambda t}(Ae^{pt}+Be^{-pt}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $p=(\lambda^2-n^2)^{\frac{1}{2}}$.

3. Let an angular velocity, ω, be given to the coil in its position of rest. Then we have

$$\left.\begin{aligned}\theta&=0\\ \dot{\theta}&=\omega\end{aligned}\right\}\text{at the time } t=0.$$

Substituting these values in (3) and its first derivative, we get

$$A=-B=\frac{\omega}{2p}.$$

The position and the velocity at any time t are thus given by

$$\theta=\frac{\omega}{2p}e^{-\lambda t}(e^{pt}-e^{-pt})=\frac{\omega}{2p}\cdot P, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\dot{\theta}=\frac{\omega}{2p}e^{-\lambda t}\{-(\lambda-p)e^{pt}+(\lambda+p)e^{-pt}\}. \quad . \quad . \quad . \quad (5)$$

From (5) we deduce that $\dot{\theta}=0$ and therefore θ will be a maximum at the time t_1 where

$$t_1=\log_e\left(\frac{\lambda+p}{\lambda-p}\right)^{1/2p}.$$

The maximum value of θ is θ_0 where

$$\theta_0=\frac{\omega}{n}\cdot\left(\frac{\lambda-p}{\lambda+p}\right)^{\lambda/2p}.$$

The ratio of the second term on the right-hand side of (4) to the first is e^{-2pt}. This becomes smaller and smaller as t increases. Hence, after a certain time, the position of the coil will be given with sufficient accuracy by

$$\theta=\frac{\omega}{2p}\epsilon^{-(\lambda-p)t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

4. To test relation (6) observations were made on a galvanometer (N.B. No. 30,532), the moving coil of which was wound on a narrow non-conducting frame, and had a resistance of about 800 ohms. The suspension was phosphor-bronze strip, and the field was approximately uniform. The period of oscillation on open circuit was 12·32 seconds. The air damping was small, and its effect has been left out of account throughout. The observations showed that when the galvanometer was deflected and the circuit then closed through a small resistance the time the deflection took to fall to half its value was always the same and was independent of t (provided the deflection was not observed too near the turning point). In other words, if θ, the deflection, be a function of t, say $f(t)$, the value of $f(t+\tau)$, where τ is a certain interval, was always equal to $\frac{1}{2}f(t)$, no matter what the value of t be. This shows that $f(t)$ must be a simple exponential function of the

time, of the form $Ae^{-\kappa t}$, where $e^{-\kappa\tau}=\frac{1}{2}$. The observations confirm the form of relation (6).

5. Since
$$\theta=\frac{\omega}{2p}\cdot e^{-(\lambda-p)t},$$
when t exceeds a certain value, and
$$\frac{\theta}{2}=\frac{\omega}{2p}e^{-(\lambda-p)(t+\tau)},$$
we have, by combining the two, $\lambda-p=\frac{\log_e 2}{\tau}$.

Expanding p in powers of n^2/λ^2, and neglecting all terms after the second, we get
$$\frac{1}{2}\frac{n^2}{\lambda}=\frac{0{\cdot}6931}{\tau}$$
or
$$R\tau=\frac{A^2H^2}{I}\cdot\frac{0{\cdot}6931}{n^2}=\text{constant}. \quad . \quad . \quad . \quad . \quad (7)$$

To test this relation, observations were made with the galvanometer mentioned in § 4. The values of τ observed with different external resistances are given in the second column of the following table; the third column gives the product $R\tau$.

R − 800	τ.	$R\tau$ (approx.).
0 ohms	17·60 seconds	14,100
100 ,,	15·77 ,,	14,200
200 ,,	14·47 ,,	14,500
300	12·78 ,,	14,100
400 ..	11·47 ,,	13,800
500 ..	10·57 ,,	13,700
600 ..	10·00 ,,	14,000

The results given in the above table are, on the whole, in fair agreement with the deduction expressed in (7). When the external resistance is more than 600 ohms, relation (7) is no longer sufficiently accurate; higher powers of n^2/λ^2 must in that case be retained in the expansion of p.

6. To illustrate the results obtained in § 3, consider the case of the galvanometer described in § 4, when the outside resistance is negligible. Since its period on open circuit is 12·32 seconds, this gives us $n^2=0{\cdot}2601$, and (7) becomes
$$R\tau=2{\cdot}665\,.\,\frac{A^2H^2}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since $R=800$ ohms and $\tau=17{\cdot}60$ seconds, we get from (8)

$\lambda=3{\cdot}302$ and $p=3{\cdot}262$. Substituting these values in the expression for t_1 in § 3, we get

$$t_1=0{\cdot}782 \text{ second.}$$

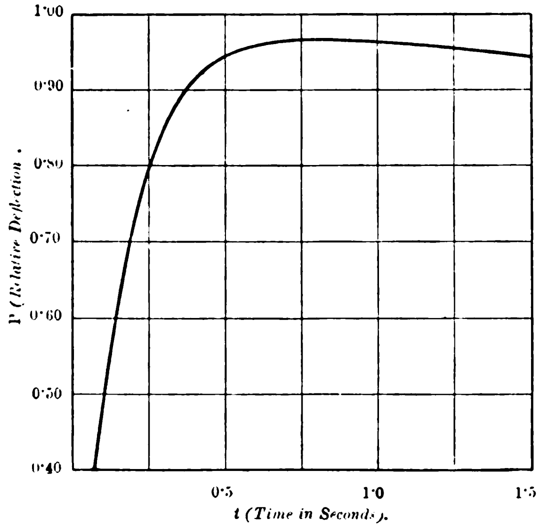
The coil always comes to rest in this time, 0·782 second, whatever be the angular impulse given to it. The throw is given by

$$\theta_0=0{\cdot}148\times\omega.$$

The following table gives the value of P, the variable part of the expression for θ in (4) for different values of t:—

t.	P.	t.	P.	t.	P.
0·1 sec.	0·487	0·6 sec.	0·957	1·5 sec.	0·942
0·2 ,,	0·723	0·7 ,,	0·962	2·0 ,,	0·923
0·3 ,,	0·848	0·8 ,,	0·963	5·0 ,,	0·819
0·4 ,,	0·912	0·9 ,,	0·962	10·0 ,,	0·670
0·5 ,,	0·943	1·0 ,,	0·960	...	...

The values are plotted in the accompanying diagram. They show that the galvanometer coil moves comparatively rapidly



to its maximum deflection and then slowly returns towards its position of equilibrium; this is in accordance with observation.

7. Let the flux through the search coil change, and let $E=\dfrac{dN_2}{dt}$

denote the E.M.F. induced in it. Then we have, as in the theory of the fluxmeter,

$$Ri+AH\,.\,\dot{\theta}+L\frac{di}{dt}=E$$

for the current and

$$I\ddot{\theta}-AHi+c\theta=0$$

for the motion of the galvanometer coil.

Substituting for i in the second equation the value obtained from the first and integrating between the limits $t=0$, the time when the flux begins to change, and $t=t_1$, the time when the galvanometer comes to rest at its maximum deflection, we get

$$[I\dot{\theta}]_0^{t_1}=\Big[-c\int\theta dt+\frac{AH}{R}(\int Edt-AH\,.\,\theta-Li)\Big]_0^{t_1}.$$

Now, $\dot{\theta}=0$ and $i=0$ at times $t=0$ and $t=t_1$, while $\theta=0$ at time $t=0$ and $\theta=\theta_0$ at time $t=t_1$. Remembering that $\int_0^{t_1}Edt=N_2$, the total change in flux through the coil, we get

$$N_2=AH\,.\,\theta_0+\frac{Rc}{AH}\int_0^{t_1}\theta dt. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In the fluxmeter the factor c is so small that the second term on the right-hand side of (9) is negligible compared with the first. In the ordinary moving-coil galvanometer this is not necessarily the case.

The ratio of the second term to the first is $\frac{\int_0^{t_1}\theta dt}{\theta_0}\cdot\frac{Rc}{A^2H^2}$. If this ratio be constant and independent of N_2, then N_2 will be proportional to θ_0. The factor $\frac{Rc}{A^2H^2}$ depends on the galvanometer and the resistance of the search coil, and is evidently constant with a given search coil. To find the value of $\frac{\int_0^{t_1}\theta dt}{\theta_0}$ we will assume that the change in flux (N_2) has been completely effected before the galvanometer coil has appreciably moved from its position of rest. The effect of this will be to give an angular impulse to the coil; its subsequent motion will then be given by (4). The value of $\int_0^{t_1}\theta dt$ will

depend only on λ, p and ω, and can be determined from (4), while θ_0 also depends on λ, p and ω alone. The ratio $\frac{\int_0^{t_1}\theta dt}{\theta_0}$ depends on λ and p, and does not involve ω.

Hence the ratio is independent of N_2, and if the flux change be sudden *the throw will be proportional to* N_2.

8. The ratio of the two terms in (9) is readily evaluated in any given case. With the galvanometer used for the observations recorded in §§ 4-5 and a search coil of negligible resistance, the value of $\frac{Rc}{A^2H^2}\left(=\frac{n^2}{2\lambda}\right)$ is 0·039. The value of $\int_0^{t_1}\theta dt$ is most easily obtained from the results given in § 6. These give for $\int_0^{t}\theta dt$ the value 0·63 θ_0. Thus the ratio is 0·024. Hence, with the given galvanometer and a search coil of negligible resistance, the throws are about $2\frac{1}{2}$ per cent. less than they would be with an infinitely weak control.

ABSTRACT.

The author first considers the mathematical theory of a moving coil galvanometer in which the damping is such as to make the motion non-oscillatory; then an account is given of some observations on a galvanometer which confirm some of the deductions from the theory, and the results obtained are applied to find the relation between the galvanometer throw and the change of flux in the search coil which produces it.

DISCUSSION.

Prof. C. H. LEES thought it likely that with a highly damped galvanometer the creeping at the end would create a difficulty as to what reading to take.

Prof. T. MATHER stated that the creep presented no difficulty, since it was easy to get the maximum deflection reached. The real difficulty was in getting the true zero.

Mr. A. CAMPBELL mentioned that although the case of the strongly damped galvanometer when used with a search coil did not appear to be treated in any English text-book, he had found it described in Kohlrausch's "Lehrbuch der praktischen Physik" (10th edition, 1905).

VIII. *A Note on Aberration in a Dispersive Medium and Airy's Experiment.* By JAMES WALKER, *M.A.*

IN a Paper entitled "Aberration in a Dispersive Medium"* Lord Rayleigh accepts the view of Ehrenfest† that in the case of aberration we have to deal with a peculiarity impressed on a wave-front and that in consequence the angle of aberration is v/U, v being the velocity of the earth in its orbit and U the group velocity, instead of v/V, where V is the wave velocity. This being so, it becomes necessary to consider Airy's experiment, in which he found the same angle of aberration with a telescope filled with water as with one that contained air.

In his explanation of this experiment Lord Rayleigh replaces the telescope by two perforated screens moving together with the velocity of the earth, the space between being occupied by water at rest relatively to the screens, and calculates the angle through which the hole in the hinder screen must be displaced, in order that light from a star falling normally on the foremost screen may pass through the system.‡

If in this explanation we make the changes rendered necessary by the view that it is the group-velocity with which we are concerned, we find that the angle of aberration is $\mu^{-2}v/U$ measured in water, corresponding to an angle $\mu^{-1}v/U$ measured in air.

The same result may be obtained from an analytical solution. Let us take the anterior screen as the plane of xy, the axis of x being drawn in the direction of its motion, and let the centre of the hole of width $2d$ be at $x=0$ at time $t=0$. Then at the point x just behind the screen the luminous disturbance will commence at time $(x-d)/v$ and end at time $(x+d)/v$.

Now $$\frac{2}{\pi}\int_0^\infty \frac{\sin \alpha}{\alpha} \cos \frac{vt-x}{d} \alpha d\alpha \quad . \quad . \quad . \quad . \quad . \quad (1)$$

has the value unity when t lies between $(x-d)/v$ and $(x+d)/v$

* "Phil. Mag. (6), XXII., 130 (1911).

† "Ann. d. Physik.," (4), XXXIII., 1571 (1910).

‡ "Nature," XLV., 499 (1892). "In consequence of the movement of the water the wave after traversing the first aperture is carried laterally with the velocity $v(1-\mu^{-2})$ and this is to be subtracted from the actual velocity v of the aperture in the posterior screen. The difference is $\mu^{-2}v$. The ratio of this to the velocity of light in water (V/μ) gives the angular displacement of the second aperture necessary to compensate for the motion. We thus obtain $\mu^{-1}v/V$. This angle being measured in water corresponds to v/V in air, so that the result of the motion is to make the star appear as if it were in advance of its real place by the angle v/V precisely as would have happened had the telescope contained air or vacuum instead of water."

and is zero when t is without these limits. If, then, $\cos nt$ represents the vibration incident on the screen, the vibration just behind it will be

$$\frac{2}{\pi}\cos nt\int_0^\infty \frac{\sin\alpha}{\alpha}\cos\frac{\alpha}{d}(vt-x)d\alpha, \quad . \quad . \quad . \quad (2)$$

which represents an aggregate of terms such as

$$a\cos m(vt-x)\cos nt, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which mv will be small compared with n, for $\sin\alpha/\alpha$ becomes insensible long before $\alpha v/d$ becomes comparable with n, provided d is not very small.

The expression (3) is the vibration just behind the screen considered by Lord Rayleigh in the Paper above cited in the case of a stagnant dispersive medium. Taking as the vibration at a finite distance from the screen

$$\phi=\tfrac{1}{2}a\cos\{(n+mv)t-mx-k_1z\}+\tfrac{1}{2}a\cos\{(n-mv)t+mx-k_2z\}, \quad (4)$$

where k_1, k_2 are determined so as to satisfy in each case the general differential equation of propagation,

$$\frac{\partial^2\phi}{\partial t^2}=V^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which, the medium being dispersive, V must be given values V_1, V_2 when the coefficient of t is $n+mv$ or $n-mv$, he shows that the angle of aberration due to v is

$$-\frac{x}{z}=\frac{k_1-k_2}{2m}^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In the case of a medium moving with speed v in the direction of x, we have instead of (5)

$$\frac{\partial^2\phi}{\partial t^2}+2v(1-\mu^{-2})\frac{\partial^2\phi}{\partial t\partial x}=V^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right),$$

in which suffixes (1) or (2) are to be assigned to V and μ when the coefficient of t is $n+mv$ or $n-mv$. Thus

$$V_1^2(m^2+k_1^2)=(n+mv)^2-2mv(1-\mu_1^{-2})(n+mv)\doteqdot(n+\mu_1^{-2}mv)^2,$$
$$V_2^2(m^2+k_2^2)=(n-mv)^2+2mv(1-\mu_2^{-2})(n-mv)\doteqdot(n-\mu_2^{-2}mv)^2,$$

whence the difference k_1-k_2 may be regarded as corresponding

* k has been substituted for μ in the above, so as to retain μ for representing the refractive index.

to a change in the coefficient of t from $n+\mu_1^{-2}mv$ to $n-\mu_2^{-2}mv$, the medium being at rest. Hence, denoting the general coefficient of t by σ, of which k is a function,

$$k_1-k_2=(\mu_1^{-2}+\mu_2^{-2})mv\frac{dk}{d\sigma}\doteq 2\mu^{-2}mv\frac{dk}{d\sigma}=\frac{2\mu^{-2}mv}{U}:$$

and accordingly $$-\frac{x}{z}=\mu^{-2}\frac{v}{U}$$

represents the angle of aberration in the medium, to which corresponds the angle $\mu^{-1}v/U$ measured in air.

Taking $$V=V_0(A+Bk^2)=V_0\left\{A+4\pi^2B\left(\frac{\mu}{\lambda}\right)^2\right\},$$

where λ represents the wave-length *in vacuo*, we have

$$U=\frac{d(kV)}{dk}=V\left\{1+4\pi^2B\,.\,2\mu\,.\left(\frac{\mu}{\lambda}\right)^2\right\},$$

and calculating B from the data for

$\lambda=6{\cdot}562\times10^{-5}$, $\mu=1{\cdot}3312$, and for $\lambda=4{\cdot}311\times10^{-5}$, $\mu=1{\cdot}3406$, we find $$4\pi^2B=-9{\cdot}47\times10^{-12},$$
whence for sodium light ($\lambda=5{\cdot}893\times10^{-5}$, $\mu=1{\cdot}3331$)
$$U=V(1-0{\cdot}0129)\text{ or }V/U=1{\cdot}013,$$
so that the angle of aberration is increased by about 1 per cent.

Considering the difficulties of Airy's experiment, it seems very unlikely that so small a variation as this could be detected, and, in fact, there appears to be an uncertainty of about this amount in the determination of the angle of aberration under ordinary circumstances.

ABSTRACT.

The view recently adopted by Lord Rayleigh that in the case of aberration we are concerned with the group-velocity instead of with the wave-velocity, makes it necessary to consider the experiment of Airy, in which he measured the angle of aberration with a telescope filled with water.

A modification of Lord Rayleigh's explanation of this experiment leads to the result that the angle of aberration thus determined corresponds to an angle $\mu^{-1}v/U$ measured in air. The same result is obtained from an analytical investigation, and a numerical calculation shows that the increase in the angle is about 1 per cent.—an amount that is probably too small to be detected.

IX. *The Thermal Expansions of Mercury and Fused Silica.*

By F. J. HARLOW, *A.R.C.S., B.Sc., Assistant Lecturer in Physics, Sir John Cass Technical Institute.*

RECEIVED NOVEMBER 11, 1913.

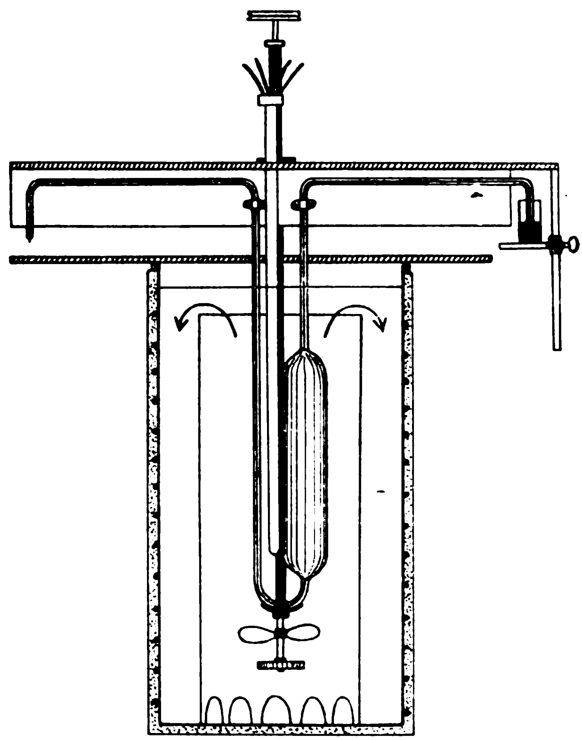
Introductory.—In a previous communication on the subject of the cubical expansion of fused silica* an account was given of observations which had been made on the apparent coefficient of expansion of mercury in fused silica by the weight thermometer method ; these observations were restricted to the temperatures of 0°C., 100°C. and 184°C. In view of the importance of establishing the correct values of the absolute coefficient of expansion of mercury and silica, it was thought that the extension of these observations to other temperatures would be useful. The results given in the following Paper were obtained with two of the cylindrical weight thermometers previously used, and comprise readings at frequent intervals from 0°C. to 300°C. The apparent coefficients for the temperature ranges 0°C. to 100°C., and 0°C. to 184°C. are in good agreement with the previous determinations, although an entirely different method of heating was adopted.

At the suggestion of Prof. Callendar, to whom I am indebted for guidance in this work, a spherical bulb was also used, as the earlier observations appeared to indicate that one could not assume the silica to be isotropic ; and if this were the case one would scarcely expect to get the same value for the apparent coefficient of expansion of mercury as with the cylindrical bulbs. The nature of the spherical bulb, however, which was not constructed specially for the experiment, did not allow of a continuous set of observations being made conveniently, so that results for the fundamental interval only were obtained. These, as will be seen, did not differ by more than 1 part in 18,000 from similar observations with the cylindrical bulbs, so that, in so far as this experiment has weight, the cubical coefficient of expansion may be considered, for these bulbs at any rate, to be three times the linear. One has thus felt

* The Cubical Expansion of Fused Silica, &c., "Proc." Phys. Soc., December, 1911.

justified in including in this Paper values of the coefficients of absolute expansion of mercury deduced from the observations of the apparent coefficients.

The Heating Bath.—In order that observations might be made at any desired temperature, the method of heating the bulbs by means of an electrically-heated oil bath was adopted. The arrangement is shown in the accompanying figure. A



WEIGHT THERMOMETER IN HEATING BATH.

cylindrical copper vessel 30 cm. by 15 cm. was covered with asbestos paper, and around this were wound two coils of Eureka wire in parallel, the whole being lagged with asbestos cloth. Inside this vessel was fixed a concentric cylinder extending from the bottom to within about 5 cm. of the top. This was slotted at its base so as to allow a free circulation of the oil. The bulb of mercury, the filling of which under a

vacuum has been fully described in the earlier Paper already referred to, was suspended at the centre of the cylinder by a hollow steel rod carrying a support for it at its lower end. The bulb stems were held as shown by a wooden cross-piece, also attached to the hollow steel rod. This cross-piece carried in addition a mercury thermometer for measuring the temperature of the exposed stems, and at its end a small adjustable platform to carry the overflow vessel. The advantage of using a hollow steel rod was, first, that the conduction of heat along it from the bath was small; and, secondly, that it served to carry a narrow rod for driving the stirrer. The stirrer, which was of the screw type, was driven by gear wheels almost vertically beneath the bulb. The oil was thus caused to circulate rapidly round the bulb, and at the same time a continuous flow was maintained up the central cylinder and down past the heated surface of the outer, as indicated by the arrows. The bath was closed at the top with sheets of uralite suitably cut for the supporting rod, bulb stems, and thermometer to pass through. These sheets extended outwards as far as possible in order to diminish the heating of the exposed stems by convection from the sides of the vessel.

The Observations.—The temperature of the oil bath was recorded by a platinum thermometer of about 5 ohms fundamental interval placed in the bath with its coil alongside the silica bulb. The temperature could be read by means of the bridge and galvanometer used to 1/500th of a degree at the lower temperatures and with proportionate accuracy at the higher. When the conditions were favourable, by a careful adjustment of the heating current the bath could be maintained to within 1/100th of a degree or so of any desired temperature for several minutes. The length of time usually allowed for a reading, the first at the particular temperature, was about an hour. The heating-up was performed as rapidly as possible, and then, when the conditions had become more or less steady, the temperature was kept within about a tenth of a degree of the desired temperature for some considerable time. When it was thought that the mercury in the bulb had attained the temperature of the bath, the final careful adjustment was made. It was frequently possible to keep the spot of light dead steady for a minute or two. If the temperature had not varied appreciably for several minutes the overflow vessel was removed and weighed. The temperature of the bath was then allowed to fall slightly to prevent loss of mercury

by a possible rise of temperature. A second reading was often taken as a check, and for this a quarter of an hour or so was found to be sufficient for the conditions to have become steady again.

The fact that there was good agreement between the readings when the desired temperature was approached, sometimes from above and sometimes from below, showed that there was no error due to a temperature lag of the mercury behind that of the bath. In order to test the efficiency of the stirring, readings were also taken with the direction of flow of the oil reversed. No systematic changes in the readings could be detected, which proves that the stirring arrangement was entirely satisfactory.

The readings above 100 deg. were made with a heavy lubricating oil, which was sufficiently fluid at the higher temperatures to allow of a vigorous stirring; for those at and below 100 deg. paraffin oil was found to be more suitable.

Ice readings were taken at frequent intervals during the course of the experiments. After being allowed to cool down to atmospheric temperature, the bulb and its attachments were withdrawn and cleaned, and then suspended in a glass cylinder which was subsequently packed with ice shavings. The vessel was provided with an outlet at its base, so that any excess water could be drained away. The constancy of the ice readings, even after heating to 300 deg., proved the absence of thermal hysteresis, thus bearing out the statements made in the previous Paper concerning the suitability of silica as the envelope for a thermometer.

The larger of the two cylindrical bulbs used was the only one suitable for observations at the lower temperatures on account of the smallness of the stem exposure correction, which, for the small bulb, formed an appreciable fraction of the overflow, the tubes of this being, unfortunately, somewhat wider than those of the large bulb.

The observations on the fundamental interval with the spherical bulb were made with ice and steam in the usual manner, this bulb, as has already been mentioned, not being suitable for observations with the oil bath.

The Apparent Coefficients of Expansion of Mercury in Silica.—A summary of the readings and calculated apparent coefficients is given in Tables I., II. and III. for the three bulbs; Table IV. contains the collected results.

TABLE I.—*Large Cylindrical Bulb.*

Mass of mercury filling to mark at 0°C. 1,912·70 gms.

Temp. °C.	Date.	Stem correction in gms.	Corrected overflow in gms.	Apparent coeff. $\times 10^8$.
	1913.			
29·993	January 24th	0·0003	10·3048	...
	April 1st	0·0003	10·3022	...
	April 4th	0·0003	10·3042	...
	April 4th	0·0003	10·3035	...
	April 11th...........	0·0003	10·3063	...
			10·3042	18,060
49·993	February 28th	0·0007	17·1226	...
	February 28th	0·0007	17·1246	...
	March 18th	0·0007	17·1202	...
	March 28th	0·0008	17·1209	...
	April 4th	0·0006	17·1242	...
			17·1225	18,068
74·993	February 21st	0·0013	25·5894	...
	February 28th	0·0012	25·5883	...
	March 4th...........	0·0014	25·5877	...
	March 18th	0·0013	25·5891	...
	March 18th	0·0014	25·5883	...
			25·5886	18,081
99·991	January 14th	0·0016	34·0044	...
	February 18th	0·0019	34·0067	...
	March 4th...........	0·0018	34·0047	...
	April 10th...........	0·0020	34·0050	...
	May 26th	0·0026	34·0066	...
			34·0055	18,102
184	May 26th	0·0030	62·0332	...
	May 30th	0·0030	62·0376	...
	May 30th	0·0030	62·0368	...
	June 2nd	0·0028	62·0395	...
			62·0368	18,218
200	May 30th	0·0030	67·3456	...
	June 2nd	0·0030	67·3480	...
			67·3468	18,248
250	May 30th	0·0050	83·9148	...
	June 2nd	0·0052	83·9188	...
			83·9168	18,355
300	May 27th	0·0060	100·5001	...
	May 30th	0·0060	100·5063	...
	June 2nd	0·0061	100·5082	...
			100·5049	18,487

Ice readings :—	May 29th	104·8871
	May 31st..................	104·8869
	June 3rd	104·8876

TABLE II.—*Small Cylindrical Bulb.*

Mass of mercury filling to mark at 0° C. 1,154·66 gms.

Temp. °C.	Date.	Stem correction in gms.	Corrected overflow in gms.	Apparent coeff. × 10^8.
	1912-1913.			
100	November 15th …	0·0028	20·5268	…
	November 19th …	0·0028	20·5282	…
	June 30th	0·0050	20·5279	…
	July 4th	0·0050	20·5297	…
	July 10th	0·0042	20·5336	…
			20·5292	18,101
140	July 4th	0·0074	28·6106	…
	July 11th	0·0068	28·6137	…
			28·6122	18,150
184	June 30th	0·0084	37·4525	…
	July 4th	0·0090	37·4528	…
	July 11th	0·0090	37·4550	…
			37·4534	18,220
200	July 4th	0·0096	40·6547	…
	July 11th	0·0110	40·6587	…
			40·6567	18,248
250	June 30th	0·0118	50·6676	…
	July 4th	0·0116	50·6622	…
	July 11th	0·0116	50·6627	…
			50·6642	18,357
300	June 30th	0·0136	60·6746	…
	July 4th	0·0140	60·6849	…
	July 11th	0·0144	60·6879	…
			60·6825	18,490

Ice readings :— June 27th .... 75·9104
July 3rd .... 75·9106
July 10th .... 75·9086
July 16th .... 75·9090

TABLE III.—*Spherical Bulb.*

Mass of mercury filling to mark at 0° C. 934·66 gms.

Temp. of steam.	Date.	Stem correction in gms.	Corrected overflow in gms.	Apparent coeff. × 10^9.
99·879	June, 1912	0·0018	16·5983	181,017
99·847	,, ...	0·0018	16·5931	181,016
99·815	,, ...	0·0020	16·5861	180,998
100·067	,, ...	0·0018	16·6295	181,022
100·159	,, ...	0·0021	16·6441	181,020
				181,016

TABLE IV.—*Collected Results.*

Temp. range.	Apparent coeff. × 10^8.				
	Large bulb.	Small bulb.	Spherical bulb.	Mean of 1911 observations.	Final mean.
0° to 30°	18,060	...	...	...	18,060
50°	18,068	...	...	...	18,068
75°	18,081	...	...	...	18,081
100°	18,102	18,101	18,102	18,104	18,102
140°	...	18,150	...	...	18,150
184°	18,218	18,220	...	18,221	18,220
200°	18,248	18,248	...	...	18,248
250°	18,355	18,357	...	...	18,356
300°	18,487	18,490	...	...	18,489

Where more results than those quoted were obtained, representative readings are given which have the same average as the larger number of readings. The percentage accuracy is not so great at the lower temperatures as at the higher on account of the smallness of the overflow, but above about 50 deg. the results can be regarded as being accurate to 1 part in 18,000, the accuracy to which one attempted to work throughout. It will be noticed that the readings at the higher temperatures show a gradual increase, not sufficient, however, to affect the results appreciably, This was found to occur invariably when the bulb was heated repeatedly to the higher temperatures, and even for the lower temperatures after continued use. This peculiarity was traced to the expelled mercury carrying back with it a slight amount of contamination, which was most probably either oxide or dissolved gas. It was not sufficient to affect the ice readings appreciably, but caused the subsequent overflows to be larger, particularly those at the higher temperatures. At one time, when the overflows had become much higher than usual after a number of readings had been taken, on putting the bulb under a vacuum at 100 deg. small bubbles of gas appeared around the top part of the bulb. After allowing the mercury to cool down to 0 deg., pumping off this gas and then re-heating while it was still under the vacuum, the normal readings were again obtained. It is probable, therefore, that the lower readings of those quoted are the more accurate, since these were obtained immediately after steps had been taken to remove any trace of contamination either by re-filling or by the method just described. The good agreement obtained between the results with the two bulbs was considered to be a sufficient guarantee of the accuracy of the

observations, especially as the stem exposure correction was so large in the case of the smaller bulb.

The Coefficients of Absolute Expansion of Mercury.—From the above observations on the apparent coefficients of expansion, most of which, as already mentioned, are considered to be accurate to 1 part in 18,000, one could deduce reliable values for the coefficients of absolute expansion of mercury, if the expansion of the silica bulbs were known with certainty. Since the excellent agreement between the results with cylindrical and spherical bulbs seems to warrant the conclusion that the bulbs were isotropic, observations of the linear coefficient alone would be necessary. The Fizeau method appears to be the only one suitable for this purpose, but, unfortunately, the bulbs were too wide for the apparatus at one's disposal.

Mr. A. Eagle, however, has carried out measurements with this apparatus on a narrower tube made by the Silica Syndicate, from whom the bulbs were also obtained. The tube and bulbs will, therefore, most probably have been made in the same way and from similar material, and should, therefore, agree in their thermal expansion. Eagle's values, which have not yet been published, cover the temperature range 0 deg. to 120 deg., and are in excellent agreement with the results obtained by Prof. Callendar by the interference method,* on a rod of silica obtained from the same firm.

Prof. Callendar has kindly furnished me with the equation which represents his results over the range 20°C. to 300°C. This is

$$a_0{}^t=\left\{78{\cdot}0-\frac{8650}{t+175}\right\}10^{-8},$$

$a_0{}^t$ being the coefficient of linear expansion between 0°C. and t°C.

The values of $a_0{}^t$ calculated by means of this equation for the various temperatures used in the weight thermometer experiments, and also the deduced values of the coefficient of absolute expansion of mercury, are given in Table V. For comparison, the values obtained from the equations given by Callendar and Moss† and by Chappuis‡ for the absolute expansion of mercury, are also included.

* "The Expansion of Vitreous Silica." "Proc." Phys. Soc. June 15, 1912.

† Phil. "Trans.," January, 1911.

‡ "Procès-Verbeaux." Comm. Int. des Poids et Mesures, 1903.

TABLE V.—*The Absolute Coefficient.*

Temp. range.	Linear coeff. of expansion of silica $\times 10^8$. Callendar.	Absolute coefficients of expansion of mercury $\times 10^9$.		
		Callendar and Harlow.	Callendar and Moss.	Chappuis.
0° to 30°	35·8	18,168	18,095	18,171
50°	39·6	18,188	18,124	18,183
75°	43·4	18,213	18,163	18,211
100°	46·6	18,244	18,205	18,254
140°	50·5	18,305	18,280	...
184°	53·9	18,387	18,371	...
200°	54·9	18,419	18,406	...
250°	57·6	18,537	18,525	...
300°	59·8	18,678	18,657	...

It will be observed that there is a fairly good agreement with Chappuis' values, which were also obtained by the weight thermometer method, but a considerable discrepancy from those of Callendar and Moss obtained by the absolute method; the discrepancy, however, gets less as the temperature rises.

The coefficients of expansion of silica given by Prof. Callendar's equation are rather larger than those obtained by Randall.* If these are used there is quite a good agreement with the results of Callendar and Moss at the higher temperatures.

The large discrepancy which exists between the results of the absolute and weight thermometer methods at the lower temperatures is difficult to explain. It seems that there must be some hitherto undiscovered systematic error in one of the methods, and in view of the importance of a knowledge of the correct thermal expansion of mercury in thermometry of precision, the subject undoubtedly calls for further investigation.

In conclusion, I desire to thank Prof. Callendar for the help and advice he has given me in this work, and the Governors of the Imperial College of Science for permission to work in their laboratories.

ABSTRACT.

A more complete set of observations of the relative coefficients of expansion of mercury in silica than those previously published are obtained by the use of an electrically heated oil bath. The observations comprise readings at frequent intervals up to 300°C., and are in good agreement with the earlier observations, which, when applied to the values of the absolute expansion of mercury given by Callendar and Moss, seemed to show that the cylindrical bulbs used were not isotropic. Further experiments made with a spherical bulb negatived this conclusion. Tables are included in the Paper giving representative observations and the final results. From the values

* "Phys. Rev.," XXX., p. 216, 1910.

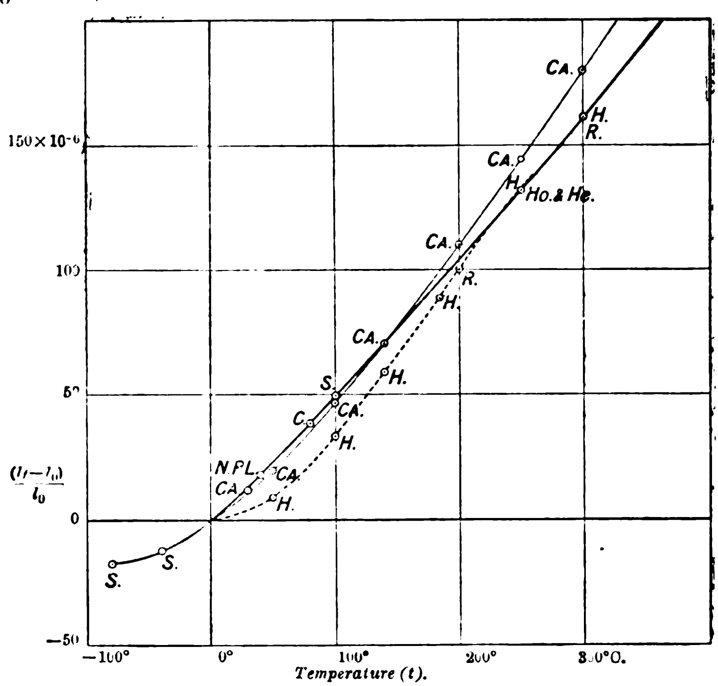
of the coefficients of expansion of silica determined by Prof. Callendar upon specimens similar to those from which the bulbs were made, the coefficients of absolute expansion of mercury are calculated. These over the range of the experiments of Chappuis, who used a similar method, but with bulbs of *verre dur*, are in fair agreement with his results, but differ appreciably from the results of Callendar and Moss obtained by the absolute method. It seems, therefore, that there is some undiscovered fundamental difference in the two methods.

DISCUSSION.

Dr. J. A. Harker said that in Chappuis' experiments bulbs both of platinum-iridium and *verre dur* were used, and no indication was obtained of anomalous expansion of the *verre dur*. This, he thought would have shown up in the expansion of water, as afterwards determined by Chappuis with the same apparatus.

Mr. F. E. Smith believed that despite the agreement between Harlow's results for the coefficient of expansion of mercury with those of Chappuis and others, the results of the hydrostatic method should have precedence. There were always films of moisture on glass and silica which were not removed, even by filling in vacuo, under a temperature of 400°C. This would account for the observed discrepancy, which, as would be expected, disappeared at higher temperatures.

Dr. G. W. C. Kaye exhibited some curves in which were collected the results of various experimenters on the expansion of mercury, including those of Harlow, and indicated the best mean curve obtainable from them.



C. Chappuis, 1903. H. Harlow and Callendar and Moss. Ho. & He. Holborn and Henning, 1908. R. Randall, 1910. S. Scheel, 1907. CA. Callendar, 1913.

Mr. A. Campbell thought the effect, if any, of surface films of moisture would be evident from results taken with bulbs of different size, since the volume of the film only increased as the square of the dimensions. Mr. Harlow's results appeared to show that the effect was very small.

Mr. Sears called attention to the part played by the late Mr. Donaldson in connection with the discussion on the subject of the coefficients of expansion of fused silica and mercury, and expressed regret that his untimely death had prevented him from carrying out the further experiments which he had intended. Mr. Sears asked leave to re-exhibit two of the lantern slides prepared by Donaldson to illustrate the Paper read by him before the Society in March, 1912, together with a new slide displaying certain of the figures in the subjoined table. In this table, columns 1, 2 and 3 are the same as the columns of Mr. Harlow's Table V., and are reproduced only for convenience of comparison. Column 4 gives the figures which Mr. Harlow would have obtained for the coefficient of expansion of mercury if he had taken for the coefficient of fused silica the values given by Kaye ("Phil. Mag.," October, 1910) as the result of a general survey of the work of all observers up to that date. This was the basis of the formula suggested by Donaldson, and the speaker called attention to the very close agreement between Donaldson's values (column 5) and those actually found by Harlow in the range 0°C.–200°C. Beyond this range Donaldson had not considered it safe to extrapolate with the material then at his disposal, and he had suggested in his Paper the desirability of the experiments which Mr. Harlow had now carried out. Column 6 of the table, as exhibited at the meeting, gave a series of values calculated by the speaker from Harlow's new figures, with the aid of a quartic formula instead of the cubic, to which the paucity of the previous observations had compelled Donaldson to limit himself. This quartic was calculated on the basis of Kaye's value for the coefficient of expansion of silica, partly because this was the basis on which Donaldson worked, and partly because Prof. Callendar's Paper had not been circulated before the meeting, so that the speaker had not known what weight to attach to the formula given in Harlow's Paper. After hearing Prof. Callendar's Paper he was of opinion that the time had now come when, in cases of this kind, the coefficient of expansion of the actual silica vessel used ought to be determined.

Temperature range.	1. Chappuis, 1907.	2. Callendar and Moss, 1911.	3. Harlow, 1913. (Callendar, silica).	4. Harlow, 1913. (Kaye, silica).	5. Donaldson, 1912 (cubic).	6. Sears (Quartic)
0°C.– 30°C.	18,171	18,095	18,168	18,187	18,170	**18,174**
0°C.– 50°C.	18,183	18,124	18,188	18,201	18,192	18,194
0°C.– 75°C.	18,211	18,163	18,213	18,223	18,222	18,221
0°C.–100°C.	18,254	18,205	18,244	18,251	18,254	**18,251**
0°C.–140°C.	...	18,280	18,305	18,305	18,311	18,306
0°C.–184°C.	...	18,371	18,387	18,380	18,380	18,379
0°C.–200°C.	...	18,406	18,419	18,410	18,407	**18,410**
0°C.–250°C.	...	18,525	18,537	18,523	18,497	18,522
0°C.–300°C.	...	18,657	18,678	18,660	18,597	**18,663**

$\times 10^{-8}$

Note added since the Meeting.

Column 6 of the table has been revised from that shown at the meeting as follows: On consideration, there appeared still to be some doubt as to

whether it were better to take Kaye's values or Callendar's for the coefficient of expansion of silica when working up Harlow's observations. This for two reasons: In the first place, Callendar and Eagle's experiments, though made with pieces of silica supplied from the same source, were not actually done with the vessels used by Harlow; and, secondly, Callendar's values lie sufficiently near to Kaye's curve to be included amongst the other observations on which that curve was based without seriously distorting it. On the whole, therefore, it seemed fairer to adopt this latter procedure, and the new quartic from which column 6 of the table was calculated was accordingly obtained by using a mean between Callendar's and Kaye's values for silica in working up Harlow's results for mercury. The mean of the figures so obtained and those given by Chappuis was then taken at 0°C. and 100°C. (the two points at which Chappuis' values are most firmly established), and the mean of the Harlow and Callendar-Moss figures at 200°C. and 300°C. (the range of temperature where these two series of results are in reasonably good agreement). The quartic was calculated so as to give the four values thus obtained (distinguished by black type in the table). The formula so obtained seems to represent most fairly the results of all the work done on the subject up to the present. After the remarks which he made at the meeting, Prof. Callendar would presumably agree that, pending some further investigation, the Callendar-Moss values at the lower temperatures must be regarded as being affected by some unexplained source of experimental error.

The formula from which column 6 has finally been calculated is—

$$V = V_0[1 + 10^{-6}\{181{\cdot}456t + 0{\cdot}009{,}205t^2 + 0{\cdot}000{,}006{,}608t^3 + 0{\cdot}000{,}000{,}067{,}320t^4\}].$$

It may be noticed, before leaving the subject, that this formula has the further slight advantage of agreeing better with Chappuis' values over the range −20°C. to 0°C. than did Donaldson's cubic.

Prof. CALLENDAR, in reply, said they had a bulb made with concentric cylinders inside to test for surface film effects. No effect was detected.

X. *Some Characteristic Curves and Sensitiveness Tests of Crystal and Other Detectors.* By PHILIP R. COURSEY, *B.Sc., Assistant in the Electrical Engineering Laboratory, University College, London.*

RECEIVED NOVEMBER 25, 1913.

ALTHOUGH much work has been done on the comparative sensitiveness of various types of radio-telegraphic detectors, many questions still remain unsettled.* It has been known for some time in the case of the oscillation valve detectors invented by Dr. J. A. Fleming that if the second differential of the characteristic, or volt-ampere curve, is plotted it bears a considerable resemblance to the curve of sensitiveness of the valve to wireless signals.†

The tests described below were undertaken with a view to finding out whether any similar relation could be traced in other detectors than the valve, such as the crystal receivers so commonly employed in the wireless stations of to-day.

The detectors examined in this manner were ; Carborundum, electrolytic, galena, molybdenite, "perikon" (zincite-chalcopyrite), tellurium-aluminium, and zincite-bornite, while curves were also obtained for the Fleming valves, both carbon and metal filament, and partial tests made on carbon-steel, chalcopyrite, bornite, zincite, silicon, tellurium-zincite, and other detectors, but complete curves for the latter were not obtained, in many cases on account of their relative insensibility. The Marconi magnetic detector, belonging to a different class to the above, was used as a standard of reference

* "The Comparative Sensitiveness of Some Common Detectors Used in Radio-telegraphy," by L. W. Austin, "Bulletin" of Bureau of Standards, Washington, Vol. 6, pp. 527-542 : and "Electrical Review and Western Electrician," Vol. 58, pp. 294-296, February, 1911.

"Electrothermal Phenomena at the Contact of Two Conductors, with a Theory of a Class of Radio-telegraphic Detectors," by W. H. Eccles, "Proc." Physical Society of London, Vol. 25, pp. 273-293.

"Crystal Rectifiers," by G. W. Pierce, "Physical Review," Vol. 29, pp. 478-484 ; "Science Abstracts," Vol. 13A, No. 163 and Vol. 12A, No. 1295.

"Coherers," by W. H. Eccles, "Phil. Mag.," Vol. 19, pp. 869-888 ; "Electrician," Vol. 65, pp. 724-727 and 772-773, August, 1910.

"Theory and Practice of Wireless Detectors as at Present Used," by S. M. Powell, "Electrical Review," Vol. 68, pp. 11-13 and 72-75.

† "The Principles of Electric Wave Telegraphy and Telephony," by Dr. J. A. Fleming, 2nd edition, p. 480.

for the measurement of sensitiveness in each case. Magnetic or closed-circuit transmitters and receivers were used throughout to minimise interference with other apparatus in the laboratory.

Preliminary experiments (particularly with the carborundum detector) were carried out, using the "tilting-coil" method of testing detectors,* employing a square coil of 2 ft. side, and eight turns at the transmitter, included in an oscillation circuit containing a condenser and a quenched spark gap of the rotary (modified Peukert) type, described in the "Proceedings" of the Physical Society of London,† fed from an induction coil with an automatic sender in the primary circuit. At the receiver a rectangular coil, pivotted horizontally, was connected to a variable air condenser to tune it to the transmitter, while the detector under test was connected in series with a double head-piece 2,000-ohm telephone receiver, as a shunt to the tuning condenser. In the same circuit was included a potentiometer to inject a variable direct-current boosting voltage, with a microammeter to measure the current passing, and a high-resistance moving-coil voltmeter to measure the voltage impressed on the circuit by the potentiometer.

The sensitiveness of the detector was measured relatively in these tests by finding the angle between two positions of the tilting-coil at which the sounds in the phones just ceased—*i.e.*, the "angle of silence"—and taking the sensitiveness as proportional to the co-secant of half of this angle—that is, as inversely proportional to the projected area of the receiving coil normal to the wave. This, it was found, does not give strictly accurate results, as with very sensitive detectors it was possible to obtain sounds in any position of the coils—even when the transmitting and receiving coils were at right angles.

It was later found necessary to measure the actual voltage on the terminals of the crystal or detector in use, on account of the drop in the telephones, &c., which amounted in some cases to a considerable fraction of a volt. This was most conveniently carried out by means of a potentiometer, and for this purpose a special one was constructed enabling voltages up to

* "The Production of Steady Electrical Oscillations in Closed Circuits, and a Method of Testing Radio-telegraphic Receivers," by J. A. Fleming and G. B. Dyke, "Proc." Physical Society of London, Vol. 21; and "Phil. Mag," May, 1909.

† "The Measurement of Energy Losses in Condensers Traversed by High-Frequency Electric Oscillations," by J. A. Fleming and G. B. Dyke, "Proc." Phys. Soc. Lond., Vol. 23, p. 117, 1910.

25 or 30 volts to be measured with an accuracy of 0·001 volt or less. The microammeter employed for the current measurements took the form of a Paul single-pivot galvanometer used shunted, when so required, by a resistance box, the currents corresponding to the deflections being obtained from a calibration of the instrument by means of a potentiometer.

On account of the unreliability of the sensitiveness measurements made with the "tilting-coil" method, experiments were conducted with a view to finding a more satisfactory method. The scheme finally used consisted of two flat spiral coils mounted vertically, and having 15 turns of 3/22-wire each, of mean radius 7·25 cm., and inductance 36,000 cm., one coil being fixed (used as the transmitter) and the other mounted so that it could be moved, in guides, to varying distances from the fixed coil. The range of motion of the two coils was about 1 metre.

These coils could be accurately calibrated by sending known alternating currents through the transmitting coil, and measuring the current induced in the receiving coil at different distances from the transmitter. This was done with alternating current at about 1,000 cycles per second from a high-frequency alternator, and the calibration curve of the coils was plotted as the ratio $\frac{\text{received current}}{\text{transmitting current}}$ against scale distance between the coils. The current in the transmitting coil was measured on suitable (calibrated) thermal ammeters,* the received currents being obtained on a vacuum thermo-ammeter and sensitive galvanometer.

The calibration so obtained gives a regular curve, but one which does not follow a very simple law, as the index of the curve (deduced from log. curves) varies from —1 to —3, at short distances the received current falling off inversely as the distance, at slightly greater distances inversely as the square, and at still greater distances at an increasing power which tends towards the cube as a limit.

When obtained in this manner, the above calibration curve also gives the ratio of the voltages on the terminals of the transmitting and receiving coils, and hence serves to compare the sensitiveness of two detectors by drawing out the moving coil until the sounds in the telephone connected to the detector under test are just extinguished in each case, the curve then

* *See* "Journal" Inst. Elec. Engineers, Vol. 44, p. 352.

giving the ratio of the oscillatory voltages impressed on the detectors; and hence, if one of them (say, the Marconi magnetic detector) is taken as a standard, the relative sensitiveness of the other can be determined.

As source of oscillations, experiments were conducted with various arrangements of buzzers and buzzer contacts (any kind of spark-gap being quite out of the question on account of the short distance between the coils), but very successful results as far as stability and reliability goes could not be obtained. The most satisfactory arrangement, finally adopted, was a small eight-part commutator mounted on the shaft of a small motor running at about 3,000 revs. per min., with two gauze brushes pressing on it, and connected as shown in Fig. 1, the brushes being arranged to give alternate periods of "make" and "break." This arrangement was found to be much more

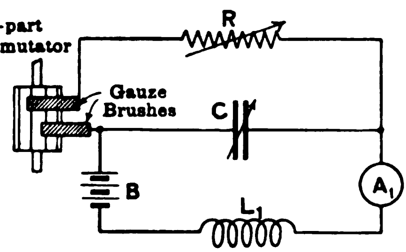


Fig. 1.—Transmitting Connections.

steady in operation than the buzzer method employed by Austin for similar purposes.*

The strength of the signals transmitted could be regulated by means of the resistance R, and kept constant during a test by the indications of the thermal ammeter A_1. L_1 is the transmitter of the sliding coil arrangement and L_2 the receiver.

At the receiver provision was made for quickly changing over from the Marconi magnetic to the crystal or other detector under test, and vice versa, and also for changing over the telephones from one to the other (Fig. 2). It was found advisable to insert a key at K_1 to determine the exact position of silence in the 'phones. The crystal detectors were connected across D_1 and D_2, and when valves were being used, the valve

* "The Comparative Sensitiveness of Some Common Detectors of Electrical Oscillations," by L. W. Austin, "Bulletin" of the Bureau of Standards, Washington, Vol. 6, p. 528.

filament was put across V_{f-} and V_{f+}, and the plate connected to V_p. The moving coil voltmeter V served to adjust the voltage on the filament to the correct value, by means of the variable resistance R_3 Western Electric 2,000-ohm double headpiece telephones were used for all the tests.

The majority of the crystals were clamped in a holder comprising two brass plates, through a hole in one of which a portion of the surface of the crystal is exposed, on to which a contact of metal or another crystal can be pressed.

At the commencement of each test the strength of the transmitted signals was adjusted until the signals received on the

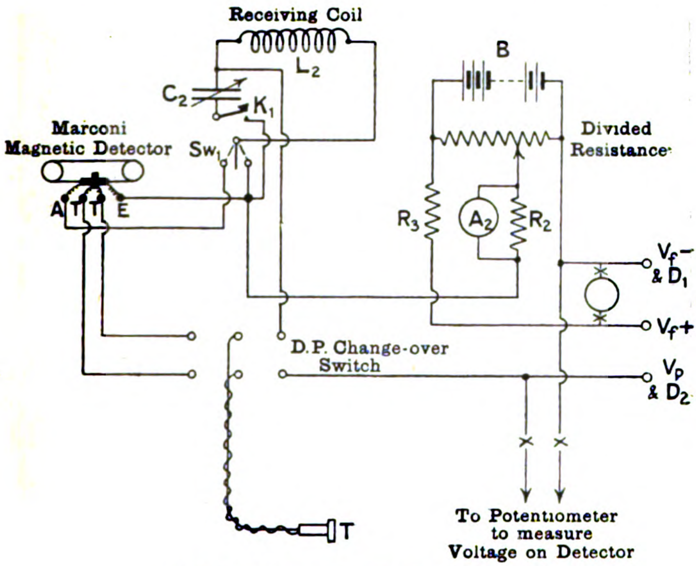


FIG. 2.—RECEIVING CONNECTIONS.

magnetic detector were just extinguished at a predetermined point on the scale of the sliding coils, and the sensitiveness tests then made on the second detector at various boosting voltages, the currents being measured in each case to enable the characteristic curve to be plotted. In this manner the "personal equation" due to the varying sensitiveness of the ear was eliminated.

As a preliminary experiment with this method the sensitiveness of a series of detectors, used as rectifiers only—*i.e.*, without boosting voltage—was measured, the most sensitive point

on each crystal being found for this (*see* table below). This shows the " Perikon " or zincite-chalcopyrite detector to be the most sensitive of those tested when used as a simple rectifier. The table also gives the maximum measured values of the sensitiveness for the various detectors tested when used with boosting voltages.

Table showing the Relative Sensitiveness of Various Detectors.

Detector.	Maximum sensitiveness when used as simple rectifiers.	Maximum measured sensitiveness with boosting voltage.
Marconi magnetic, assumed as unity	...	...
Molybdenite-copper point	2·15	2·15
Graphite-steel point	1·26	...
Carborundum, one end set in solder	0·50	0·60
Carborundum, not set in solder	0·325	1·0
Galena-plumbago	8·35	12·6*
Zincite-copper point	6·6	...
Zincite-brass point	3·43	...
Chalcopyrite-copper point	0	...
Chalcopyrite-brass point	0	...
" Perikon " (zincite-chalcopyrite)	10·5	12·1
Bornite-copper point	1·0	...
Bornite-carbon point	0	...
Zincite-bornite	7·4	7·4
Fleming carbon-filament valve (No. 12)	...	1·0
Fleming 12-volt metal-filament valve	...	1·0
Fleming 15¼ volt metal-filament valve (No. 33)	...	12·0
Tellurium-aluminium	...	0·9
Electrolytic (German make)	...	1·0
Electrolytic-nitric acid	...	4·0

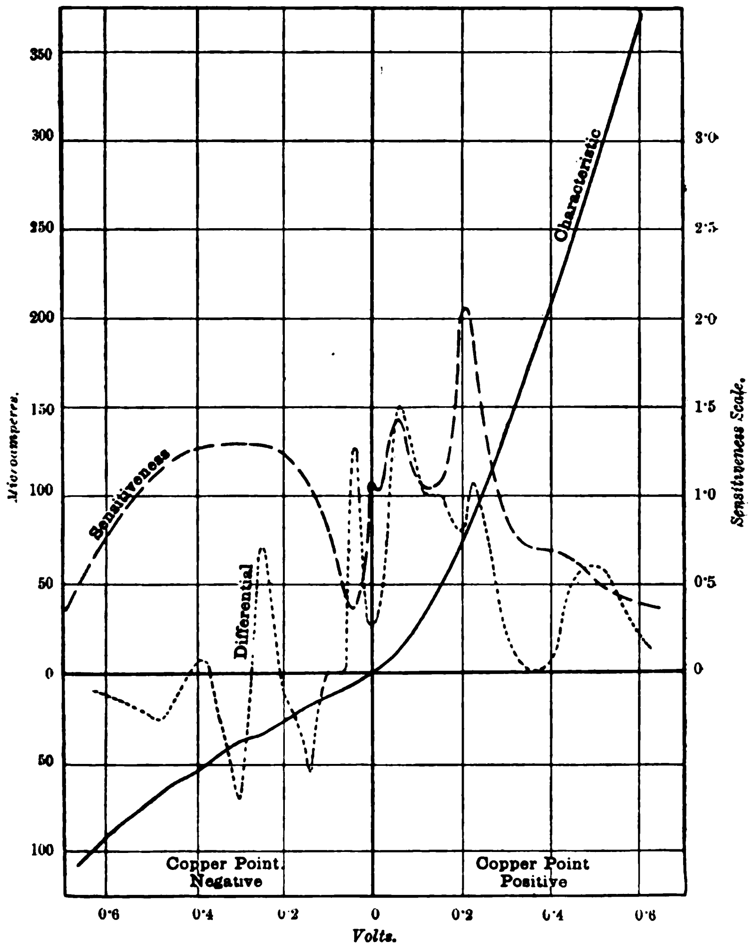
* NOTE.—If the *general shape* of the sensitiveness curve for the galena detector may be assumed to remain the same for very light contact as for firmer contact (with which the above measurements were taken), the maximum *estimated* sensitiveness for this detector with very light contact is *of the order of* 60 *to* 70, as compared with the magnetic detector.

Since the exact values of the current, voltage, &c., obtained in these tests for the various detectors will necessarily differ with different crystals, tables giving the results of the measurements are not given, but sample curves are included to show the general form of these results, as the *general shapes* of the curves will probably remain the same for the same substances.

The conditions of these tests should be noted—viz., all the detectors were tested with the same telephone receivers, of 2,000 ohms resistance, and hence the figures given do not necessarily represent those of the best possible conditions, the results being, therefore, not strictly comparable with those of,

say, Dr. Austin, in which the best resistance of telephone was chosen for each detector, conditions not always obtainable in practice.*

It should be noted that whilst the numbers given opposite

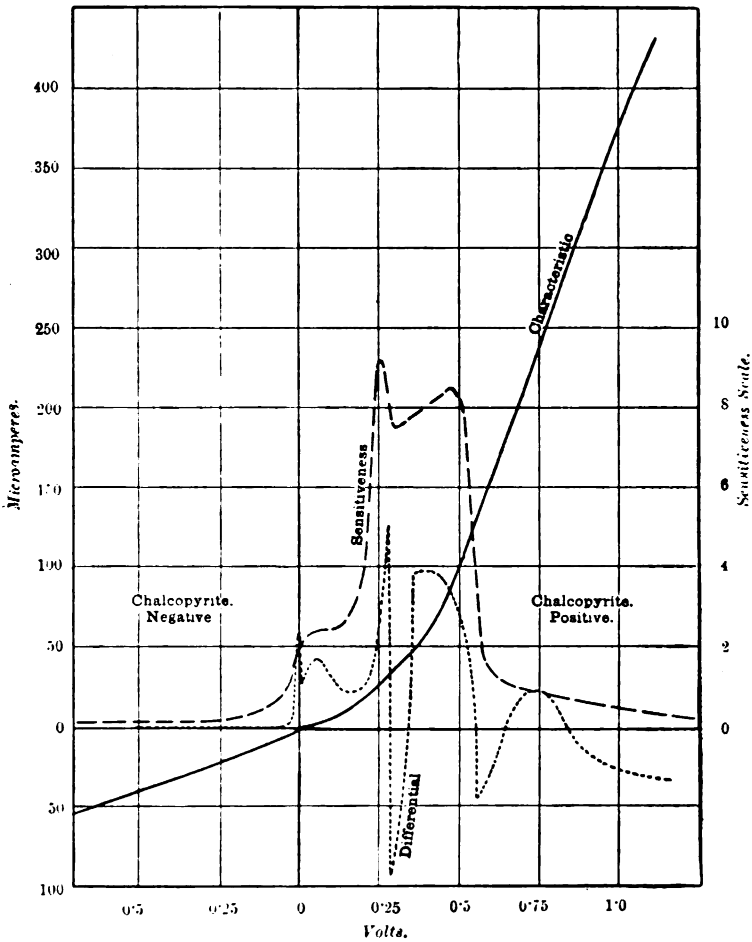


MOLYBDENITE DETECTOR.

each detector in this table indicate the relative sensitivity of these detectors to one another, the true relative sensitiveness

* "The Comparative Sensitiveness of Some Common Detectors used in Radiotelegraphy," by L. W. Austin, "Bulletin" of the Bureau of Standards, Washington, Vol. 6, pp. 527-542, November, 1910.

of the magnetic detector when worked under its own special best conditions would be expressed by a number greater than unity, since it belongs to a different class, being a "current" detector where the others are all "potential" detectors, and,



PERIKON DETECTOR.

therefore, not strictly comparable with them under the same conditions of testing apparatus, its use in these tests being merely for a convenient and reliable standard of comparison for adjusting the apparatus previous to each experiment.

Notes on Curves.

In comparing the sensitiveness and differential curves here given, it should be remembered that as far as the operation of the detector is concerned, it does not matter whether the ordinate of the differential is positive or negative (*i.e.*, whether the current through the telephone is increased or decreased by the application of the oscillation), and hence all negative ordinates of the second differential curve should be reversed in sign and a species of envelope curve drawn in. This curve should then be compared with the sensitiveness curve for any points of resemblance.

Curve I.—Molybdenite-Copper Detector.

On the whole a general sort of agreement may be seen between the sensitiveness and second differential curves, with the exception of the minima in the differential at 0·36 volt positive, and in the neighbourhood of zero voltage, which have no counterparts in the sensitiveness.

Curve II.—" Perikon " or Zincite-Chalcopyrite Detector.

In general shape there is a pretty good agreement between the two curves for this detector. A series of sensitiveness curves taken with this detector show how much the shape of this curve can vary with the particular contact and crystal employed, but, although differing in details of shape and size of maxima, they all show in general form a maximum semsitiveness with about 0·3 to 0·5 volt, chalcopyrite positive.

Curve III.—Zincite-Bornite Detector.

The agreement between the two curves is not quite so good in this case, but the differential, however, shows up the general features of the sensitiveness curve, with the exception of the maximum at 3·5 volts, which is relatively too large.*

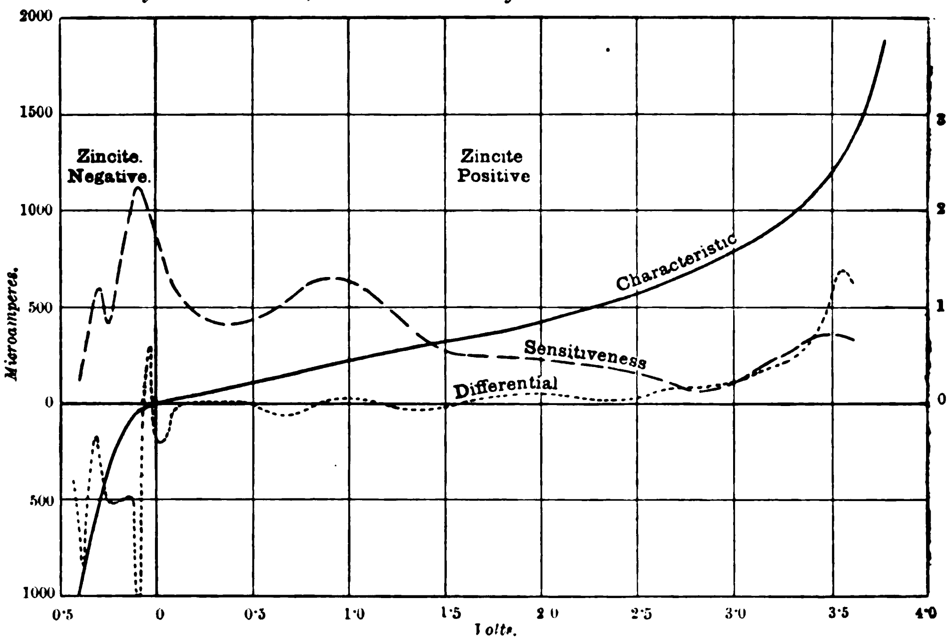
It was found with this detector that subjecting it to a complete cycle of impressed or boosting voltage had a fairly definite effect on the sensitiveness curve, the limits of the cycle being about those of practical working for this detector.

The cycle was commenced at 3·6 volts, zincite +, and carried through zero to — 0·4 volt, and then back to + 3·6 volts, the starting point of the cycle.

It was found that the maximum sensitiveness was unaffected by the cycle, but that a remarkable " kink " occurred in the curve before this maximum sensitiveness was reached on the return part of the cycle, which is not found at all in the first

* *See* also note to Curve VI. for Fleming valve.

half of the curve. This effect was observed on several crystals. It should also be noted that besides affecting the shape of the sensitiveness curve as above, the characteristic curve in the case of most detectors is also altered to a greater or less degree depending on the nature of the crystal employed, and consequently in most instances the characteristic curve for the cycle encloses an area, which is somewhat suggestive of a hysteresis effect, as also noticed by Pierce.*



ZINCITE-BORNITE DETECTOR.

The Tellurium-Aluminium Detector.

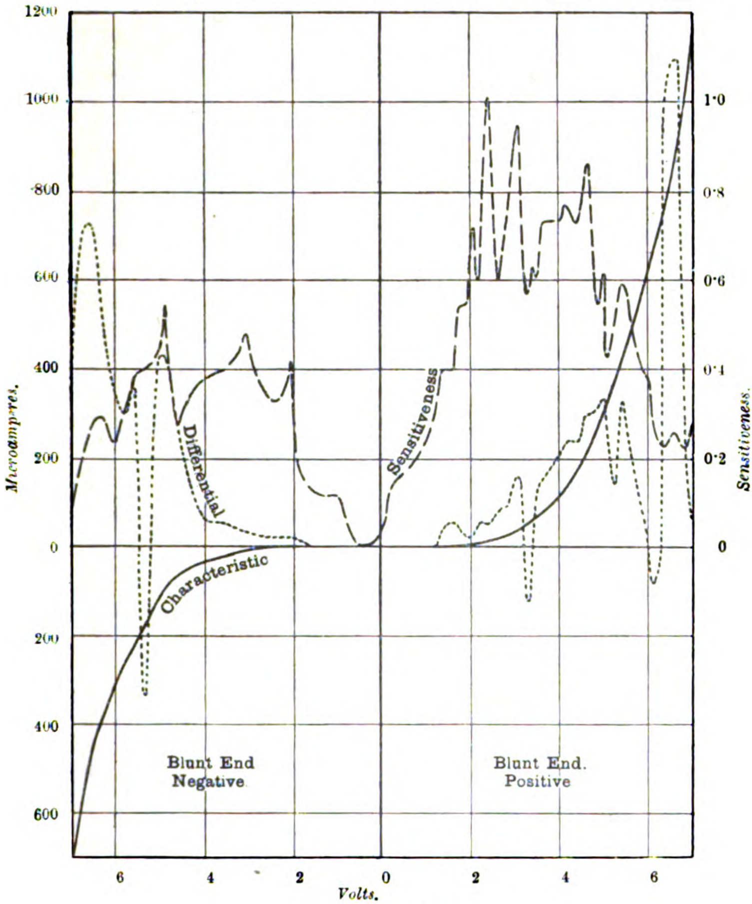
This detector does not show much similarity between the sensitiveness and differential curves; in fact, the largest ordinates of the differential occur where the sensitiveness is zero. This seems to indicate that more than one action is at work, and that in all probability they are acting in opposition, at all events, at zero voltage—otherwise it is difficult to see why such a sudden change of curvature of the characteristic as occurs

* "Crystal Rectifiers for Electric Currents and Electric Oscillations," by G. W. Pierce, "Physical Review," Vol 25, pp. 31-60; also "Conduction of Electricity at Contacts of Dissimilar Solids," R. H. Goddard, "Phys. Review," Vol. 34, pp. 423-451; "Electrician," Vol. 69, pp. 778-781, Aug., 1912.

there should not produce a considerable sensitiveness at that point. This detector also showed a marked tendency to behave like a filings coherer—the reception of a signal often causing its resistance to decrease very considerably.

Curve IV.—Carborundum.

The period of disturbance of the differential roughly coincides with the period of sensitiveness of this detector, but for

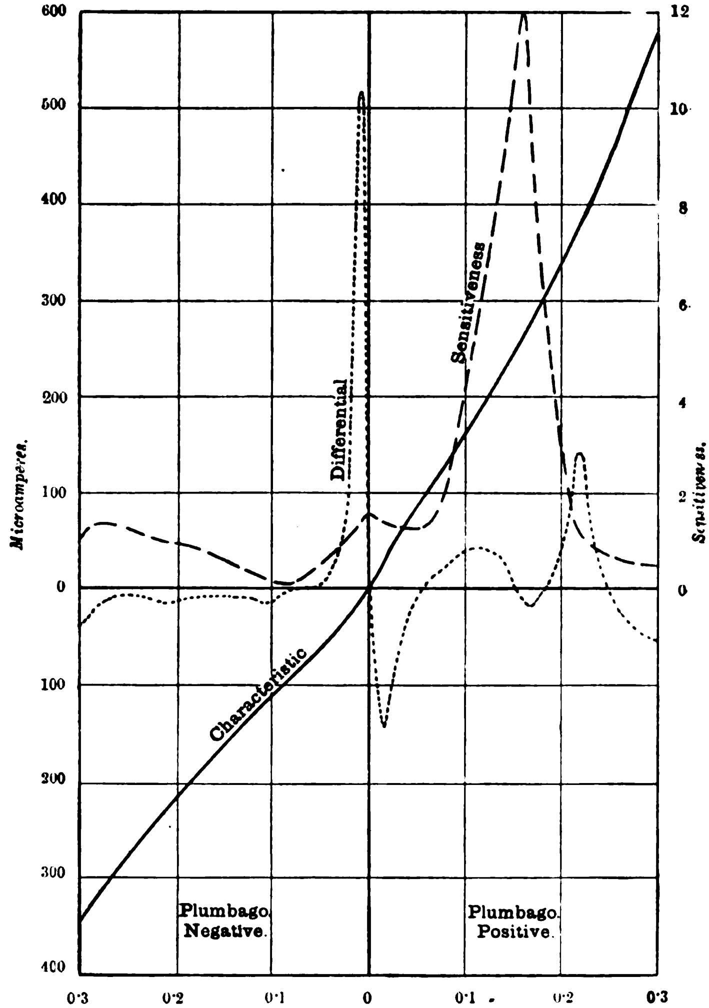


CARBORUNDUM.

both positive and negative voltages the differential shows large ordinates for the high voltages, where the sensitiveness is falling off.

Curve V.—Galena-Plumbago Detector.

Similar remarks apply to this as to the tellurium-aluminium detector.

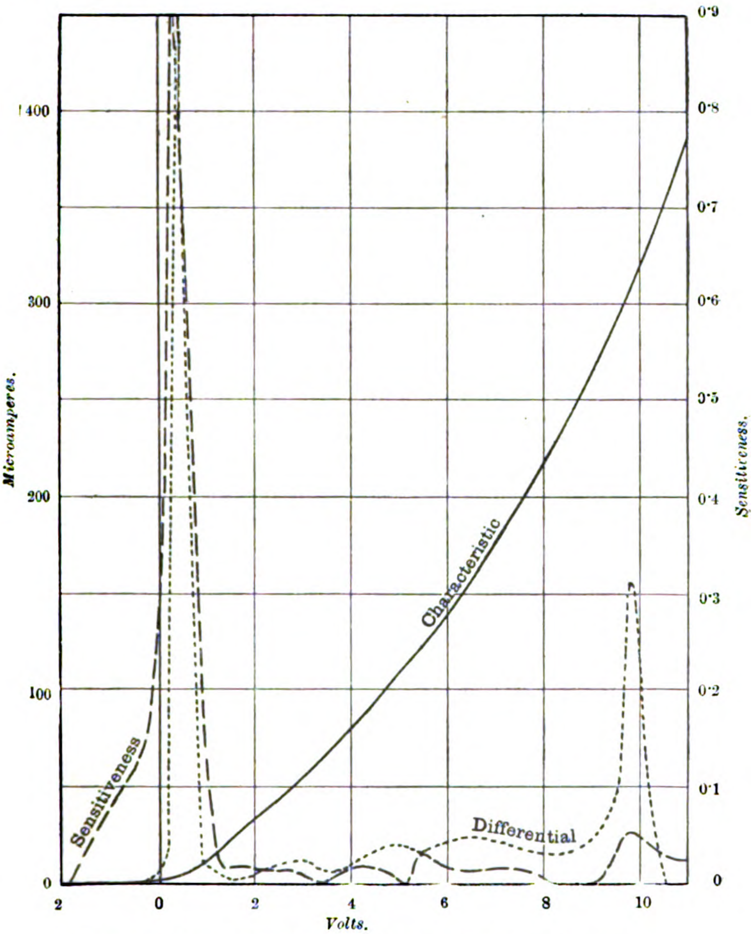


Galena Detector.

Electrolytic Detector.

The detector tested was supplied by a German firm, and was totally enclosed and sealed. Apart from a large ordinate at zero voltage, where the sensitiveness is zero, the region of

disturbance of the differential roughly agrees with the region of maximum sensitiveness of the detector, although individual details do not show up at all. A remarkable bend in the characteristic occurs at zero voltage, although the sensitiveness is there zero.



FLEMING OSCILLATION VALVE—CARBON FILAMENT (12-VOLT).
Voltage measured from Plate to Negative Leg of Lamp Filament.

Electrolytic Detector, with Nitric Acid Electrolyte.

This detector consisted of Wollaston wire dipping into nitric acid. The sensitiveness reached four times the figure for the

previous detector, and a great difference between the shapes of the characteristic curves for these two detectors was noted. Otherwise similar remarks apply as to the previous detector.

Curve VI.—Fleming Carbon-Filament Oscillation Valve.

A curve taken on a 12-volt carbon filament valve is appended to show the agreement that can be obtained between the sensitiveness and differential curves in this case, the similarity between the two curves being very great with the exception of the peak near 10 volts.

This non-agreement between the two curves at high voltages (and, therefore, large currents), which takes the form of the differential curve indications being relatively too large, and which may also be noticed in some of the other curves, may be caused by a falling off of the sensitiveness of the telephone to small variations when large steady currents are passing through it.

To sum up, the curves show that in the majority of cases the second differential of the characteristic gives a good indication of the sensitiveness of the detector, as in the case of the oscillation valve, but that some detectors, notably the galena and electrolytic, seem to show that there are other actions at work which may oppose the one depending on the curvature of the characteristic. Since this also occurs with one of the electrolytic detectors it perhaps points out that this other action may be electrolytic in nature.

An appendix gives reference to a number of recent Papers bearing on this subject, and may perhaps be of some use to those interested in the development of these crystal detectors.

Finally, my best thanks are due to Dr. J. A. Fleming, in whose laboratories at University College, London, and under whose guidance the above tests were carried out.

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ABSTRACT.

The Paper describes some experiments recently conducted on different types of wireless detectors, which were undertaken with a view to finding out whether any definite relation could be traced between the sensitiveness and characteristic (or, volt-ampere) curves of a detector, such, for example, as has been known for some time to exist in the case of the Fleming oscillation valve detector between the sensitiveness curve, and a curve plotted from the second differential of the characteristic of the valve.

A table giving the maximum measured sensitiveness of the various detectors tested both when used as simple rectifiers as well as with a steady boosting voltage, shows that of the ordinary crystal detectors the "Perikon" or zincite-chalcopyrite combination has the greatest sensitiveness for ordinary stable adjustments, but that the galena-plumbago detector may greatly surpass it when used with extremely light contact, although in this condition it is of necessity not so reliable as the Perikon, which can be used with a fairly good contact pressure between the crystals.

Sample curves for some of the most common detectors are included in the Paper and show that in some cases a fairly good agreement exists between the sensitiveness curve of a detector and the second differential of its characteristic, this being most notably the case in the more stable of the crystal detectors, but at the same time it is abundantly evident that the flexure of the characteristic curve cannot be the only cause of the response of a detector to wireless signals, but that at least a second action must also be present, as in some cases—notably the electrolytic detector—it was observed that the

maximum ordinates on the second differential, *i.e.*, the point of greatest change of flexure of the characteristic curve, were at places where the measured sensitiveness was either zero or extremely small, showing that there are probably at least two actions opposing one another at this point. As this is most prominent in the case of the electrolytic detector, it perhaps suggests that this additional action when present in other detectors is electrolytic in nature, or that the received oscillations when superimposed on the direct-current boosting voltage partake of the properties of some " trigger " action (such as in the Zehnder trigger cymoscope). This view is supported by experiments with detectors of the tellurium-aluminium type.

In the course of the tests it was found necessary to spend some time in devising a suitable and reliable means of testing these detectors in a quick and easy manner. The " tilting coil " method, due to Dr. J. A. Fleming, described before this Society in 1909, was used as a starting point in the investigation, but was found not to be altogether satisfactory as with very sensitive detectors no position of silence could be found, thus demonstrating the inaccuracy involved in taking the received energy as proportional to the projected area of the receiving coil parallel to the plane of transmitter. The arrangement finally adopted, which may be called the " sliding coil " method, consists of two flat spiral coils mounted vertically, one (the transmitter) being fixed and the other capable of sliding horizontally along a scale.

The transmitting coil was excited by a buzzer contact, and the receiving coil was tuned to the same frequency by means of a variable air condenser. The detector and receiving telephones were connected across this condenser. This method has the advantage of enabling a very accurate calibration of the coils to be obtained using measured high-frequency alternating current. The calibration curve is then used to find the sensitiveness of any detector under test by drawing out the receiving coil until silence is obtained in the telephones and taking one detector (in these tests the Marconi magnetic) as a standard of comparison in each case. In this way the " personal equation " due to the experimenter and variations of ear sensibility may be almost completely eliminated.

DISCUSSION.

Prof. J. A. Fleming thought it was impossible to avoid speculation in connection with these interesting effects. He thought the real cause of the asymmetry in the case of valve-detectors was the emission of electrons from the hot filament. The laws governing this emission had been shown to be analogous to those governing the evaporation of water. Consequently it could be increased by applying a negative pressure. Some of the irregularities observed might be the equivalent phenomenon to boiling with bumping. He thought it would be of interest if Mr. Coursey's experiments could be repeated at different temperatures.

Mr. W. Duddell thought the investigation was extremely difficult, as one could not be certain that a given pair of crystals would always work in the same way. As far as the crystal detectors were concerned, he did not think the relation between the sensitiveness and the second differential of the characteristic curve was very clear. The determination of the second differential curve was very difficult unless the original observations could be reproduced very accurately.

Dr. W. Eccles thought that the agreement between the characteristic curves given in the Paper and those given for the same pairs of substances by previous experimenters showed that the shapes of the curves were undoubtedly dependent on the physical properties of the substances more than on the configuration of the contact. A theory of the cause of the shape of the characteristic curve had been published. The present Paper dealt, however, with quite a different matter, namely the connection between the shape of the curve and the sensitiveness of the detector; the theory of this was also known and was chiefly a matter of geometry. This geometrical theory yielded the following conclusions: First, if the sensitiveness of a detector was measured by the proportion of alternating current rectified, the sensitiveness was greatest near sharp bends of the characteristic curve, that was to say, where the second differential coefficient was largest, though not exactly at that point. Second, if the sensitiveness was measured by an energy method, it was a maximum at a point between the steepest part of the curve and the maximum of the second differential coefficient. It was probable that the first type of sensitiveness was involved in this Paper, and thus on the whole the geometrical theory was corroborated.

Mr. D. Owen asked what the measure of sensitiveness adopted by the author was; also if he could furnish figures as to the actual voltage across the contact at limiting silence in the telephone. He would suggest that prediction of the sensitiveness of a particular individual detector might be more simply made from inspection of an *alternating* voltage characteristic—that was, from a graph in which the direct current through the contact under an alternating low-frequency voltage was plotted as ordinate against the alternating volts as abscissa. This amounted to integrating the two distinct effects observed in direct voltage characteristics, namely, the non-linear property and the asymmetry for positive and negative voltages. Sensitiveness of a detector might be expected to be measured simply by the slope of the straight line joining the origin to a point on the alternating characteristic. A slide was shown of the knee-shaped alternating characteristics obtained from a galena-chalcopyrite contact.

Dr. R. S. Willows thought the characteristic curve given for the oscillation valve was not a good one for the purpose. In his experience of the valve, the detector was most sensitive when the voltage was boosted up until it was just at the point of producing ionisation by collision, to which the second rapid rise in the curve was due. By making use of the electronic emission from hot lime a better cathode was obtained than in the case of the carbon filament.

Prof. G. W. O. Howe thought too much weight had been attached to slight changes in the characteristics of the crystal detectors. A minute alteration in the point of contact greatly altered the properties of the detector. It would be interesting to hear if Mr. Coursey had repeated his observations. Sometimes a polarising voltage greatly improved the sensitiveness while the slightest shifting of the point might render the polarising voltage quite useless. Since they were so sensitive to slight changes in the conditions, much stress should not be laid on little variations in the characteristic. If two detectors were used side by side, it might happen that one of them was most sensitive to waves sent out from one station, while the other might be most sensitive to those from some other station.

Mr. E. H. Rayner thought that some of the irregularities in the characteristics might easily be due to vibration and similar disturbances. He thought it would be useful if the author gave some idea of the resistance of the detectors at maximum sensitiveness, so that one could see if the telephone was doing justice to the detector.

Prof. Fortescue had also found crystal detectors very sensitive to

disturbances, such, for example, as the passing of a motor 'bus near the laboratory, and he thought that slight changes from one characteristic to another, rather than real changes in the characteristic, would account for some of the irregularities shown.

The AUTHOR, in reply, agreed with Dr. Fleming that it would probably be of great value to conduct experiments similar to those described in the Paper at various temperatures, both above and below the ordinary, as in this manner a better insight might be obtained, from the experimental point of view, as to the part played by thermal effects at the contact of the two crystals. The fact that it was unnecessary to have at least one of the contacts of low thermal conductivity was supported by the tests on the tellurium-aluminium detector, in which both materials were metals. These tests seemed to show, however, that the mode of operation of such detectors differed considerably from that of the more ordinary crystal detectors, the type of curves obtained more resembling those of the electrolytic and other detectors in which very little agreement could be traced between the sensitiveness and differential curves. The response of the detector under oscillations seemed in this case to resemble that of a filings coherer, again suggesting something in the nature of a "trigger" action. In reply to Mr. Duddell and Prof. Howe steps were taken in the tests to ascertain to what extent the characteristic curves could be repeated, and it was found that with the "good-contact" detectors, or those operating with moderately firm contact between the crystals, *e.g.*, the "Perikon," almost exact repetition of the curves could be obtained on different occasions, and that although changing the crystals or points of contact altered the scale of the curves, yet in general the main features were present. With the more "imperfect," or "loose-contact," detectors, however, the repetitions were not nearly so good, and it was largely with those detectors that the agreement between the two curves was not so pronounced—possibly for this reason. As mentioned by Dr. Eccles, the process of taking the second differential of the characteristic certainly did magnify the errors of experiment, but as stated in the Paper it was not intended to compare the sensitiveness curve with all the details of the differential, but to draw in a species of envelope curve to the differential. That method would obviously tend to smooth out all superfluous irregularities. The Author was also glad to note that the experimental curves given in the Paper agree with the theoretical ones deduced by Dr. Eccles in his Paper on that subject. It would be of interest if the numerical values of the constants involved in the equations there given could be determined for a particular crystal contact, and the same crystals then tested electrically, and the experimental curve so obtained compared with the theoretical one. The curve shown by Mr. Owen was of interest, but really expressed the response of a detector to signals of various strengths, and did not give the sensitiveness as defined in the Paper. In reply to Mr. Rayner, the lumps in the characteristic cannot well be due to vibration and similar disturbances in the firm contact detectors, while in testing those requiring a lighter contact precautions were taken to insulate the detectors as far as possible from mechanical shocks. The resistance of the detectors at their points of maximum sensitiveness varied from a few hundred to about 10,000 ohms or more, depending on the crystals used.

XI. *A Water Model of the Musical Electric Arc. By* W. DUDDELL, *F.R.S.*

In the model the arc is represented by a mushroom valve. The pressure of the valve on its seat is so arranged that the pressure tending to re-seat the valve diminishes very rapidly as the valve lifts. Water is admitted beneath the valve, flows through the valve into the vessel which contains it, and overflows. In order to indicate the difference of pressure on the two sides of the valve which represents the arc a glass pressure-column is introduced into the pipe leading to the valve and quite close to it. As the water overflows freely from the tank in which the valve is immersed, the pressure on this side of the valve may be taken as our zero of reference, and consequently the height of the water column in the pressure tube above or below the level of the overflow gives the pressure underneath the valve.

If water be admitted below the valve the pressure in the pressure tube rises to a high value ; finally, the valve lifts, *i.e.*, the arc is struck, but the pressure still remains high. If, however, the flow of water is increased, the valve will open considerably and the pressure below it will decrease. If nicely adjusted this effect can be made to take place over a considerable range.

If instead of connecting a pressure tube of small bore indicating the pressure on the underneath side of the valve a large diameter tube be introduced so that the water column in it has a periodic time of its own and is able to oscillate similarly to the condenser circuit shunting the arc, oscillations will be set up in this column, and if the periodic time of the liquid in this column be altered, the period of the oscillations will be altered ; this can easily be done by connecting air vessels of different capacity to the open end of the tube, so altering the controlling force acting on the water, in other words, altering the capacity of the circuit shunting the arc.

With this water model a great many of the properties of arcs both intermittent and oscillating can easily be shown. The one point of difficulty in constructing the model is to obtain a force acting on the valve which decreases rapidly when the valve lifts and which occasions no friction. So far the only successful method which the author has tried is to hang from the underneath side of the valve a piece of soft iron which nearly touches the pole of a small electromagnet. This gives a force which without any friction rapidly decreases as the valve lifts and works very well.

XII. *Some Further Experiments with Liquid Drops and Globules.* *By* C. R. Darling, *A.R.C.S.*

1. *Communicating Drops.*—An arrangement similar to that used for bringing the interior of two soap bubbles into communication is filled with orthotoluidine, and the extremities placed under water, one being at a lower level than the other. A large drop is formed on the upper branch and a small one on the lower, and on opening the communicating tap the smaller drop passes into the larger one, in spite of the tendency of the larger drop to siphon over into the smaller. When a drop of about half the diameter of the large drop is formed on the lower end, the direction of flow is reversed, the tendency to siphon over now prevailing owing to the diminished curvature of the drop formed at the lower level. A condition of equilibrium between two drops of unequal sizes can be established by trial, and an approximate value of the interfacial tension between the two liquids obtained from the curvature of the drops, and the difference of hydrostatic pressure between them. Orthotoluidine is slightly denser than water below 24°C.

2. *The Structure of Liquid Jets.*—Orthotoluidine is discharged from a cistern through a vertical tube terminating under water, the rate of flow being controlled by a tap. Owing to the slow descent of the escaping liquid, many of the features of a liquid jet, such as the breaking away of drops and their subsequent distortion, are made visible to the eye.

3. *Liquid Spheres Enclosed in a Skin of Another Liquid.*—Aniline is placed beneath a layer of water about 4·5 cm. in height, and a glass tube of 3 mm. bore, open at both ends, is passed through the water into the aniline. On raising the tube, a skin of aniline adheres to the end, and is inflated by the water in the tube, forming a sphere. On removing the tube gently, the sphere remains clinging to the upper surface of the water.

By covering water with a layer of dimethyl-aniline 2·5 cm. deep, and following the same procedure, a sphere of the latter liquid, encased in a skin of water, is formed. On withdrawing the tube, the compound sphere falls to the joining surface of the two liquids, and after remaining there a few seconds is projected violently into the water below into which the skin then merges. The resulting drop of dimethyl-aniline then

rises to the joining surface and breaks through into the upper liquid.

4. *Mixed Vapour and Liquid Drops.*—When a heavy, volatile liquid is heated below water, the vapour bubbles on escaping detach a quantity of the liquid, and the composite drops rise through the water. On nearing the surface the vapour contracts or partially condenses owing to cooling, thus increasing the density of the compound drops, which then sink; but on reaching a warmer level the former density is restored, so that the drops rise again. This process may be repeated several times before the drops reach the surface, when the vapour escapes and the detached liquid falls back into the mass at the bottom. Chloroform shows this action, but monobrombenzene gives the best results.

5. *Expanding Globules.*—When a globule of liquid is floating on the surface of water, and a drop of a second liquid is allowed to fall into it, the globule expands outwards in all directions, often with such violence as to be broken up into several portions. This is well shown when a drop of quinoline is permitted to fall on to a floating globule of dimethyl-aniline.

6. *Combination of Floating Globules.*—Scattered globules of some liquids floating upon water show no tendency to unite; in other cases, however, the contrary holds true. A striking example of the latter is obtained by pouring a quantity of orthotoluidine on to the surface of water, allowing it to break into globules, and then forming a single large globule of dimethyl-aniline on the same surface. The globules of orthotoluidine are absorbed, one by one, by the other large globule, which sends out a protuberance which joins on to the adjacent globule, and then shrinks back into the main mass. This action shows some resemblance to the movements of certain of the lower organisms.

XIII. *On Vibration Galvanometers of Low Effective Resistance.* *By* Albert Campbell, B.A.

Received December 15, 1913.

Most of the vibration galvanometers in use at the present time have moderately high effective resistance (of the order of 500 ohms). For many purposes a resistance much lower than this is desirable in order to obtain the most sensitive conditions of measurement. Confining my attention to galvanometers of moving coil type, I have recently succeeded in making coils of much lower effective resistance which still give very good current sensitivity. In the earliest vibration galvanometers which I constructed some years ago (1906) small and narrow coils were used, but very soon afterwards I increased the size of the coils in order to increase the area of the mirror and the robustness of the instrument. Due to their comparatively low sensitivity, these early galvanometers had not high effective resistance. By and by Mr. Duddell* so raised the standard of sensitivity with his single loop galvanometer, that he found a very considerable rise of effective resistance due to the back voltage, and he pointed out that this was desirable from the point of view of dynamical efficiency.

Accordingly I reverted to the lighter coils, and the results given in this Paper show the sensitivities that can be obtained by making the coil very small without at the same time unduly diminishing the size of the mirror.

To make the statement of these results clear it is necessary to give first a short account of the mathematical theory. I shall in the main follow Wenner's system,† as he has treated the subject very thoroughly.

If we assume that the damping is proportional to the angular velocity, the equation of motion of the coil is

$$mk^2\ddot{\theta}+b\dot{\theta}+c\theta=g\mathrm{I}_{\mathrm{max.}}\cos\omega t, \quad . \quad . \quad . \quad . \quad (1)$$

where the symbols have the following meanings :—

mk^2, moment of inertia,

b, damping constant,

* "Phil. Mag.," July, 1909. and "Proc." Phys. Soc., Vol. 21, p. 774.

† "Bulletin" Bureau of Standards, Vol. 6, p. 347, 1910. *See also* Butterworth, "Proc." Phys. Soc., Vol. 24, p. 75, February, 1912, and Haworth, "Proc." Phys. Soc., Vol. 25, p. 264, May, 1913.

c, control constant,
g, deflectional constant,
θ, the angular displacement at time t,
$I_{max.}$, the maximum value (in amperes) of sine wave alternating current (passing through the coil) of frequency n,
$\omega=2\pi n$.

When a steady state has been reached

$$\theta=\frac{I_{max.}\, g \sin(\omega t+\varepsilon)}{\sqrt{(c-\omega^2mk^2)^2+\omega^2b^2}}, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where
$$\cot\varepsilon=\omega b/(c-\omega^2mk^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If φ is the amplitude, 2φ being the whole angle of vibration of the coil, then

$$\varphi=\frac{I_{max}\, g}{\sqrt{(c-\omega^2mk^2)^2+\omega^2b^2}}=\frac{Ig\sqrt{2}}{\sqrt{(c-\omega^2mk^2)^2+\omega^2b^2}}, \quad . \quad (4)$$

where I is the effective value of the current.

Case 1.—If resonance (*i.e.*, maximum φ) be obtained by altering the control c, keeping n_1 the frequency of the source constant, then

$$\omega_1^2=c/mk^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\varphi=gI\sqrt{2}/\omega b \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and
$$\varepsilon=0.$$

Case 2.—If resonance be obtained by altering the frequency of the source while c is kept constant, then

$$\omega_2^2=\frac{c}{mk^2}-\frac{1}{2}\left(\frac{b}{mk^2}\right)^2=\omega_1^2-\frac{1}{2}\left(\frac{b}{mk^2}\right)^2. \quad . \quad . \quad . \quad (7)$$

Case 3.—If the current be cut off, the vibration will gradually settle down to zero, the equation of motion now being

$$mk^2\ddot{\theta}+b\dot{\theta}+c\theta=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and if the free frequency be n_0, we have

$$\omega_0^2=\frac{c}{mk^2}-\frac{1}{4}\left(\frac{b}{mk^2}\right)^2=\omega_1^2-\frac{1}{4}\left(\frac{b}{mk^2}\right)^2 \quad . \quad . \quad . \quad (9)$$

If the amplitude φ_0 at time $t=0$ falls to φ_2 at time t_2, then

$$t_2=\frac{2mk^2}{b}\log_e\left(\frac{\varphi_0}{\varphi_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When $\varphi_0/\varphi_2 = e$, then τ, which we may call the *amplitude time constant*, is given by

$$\tau = \frac{2mk^2}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence

$$\omega_0{}^2 = \omega_1{}^2 - \frac{1}{\tau^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11\text{A})$$

and

$$\omega_2{}^2 = \omega_1{}^2 - \frac{2}{\tau^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11\text{B})$$

As Prof. B. O. Peirce* has shown (for a ballistic galvanometer), we can find the value of mk^2/b immediately from equation (10) by observing the time taken for the amplitude to fall to a definite fraction (say $\frac{1}{2}$) of its initial value.

With the very small damping occurring in practice, ω_0, ω_1 and ω_2 are all nearly equal to one another. In what follows I shall consider only the resonance of Case 1.†

Let the moving coil have N turns of mean area s, the flux density in the magnet gap being $\mathfrak{B}$. Also let the following symbols be used :—

h, direct current sensitivity,‡ *mm* at 1 m. per microampere ;
σ, alternating current sensitivity, *mm* at 1 m. per microampere ;
q, alternating voltage sensitivity, *mm* at 1 m. per microvolt ;
R, " dead " resistance of coil ;
R′, effective resistance of coil.

Let the power sensitivity be taken as the square of the deflection (*mm* at 1 m.) divided by the power in micromicrowatts.

Quantities observed.—In testing the coils, the frequency of the source was kept constant (at 100 or 200 ∿ per second), a tuning fork hummer being used as the source. By altering the length and tension of the suspending strips the galvanometer was tuned to resonance and the current and voltage sensitivities σ and q were observed. The effective resistance is given by $R' = \sigma/q$. Then the direct current sensitivity was observed and also the direct current resistance R. Also, when the current was broken, observations were made upon the

* " Proc." American Acad., Vol. 44, No. 2, 1908.

† On the two kinds of resonance, *see* M. Wien, " Wied. Ann.," Vol. 58, p. 125, 1896.

‡ This is half of Wenner's sensitivity, as he defines it for *reversal* of direct current.

dying down deflection, to give τ the amplitude time constant. [This must be kept distinct from the electrical time constant of the circuit.]

Deduction of Constants.—When the coil has a vibration amplitude of φ radians, the scale deflection (*mm* at 1 m.) will be 4,000 φ. It is easy, from the equations given above, to deduce the following relations :—

$$R'=\sigma/q, \qquad (12)$$

$$g \doteq 9000(R'-R)/\sigma n.. \qquad (13)$$

Also $$g=\mathfrak{B}sN/10, \qquad (14)$$

$$c=2g/1{,}000h, \qquad (15)$$

$$mk^2=c/\omega_1^2, \qquad (16)$$

$$b=\frac{2\sqrt{2ch}}{\omega_1\sigma} \doteq \frac{2{\cdot}83c}{\omega_1\,.\,\sigma/h} \qquad (17)$$

Also $$b=2mk^2/\tau. \qquad (18)$$

We see also that

$$\sigma/h=\omega_1\tau\sqrt{2}, \qquad (19)$$

i.e., resonance magnification $\doteq$ 8·8 frequency $\times$ (amplitude time constant).

Thus from the observed quantities we can deduce all the constants (mk^2, b, c and g) of the equation of motion. Equation (14) gives an independent determination of g, when the air-gap flux density $\mathfrak{B}$ can be measured and the dimensions of the coil are known. The area s, however, cannot be very accurately measured directly. Approximate direct measurements are included in the table below. The damping constant b can also be determined by two independent methods [equations (17) and (18)].

Results of Tests.—The complete results for a number of coils are given in the following table. These have been chosen so as to show variety in number of turns, size of mirror, magnetic field and resonance frequency. For all the coils unifilar suspensions* of phosphor bronze strip were used, and in most of them the greater part of the resistance R was due to the suspension strips. The magnetic field was produced by a permanent magnet ; it could be altered from 1,650 to 2,500 by inserting a small soft iron pole piece. As the various coils were not all tested in the same region of the air-gap the values

* *See* "Proc." Phys. Soc., Vol. 25, p. 203, April, 1913.

TABLE

Coil	A.	B1.	B2.	C.	D1.	D2.	E1.	E2.	F.	G.	H.	J1.	J2.
N, turns	1·0	2·5	2·5	4·5	4·5	4·5	6·5	6·5	10·5	20·5	40	40	40
s, mean area, sq. cm. (app.)	0·12	0·07	0·07	0·04	0·06	0·06	0·10	0·10	0·10	0·09	0·17	0·07	0·07
Mirror area, sq. mm.	2·1	5·2	2·8	2·1	3·5	3·5	7·2	2·1	16	7·5	5·0	2·9	7·2
Magnet gap 𝔅 (approx.)	1,650	2,500	2,500	2,500	1,650	2,500	2,500	2,500	2,500	2,500	2,500	(1,650)	(2,700)
n_1, resonance freq. ∼ per sec.	100	100	200	100	100	100	100	100	100	100	100	100	100
h, direct current sensitivity	0·0095	0·043	0·0146	0·167	0·033	0·047	0·082	0·113	0·080	0·083	0·102	0·105	0·165
σ, alt. current sensitivity	6·9	21·7	10·0	56	21	33	37	50	50	61	130	130	160
q, alt. voltage sensitivity	0·80	1·02	0·62	1·16	1·49	1·07	0·50	0·50	0·285	0·174	0·056	0·167	0·104
Power sensitivity	5·3	22·0	6·2	65·0	31·0	35·0	18·0	25·0	14·0	10·6	7·3	21·0	17·0
R, "dead" resistance, ohms	7·1	6·0	4·6	10·5	5·7	5·7	6·0	6·0	7·0	8·8	14·2	14·5	14·5
R′, effective resist., ohms	8·6	20·1	16·1	48·0	14·1	31·0	74	100	175	351	2,300	780	1,540
σ/h, resonance magnification	725	505	685	335	650	700	450	442	620	740	1,270	1,240	970
$10^6 mk^2$, moment of inertia	10·4	6·9	4·5	1·77	5·6	7·4	10·2	7·6	19·2	31	91	26	26
$10^6 b$, damping constant	25·6	23·2	23·2	9·4	15·5	18·8	40	30	55	75	158	37	49
c, control constant	4·1	2·72	7·1	0·70	2·22	2·94	4·0	3·0	7·6	12·2	36·1	10·1	10·4
g, deflectional constant	19·6	58·5	51·8	60	36·7	69	165	169	303	503	1,840	530	860
τ, amplitude time con., secs	0·81	0·57	0·38	0·38	0·73	0·79	0·51	0·49	0·70	0·83	1·43	1·4	1·1
τ_2, calculated, secs.	0·6	0·4	...	...	0·5	0·6	...	...	...	0·6	1·0	1·0	...
τ_2, roughly observed, secs.	0·8	0·6	...	...	0·6	0·6	...	...	...	0·7	0·9	0·9	...

of $\mathfrak{B}$ are only approximate. A high degree of accuracy was not aimed at in the observations, and hence close consistency is not to be expected in the results. The coils were of various lengths, from 9 mm. to 22 mm.

Remarks on the Results.—It will be seen from the table that it is possible to obtain good current sensitivity with coils having effective resistances as low as 30 or 40 ohms. Also we see by two instances how much the sensitivity depends on the size of the mirror employed. It should be remarked that some of the coils (C for example) are of later and better design than others; thus the average result is not the best that can be obtained. A coil like B1 shows extremely high sensitivity on a low resistance bridge, while one like H is eminently suitable for high resistance or inductance measurements.

The last three lines of the table give the values of the amplitude time constant τ, and also, for several of the coils, τ_2, the time taken for the deflection to fall to half value, as calculated from the other constants and also as roughly determined by actual observations with a stop-watch. The agreement is as near as could be expected. More accurate agreement, however, was got by observing the time taken for the deflection to fall to one-tenth of its initial value.

Selectiveness.—One well-known advantage of resonance instruments is that they are *selective;* they are much more sensitive for the frequency to which they are tuned than for any other. Thus, when harmonic components are preseet in the current, their effect is small. This selective power, however, varies considerably in different instruments. From the constants of any galvanometer the selectivity can readily be estimated (*see* Wenner, *loc. cit.*).

Mr. D. W. Dye has pointed out to me that the current sensitivity is almost entirely determined by the absolute value of the motion time constant. If ω_1 gives resonance, and $\gamma\omega_1$ corresponds to any other frequency, γn_1, then the ratio of the current sensitivities at the frequencies n_1 and γn_1

$$=\sqrt{(1-\gamma^2)^2\omega_1^2(mk^2/b)^2+\gamma^2}. \quad . \quad . \quad . \quad . \quad (20)$$

Usually the first term is much the larger, and hence current sensitivity ratio

$$\doteqdot \pm(1-\gamma^2)\omega_1 mk^2/b$$

$$\doteqdot \pm(1-\gamma^2)\frac{\omega_1}{2}\times(\text{time constant}). \quad . \quad . \quad . \quad (21)$$

Also $\tau=(\text{resonance magnification})/\omega_1\sqrt{2}$.

Hence current selectivity for n_1 as against γn_1

$$\doteqdot \pm(1-\gamma^2)\frac{1}{2\sqrt{2}}\times(\text{resonance magnification}). \quad . \quad (22)$$

Accordingly the magnification (as well as the amplitude time constant) gives an immediate criterion of the selectiveness. The time constant is, however, usually much easier to observe. The table shows clearly that the more massive coils (like H) are the best in this respect. In fact, the more sluggish a galvanometer is, the more selective will it be in current sensitivity. For voltage sensitivity quite different conditions hold, which need not be discussed here; they can be immediately deduced from Wenner's formulas.

ABSTRACT.

The mathematical theory of the motion of the moving coil of a vibration galvanometer is first given (partly following Wenner), and simple relations are shown to hold between the two resonance frequencies, the free frequency and the amplitude time constant. It is also shown how all the constants of the equation of motion can be deduced from observations of the direct and alternating-current sensitivities, the alternating voltage sensitivity and the "dead" resistance. A complete table of the observed and deduced constants is given for a series of very small coils, the number of turns in these varying from 1 to 40. The current sensitivities range from 6 mm. to 160 mm. at 1 m. per microampere at 100 ∿ per second, the corresponding effective resistances being about 9 and 1,500 ohms respectively. It is pointed out that the selectiveness (for given current) due to resonance is mainly determined by the absolute value of the "amplitude time constant"; the more sluggish a galvanometer is in settling to zero the more selective will it be.

DISCUSSION.

Dr. Russell thought everyone was indebted to Mr. Campbell for popularising the resonance galvanometer. In the present Paper the use of the amplitude time constant struck him as being very neat.

Prof. G. W. O. Howe pointed out that one method of determining the no-load losses of a motor was to cut off the supply and observe the rate of slowing down. From this the losses could be calculated, and the efficiency found. This, in effect, was what Mr. Campbell did with the vibration galvanometer, and there seemed to be many analogous points in the two processes.

Mr. Campbell thought Prof. Howe's analogy was extremely good. He added that a coil tested at 50 ∿ per second gave a current sensitivity of 500 mm. at 1 metre per microampere with an effective resistance of 2,500 ohms.

XIV. *Vacuum-tight Lead-seals for Leading-in Wires in Vitreous Silica and other Glasses.* *By* Henry J. S. Sand, *Ph.D., D.Sc.*

Received January 16, 1914.

It is generally accepted that the simplest plan for sealing a metallic conductor through the walls of a glass vessel so as to obtain a vacuum-tight joint consists in employing a metal having as nearly as possible the same coefficient of thermal expansion as the glass. Thus platinum wires are almost universally made use of in the manufacture of vacuum apparatus of all kinds. Suitable nickel-steel alloys are sometimes utilised in a similar manner.

In the case of quartz glass it seems practically hopeless to obtain a metal having the same coefficient of expansion as the glass over the whole range between ordinary temperature and the melting point of the metal or of the glass, and, so far as the author is aware, no really satisfactory method has hitherto been available for obtaining vacuum-tight joints for leading-in-wires in this glass.

We are here confronted with the problem to obtain a joint in spite of the difference of thermal expansion between the materials to be joined, and two main principles appear available for attaining this object.

(1) The elasticity of the metal or of the glass may be utilised. On this principle the author has, *e.g.*, made use of small pieces of elastic steel tube, which were shrunk into lead or soda-glass and several cathode-ray tubes were made, more than three years ago, which have held their vacuum unchanged to the present time.* The applicability of intermediate glass for obtaining seals between a metal having the same expansion-coefficient as the intermediate glass, but a slightly different one from that of the vessel into which the wire is to be sealed, is in a similar manner dependent on the elasticity of the glass.

(2) The plasticity of the metal conductor may be utilised. It appears probable that a method, which was first suggested by Margot,† and further perfected by Bastian,‡ is based mainly on this principle. Here a very thin copper conductor of either

* "Chem. News," 1910, 102, 166.
† Swiss patent. 14288.
‡ Bastian and Calvert patent, 22911/06.

circular or flat cross-section is sealed into lead-glass, the copper evidently being sufficiently plastic to adjust itself to the changes of shape undergone during cooling by the glass.

A method, which appears not to conform to either of these principles, by means of which copper wires were sealed into soda-glass and platinum tubes and wires into Jena-glass has been described by Burnside.* Here the leading-in-wire and the glass are simultaneously chilled by successive short immersions in suitable oils. It is not quite clear on what principle the efficiency of these seals is based, but it seems possible that the solidification of the glass, which, as is known, may be better described as a continuous increase of viscosity, lags behind the decrease of temperature. It may also be taken as certain that the glass in these seals is in a state of very considerable strain. Burnside believes his method to be applicable also to quartz-glass, but it is not apparent how it can be adapted to this purpose. In these seals, as also in those described under (2), it is possible that the volume compressibility and expansibility of the metal may play a certain role.

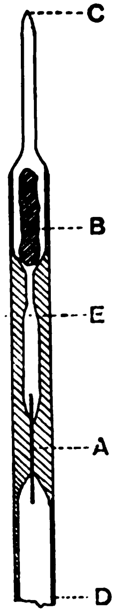
The seals now to be described depend for their efficiency on the high plasticity of lead and on the very great tenacity with which a clean surface of this metal will adhere to vitreous surfaces. The firmness with which clean metal surfaces may adhere to glass when solidified in contact with it has recently been pointed out by P. E. Shaw in a communication on "Sealing Metals" to this Society.†

The method as originally practised by me consisted in making the seal in a high vacuum. The lead was fused and the molten metal was filtered from the solid oxide by allowing it to run through a capillary. It was found desirable to employ three chambers: (1) The chamber in which the metal was melted; (2) a chamber connected with this by a capillary, in which, by agitating it, the metal was freed as much as possible from gas-bubbles escaping in the vacuum from a state of solution; and (3) a chamber in which the seal was made and the metal allowed to solidify. When applying the method to quartz glass a certain amount of difficulty was experienced owing to the fact that the surface of the glass was occasionally not quite smooth, so that, as a result of the high surface-tension of the molten metal, it did not form a perfect contact. This difficulty, as well as the inconvenience resulting from the

* "The Electrician," July 4, 1913.
† Vol. XXIV., 2, p. 95.

dissolved gas, was overcome by the Silica Syndicate by forcing the molten metal into its position by the pressure of the atmosphere. The following is an account of their procedure :—

A quartz tube is shaped as shown in the accompanying sketch and a molybdenum wire placed loosely in position at A and a piece of lead at B. The air is expelled by means of a current of hydrogen and the glass closed at C. The tube is then exhausted from D to a pressure of a few millimetres. The glass at A is now softened and pinched on to the molybdenum wire. The lead in B is then melted and allowed to filter into the space



below which has been highly heated. If necessary this operation may be assisted by shaking and tapping the tube. The end C is now broken off while the metal is still molten so that the atmospheric pressure forces it well against the surface of the glass. The tube may then be cut at E before the lead has solidified and a tinned leading-in wire may be introduced into it.

When made in this manner the seals have so far never been known to fail. They have been fitted to cathode-ray tubes and mercury lamps. They have been heated to the melting

point of the lead, and on solidification of the latter were found to be intact. A quartz-mercury lamp fitted with these seals has now been on test with intermittent burning for more than 500 hours. In another successful test a quartz mercury lamp has been repeatedly immersed in a freezing mixture (about -18 deg.), and then suddenly transferred to tepid water of over 30 deg., the seals being thus subjected to a temperature shock of over 50 deg.

University College, Nottingham, January, 1914.

ABSTRACT.

The author has found that lead which has been allowed to solidify in contact with glass will, if free from oxide, form a vacuum-tight joint with the latter. Owing to the very great firmness with which the metal adheres, and owing to its great plasticity, these joints can stand temperature changes without damage. When applied to quartz, the joints are usually made inside a tube in conjunction with a molybdenum wire seal as follows: The tube is shaped so that the molybdenum wire can be placed loosely inside it. A short piece of the latter is later sealed into the quartz, whereas one of its ends projects a few millimetres into the space in which the lead seal is to be made. Connected with this space by a short capillary there is an upper chamber in which the piece of lead is placed. The air is first blown out with hydrogen, and the tube then closed at the top and evacuated to a pressure of a millimetre or two. The quartz is then softened and pinched on to the molybdenum wire. After this the lead is melted and allowed to filter from oxide through the capillary and run into the space shaped to receive it, which has been highly heated. Before it has solidified, the tube is broken at the top to allow the pressure of the atmosphere to force the lead well against the surface of the glass. The tube is then cut at a suitable place and a tinned leading-in wire introduced into the lead. Such seals have been successfully fitted to vacuum tubes, mercury lamps, &c., and when made as described have so far not been known to fail.

DISCUSSION.

Prof. C. H. Lees asked how the cost of the process compared with that of the ordinary platinum seals in the case of glass.

Dr. Sands, in reply, stated that platinum seals could be made with such small quantities of platinum that it was hopeless to try to compete with them in point of cheapness.

XV. *The Asymmetric Distribution of the Secondary Electronic Radiation produced by X-Radiation.* *By* A. J. PHILPOT, *B.Sc., Layton Research Scholar, University of London, King's College.*

COMMUNICATED BY PROF. BARKLA, F.R.S.

RECEIVED JANUARY 5, 1914.

IT is well known that when X-radiation falls upon a metal such as gold there is emitted from the surface of the metal an electronic radiation sufficient in quantity to produce a considerable ionisation in the neighbourhood of the metal. It is known further that if the metal be in the form of a thin sheet, and the X-radiation pass through the sheet perpendicular to the plane of the sheet, electrons are emitted from both sides of the sheet. It has been found, however, that the amounts of electronic radiation given off on the two sides of the sheet are not equal in such a case; but that, when due allowance has been made for the absorption of the X-radiation in the sheet itself, there is a preponderance of electronic radiation emitted in the original direction of propagation of the X-radiation. This effect has been observed by Cooksey*, Beatty,† and later again by Cooksey.‡ Similar effects have been found in photo-electric work by Stuhlmann,§ and in the production of β-rays by γ-rays by Bragg and Marsden.‖ Whilst, however, the results obtained have all shown the existence of asymmetry, there has not been good agreement as regards the extent of the asymmetry, or as regards variation of asymmetry with variation of wave-length in the exciting X-radiation. Thus Beatty found that the ratio of the ionising effect due to ejected electrons in the direction of propagation of the exciting radiation to that in the reverse direction varied from about 1·3 when the exciting radiation was the secondary X-radiation characteristic of tin to practically unity when that from copper or iron was employed. Cooksey, however, in his later Paper, found that the ratio had a constant value of about 1·18 over the same

* "Nature," Vol. LXXVII., p. 509.
† "Proc." Camb. Phil. Soc., Vol. XV., p. 492.
‡ "Phil. Mag." Vol. XXIV., p. 37.
§ "Phil. Mag." Vol. XXII., p. 854.
‖ "Trans." Roy. Soc. of S. Australia, Vol. XXXII., May, 1908.

range of exciting X-radiation. Again, Bragg and Marsden found that the lack of symmetry was less for soft γ-rays than for hard, and still less for X-rays which are very soft γ-rays. The order of magnitude of the ratio of emergence to incidence radiation ranged all the way from 20 : 1 down to unity, depending on the nature of the radiations used and the substance in which the secondary rays were excited. Considering the important bearing which this asymmetry has upon our knowledge of the nature of X-radiation it seemed advisable that further experiments should be undertaken; in this Paper such experiments are described.

The apparatus employed is shown in the accompanying diagram (Fig. 1). A beam of X-rays, limited by suitably placed screens, was directed from the anti-cathode K on to

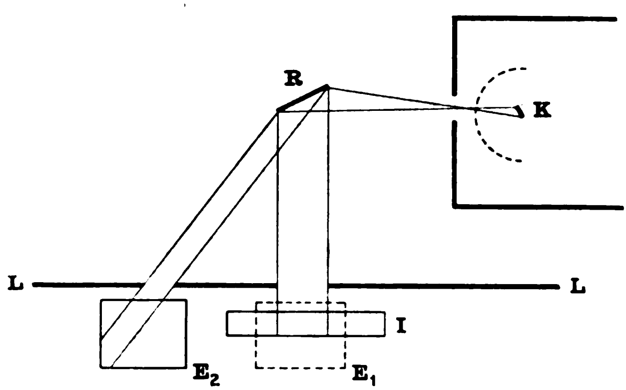


Fig. 1.

a radiator, R, of some desired element. The secondary homogeneous radiation from R was allowed, by means of suitable apertures in a lead screen, LL, to pass (1) through an ionisation chamber, I, (2) into a standardising electroscope, E_2. The ionisation chamber I (Fig. 2) used in these experiments consisted of a metal box of about 2·5 cm. depth, with square ends of about 9 cm. edge. In the centre of the front face of the box a square aperture with edge 4 cm. was cut and the opening covered with thin aluminium. By means of grooves, hollow square aluminium frames could be slid down immediately next to the front and back faces A and B. Three such frames were used during the experiments and on each was stuck a sheet of paper, which was in turn covered with two specially thick gold leaves,

each having a thickness of about 7×10^{-5} cm. Each frame, being hollow, could thus be placed so that either gold leaf or paper faced into the chamber. Between the two faces A and B was placed a square electrode of aluminium wire, crossed by meshes of exceedingly fine aluminium thread, the electrode being of such a size that only the fine mesh was at any time exposed to the radiation from R. The electrode communicated through an ebonite plug with the gold-leaf system of an electroscope, E_1. On passage of X-radiation the rate of leak of E_1 was thus a measure of the ionisation taking place in I, and this was standardised by comparison with the leak in the electroscope E_2. Both electroscopes were of the ordinary Wilson pattern.

The method of procedure was as follows: Two frames were placed in the chamber, the one at A having gold facing into the chamber, the one at B paper. The X-radiation was passed and the ionisation measured. The frames were then taken out

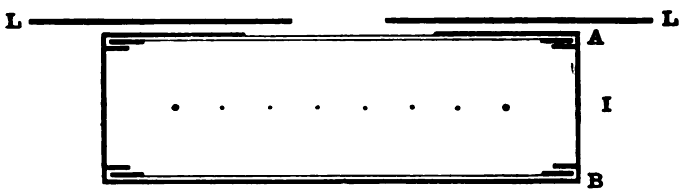


FIG. 2.

and their positions interchanged, the one previously at A being placed at B with the gold again facing inwards and vice versa. The ionisation was again taken. The frame at B was then removed and the third frame put in, having paper facing inwards. The ionisation was again taken. This third ionisation is evidently that due to all effects except the electronic radiation from the gold, this last being absorbed in the paper. Evidently, then, by subtracting this third ionisation from the first and second, we get respectively the ionisation due to the electronic radiation in the direction of propagation of the exciting X-radiation and in the reverse direction. It will be noticed that, as in both cases the same thickness of gold had been traversed, at the surface of the gold the intensity of the exciting radiation was the same, except for a slight absorption due to the passage through the 2 cm. of air in the chamber. Also the electrons were always ejected from the same gold surface, and it was to prevent injury to, or contamination of,

this surface, by contact with either of the faces of the chamber, that the third frame was used. Lastly, the thickness of gold used, $1 \cdot 4 \times 10^{-4}$ cm., acted, except for extremely penetrating radiations, as an infinitely thick gold plate as regards absorption of the emitted electrons.

A slight correction must be made due to absorption. Let λ_1 be the co-efficient of absorption of the homogeneous exciting radiation in gold. Let λ_2 be the co-efficient of absorption of the electronic radiation in gold, assuming an exponential absorption. Suppose a radiation of intensity I acting over an area A produces in a thickness dt of gold electrons which absorbed in air would give an ionisation A$Iidt$. Considering the case in which the gold faces inwards at A (Fig. 3). Let the

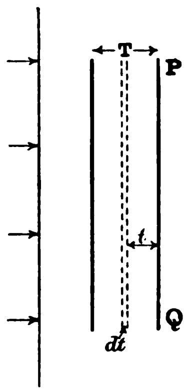


FIG. 3.

intensity at the inmost face PQ be I. If the electrons were not absorbed in the gold the ionisation due to corpuscles produced at a distance t from PQ would be

$$\mathrm{AI}e^{\lambda_1 t}\, i\, dt.$$

Since the electrons are absorbed, the actual ionisation is

$$\mathrm{AI}ie^{(\lambda_1-\lambda_2)t}\, dt.$$

The total ionisation will be

$$\mathrm{AI}i\int_0^{\mathrm{T}} e^{(\lambda_1-\lambda_2)t}\, dt,$$

where T is the total thickness of the gold.

This $=\dfrac{\mathrm{AI}i}{\lambda_2-\lambda_1}$, since $e^{-\lambda_2 \mathrm{T}}=0$. Similarly it will be easily seen

that when the gold faces inwards at B the ionisation is $\frac{AIi}{\lambda_1+\lambda_2}$; so that we must increase the ionisation with the gold at B by a multiple $\frac{\lambda_2+\lambda_1}{\lambda_2-\lambda_1}$ in order to compare it with that with the gold at A. The absorption of the exciting X-radiation in the air of the chamber has been ignored, since even with the least penetrating radiation employed the correction involved is negligible.

Four homogeneous radiations were used and the following table gives the results obtained :—

Exciting radiation.	λ_1.*	λ_2.†	$\frac{\lambda_2+\lambda_1}{\lambda_2-\lambda_1}$.	Uncorrected ratio.	Corrected ratio.
Mo. $\frac{\lambda}{\rho}$ in Al=5......	2,142	136,000	1·03	1·15	1·11
Ag. $\frac{\lambda}{\rho}$ in Al=2·5...	1,185	89,000	1·02	1·16	1·13
Sn. $\frac{\lambda}{\rho}$ in Al=1·57..	998	58,000	1·03	1·20	1·16
Ba. $\frac{\lambda}{\rho}$ in Al=0·8 ...	870	33,000	1·05	1·27	1·21

The values thus obtained do not differ in any marked degree from those obtained by Cooksey, but it seemed significant that the gradual rise in the ratio corresponded with a rise in the penetrating power of the exciting radiation. In order to ensure that this rise was not due to a slight loss of electrons to the sides of the chamber when the gold was at B (due to the slight divergence of the exciting beam), which loss would, of course, increase with the hardness of the exciting radiation, the size of the aperture was considerably diminished. Values were obtained which, within the limits of experimental error, were the same as before. Again, to make sure that the variation was not due to a slight variation in the thickness of

* Barkla and Collier, "Phil. Mag.," June, 1912.

† The value of λ_2 for *Sn* was found by taking the value for absorption in air given by Beatty ("Phil. Mag." Vol. XX., p. 324), and calculating from it, assuming the truth of Lenard's law, the value for absorption in gold. The values of λ_2 for the other radiations were obtained by taking λ_2 inversely proportional to the fourth power of the atomic weight of the radiator. (Whiddington, "Proc. Roy. Soc.," Vol. 86, Series A, p. 376.)

the gold leaves, the frames were used to absorb the radiation from copper. No variation in absorption was found.

In order to obtain still harder radiation, and also to make certain that the effect was in no way dependent on the intensity of radiation, two or three values were obtained using the primary radiation from the X-ray bulb. A deeper chamber was used, but the aperture was considerably diminished. The radiation was made to pass through a considerable thickness of aluminium before reaching the chamber, and the mass absorption co-efficient in aluminium of the radiation entering the chamber was found after each experiment :—

Mass absorption co-efficient in Al of radiation (λ/ρ)	Corrected ratio.
0·5	1·24
1·3	1·19
2·5	1·15

Here, again, the same gradual rise occurs.

SUMMARY.

The extent of asymmetry has been investigated with radiations whose mass absorption co-efficients in aluminium vary from 0·5 to 5. Although the values generally do not differ much from the value 1·18 found by Cooksey, yet in general an increase in hardness of the exciting radiation has been found to correspond with an increase of asymmetry. Such increase, however, is very slight compared with the increase in penetrating power of the exciting radiation.

In conclusion, I wish to express my great indebtedness to Prof. Barkla for the interest he has shown and the advice he has given during the course of this investigation.

XVI. *Time Measurements of Magnetic Disturbances and their Interpretation.* By C. CHREE, *Sc.D., LL.D., F.R.S.*

RECEIVED FEBRUARY 19, 1914.

§ 1. Fresh interest in the question of the simultaneity or practical simultaneity of occurrence of magnetic disturbances over the whole earth was aroused a few years ago by the advancement of a theory, due to Dr. L. A. Bauer, expressed by himself in the following words*: "Magnetic storms do not begin at precisely the same instant all over the earth. The abruptly beginning ones, in which the effects are in general small, appear to progress over the earth more often eastwardly, though also at times westwardly, as a speed of about 100 km. to 200 km. per second, so that if a complete circuit of the earth were made it would require, on the average, between seven and three minutes."

A discussion by Mr. R. L. Faris† of the apparent times of commencement of 15 magnetic storms at the five observatories of the U.S. Coast and Geodetic Survey appeared favourable to Dr. Bauer's views, giving for the mean time of propagation round the earth about 3½ minutes.

In a Paper‡ read before this Society in November, 1910, I discussed Mr. Faris's Paper critically, reaching the conclusion that his results were insufficient to confirm Dr. Bauer's theory, which appeared to me on various grounds to be somewhat improbable.

At the end of my Paper, *l.c.*, p. 56, I said: "I think Dr. Bauer's conclusions could be adequately tested by a careful comparison of curves from selected stations fairly encircling the globe, choosing, if possible, stations whose time measurements are specially reliable."

Shortly afterwards, on December 20, 1910, Dr. Bauer§ issued a circular to observatories requesting recipients to send to the Carnegie Institution of Washington "the Greenwich mean civil times for your observatory of sudden beginnings of the 15 magnetic disturbances tabulated by Mr. R. L. Faris in 'Terr. Mag.,' Vol. XV., p. 101." Dr. Bauer also asked

* "Terrestrial Magnetism," Vol. XV., 1910, p. 232.
† "Terrestrial Magnetism," Vol. XV., p. 93.
‡ Physical Society "Proceedings," Vol. XXIII., p. 49.
§ "Terrestrial Magnetism," Vol. XVI., p. 85.

for particulars of the amplitude of the disturbances in the several magnetic elements, a side of the question which I do not at present propose to touch on.

A large number of observatories responded to Dr. Bauer's appeal, and the data sent him appeared in the June and September, 1911, issues of "Terrestrial Magnetism." Some observatories—*e.g.*, Greenwich—measured both the times and amplitudes themselves; others, including Potsdam, measured the times, but sent copies of the curves to Washington, so that the amplitudes might be measured there. For my part, I sent rough tracings of the Kew D (declination) and H (horizontal force) curves, indicating what points had been taken as representing the commencements of the disturbances, because in a good many cases there seemed a possibility of confusion.

The September, 1911, issue of "Terrestrial Magnetism" contained a new set of time data for the five U.S. Coast and Geodetic stations, preceded by the following explanation from Dr. Bauer (*l.c.*, p. 200): "On March 16, 1911, the following self-explanatory request was made . . . to the superintendent of the U.S. Coast and Geodetic Survey: 'In addition to the data . . . already supplied, kindly furnish time scalings of all the disturbances, . . . taking into consideration the specimen curves from Potsdam and Kew, the tracings of which are enclosed herewith. Since the Kew curves are subject to a possible electric car disturbance, I would suggest that chief consideration be paid to the Potsdam curves, the precise point of scaling of which has been indicated by the observer (Dr. Venske).'"

Of the new data thus supplied some differ considerably from those supplied originally. The principal reason for calling attention to this matter here is because it is important to know that the final time data for the five U.S. stations were got out under the immediate direction of Mr. Faris, to whom the Potsdam curves had been supplied for guidance. This would naturally go a long way to secure that the disturbances whose times were measured really corresponded. Also, as Mr. Faris, it will be remembered, was originally on the whole distinctly in favour of Dr. Bauer's theory, the fact cannot be regarded as likely to prejudice a decision favourable to that theory.

§ 2. The June, 1911, issue of "Terrestrial Magnetism" held out hopes that "the final compilation and discussion" would follow at an early date, and though a considerable time has since elapsed I should not have intervened even now but for

the appearance of a Paper by Dr. G. Angenheister,* which discusses all the time data published by Dr. Bauer, along with some additional data obtained from quick runs of magnetographs taken at four stations in September, 1911.

Dr. Angenheister, following a similar procedure to that originally adopted by Mr. Faris, arranged the 33 stations which responded to Dr. Bauer's invitation in four groups, according to their longitude. The mean longitudes of the groups were respectively 76°, 175°, 274° and 354° west of Greenwich. Confining himself entirely to results from the H curves, Angenheister calculated the mean times of apparent commencement of each disturbance for the four groups of stations separately, and concluded that any differences in these times lay inside the limits of error to be expected with magnetographs run at the ordinary slow rate.

Dr. Angenheister's discussion will, I think, convince most people that the theory of a time of travel round the earth, from east to west or west to east, of the order of three minutes is not tenable for any of the 15 disturbances; but there remain a number of interesting points not discussed by him which I propose to consider here.

There is the question how far the time data from D and V (vertical force), which Angenheister did not utilise, agree with those from H. There is also the possibility that disturbances might travel, or appear to travel, from north to south or south to north. Magnetic disturbances are larger and more numerous in high than in low latitudes. Also, in some cases at least, they seem clearly associated with aurora, which is mainly a phenomenon of the Arctic and Antarctic regions. Supposing aurora to represent, as is generally believed, an electric discharge in the upper atmosphere, a gradual increase in the intensity of that discharge might produce a magnetic disturbance which would seem to propagate itself at a finite rate from the auroral belt towards lower latitudes.

Lastly and principally, the data seemed likely to throw a valuable light on the question of the accuracy of time measurements of disturbances as recorded by ordinary magnetographs. The 15 disturbances dealt with were all selected *after* the event, so that the records were taken under absolutely normal conditions. On any given day, of course, an individual station may be exceptionally fortunate or exceptionally unfortunate

* "Nachrichten der K. Gesell. der Wissenshaften zu Göttingen Math Phys. Klasse," 1913.

in the accuracy of the time marks on the curves, and, even in the case of as many as 15 disturbances, a more than usual amount of good or bad fortune may have been experienced at particular observatories; but deductions based on the records of 15 disturbances at some 30 stations cannot but represent a close approach to average conditions.

§ 3. Table I. deals with the problem treated by Angenheister, but in a somewhat different way, making use of D results instead of H. It arranges the observatories, as Angenheister did, in the order of their longitude, but does not divide them into groups, and gives for each, not the absolute time recorded for the commencement of the disturbance, but the difference between that time and the arithmetic mean of the times derived from all the observatories, with one or two exceptions. The exceptions include all the data from Eskdalemuir, Batavia and Mauritius. The two first of these stations were omitted, because they sent data for only two or three of the disturbances (Eskdalemuir was not in existence during the earlier disturbances), and as it was desired to intercompare the results for different disturbances this seemed the fairest course. Mauritius was omitted because a number of the Mauritius times seemed so abnormal as to suggest errors of the order of 10 minutes. If the Mauritius times are really correct, they represent something wholly abnormal. Two or three individual data from other stations were omitted for one reason or another, but the blanks in Table I. represent in all but a few cases simply the absence of D data. Toungoo, Barrackpore and Kodaikanal in particular contributed few D data, giving their principal attention to H.

The storms are numbered 1 to 15 in chronological order. The dates are given at the top of the table, but are omitted in the subsequent tables. The times given represent the arithmetic means of the accepted data from all the stations combined, and the figures in the columns below represent the differences, in minutes, between the times assigned at the individual stations and these arithmetic means. A plus sign indicates that the corresponding reading was later than the mean.

If the storms travelled round the earth at such a rate as Dr. Bauer supposed, we should have plus entries grouped together, tending towards a maximum at some one meridian, and negative values grouped together tending towards a numerical maximum at an opposite meridian. There seems no trace of this

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in Table I. The incidence of plus and minus signs appears fortuitous. The stations whose data differ most seem as often to be near stations as remote.

§ 4. Table II. is constructed on similar lines to Table I., but it deals with H times and arranges the stations in order of latitude. As in Table I., the numerical variations in the differences from the mean time shown by the returns from the different stations bear no obvious relationship to their geographical position, plus and minus signs occurring promiscuously. So far as H is concerned, there seems no more evidence of motion of disturbances along meridians than there was along parallels of latitude.

The mean times of commencement given in Table II. agree with those found by Angenheister in the case of disturbances Nos. 4, 5, 6, 8, 10 and 15. In the other cases the algebraic excess of the mean values in Table II. over Angenheister's is as follows :—

Disturbance No. ...	1	2	3	7	9	11	12	13	14
Excess in Table II....	−0·1	−0·1	+0·1	+0·3	−0·1	−0·1	−0·2	+0·4	+0·1

In the case of disturbance No. 14, the times given by Angenheister for Stonyhurst (8 h. 29·0 m.) and Eskdalemuir (8 h. 30·0 m.)—the latter at least of which was apparently not employed in calculating his mean—clearly refer to a disturbance anterior to No. 14, which was noted at various other stations, including Greenwich. The times given by Angenheister for No. 7, at Munich, and No. 10, at Sitka, differ by four and six minutes respectively from those given in "Terrestrial Magnetism," and there are some other differences which may be misprints. In the case of Nos. 8, 9, 10 and 12, Angenheister omits the data given in "Terrestrial Magnetism" for Pilar. They are, perhaps, a little outstanding; but the same is true of some other Pilar data which he employs. In the case of this station Angenheister—presumably for some definite reason—has subtracted 0·1 from the times given in "Terrestrial Magnetism." It has, however, seemed best here to follow "Terrestrial Magnetism"; in any case, the difference is immaterial.

Table III. does for V data what Table II. did for H data. The stations follow the same order as in Table II., but Wilhelmshaven, Kew, Munich and Agincourt are omitted as giving no V data. There were a good many blanks in the V data at

most stations, and some of the mean values depend on observations from only 13 or 14 stations. In disturbance No. 1 there is a suggestion of motion from south to north, the first seven entries being all plus and the last six all negative. The numerically largest values, however, both positive and negative, occur in intermediate latitudes, and the phenomenon may be quite accidental.

The Falmouth and Val Joyeux estimates for disturbance No. 13 appear abnormal, but were retained as they roughly neutralise one another in the mean.

§ 5. A comparison of two selected pairs of stations will, I think, show at once the inadequacy of Dr. Bauer's theory to explain the facts, and will bring out the true nature of the differences recorded in Tables I. to III.

The first pair of stations selected for this purpose, Potsdam and Baldwin, possess instruments similar, if not identical, in type, and the curves of the former station served as a guide to the identification of the disturbance times at the latter. The second pair of stations, Greenwich and Val Joyeux, possess magnetographs of widely different types, and the times at the two were got out absolutely independently.

The conclusion originally drawn by Mr. Faris from the data given for the 15 disturbances at the five U.S. stations was that the mean time required to travel 360° in longitude was 3·3 minutes from the D curves, 3·5 minutes from the H curves and 3·6 minutes from the V curves. Accepting 3·5 minutes as a mean of these, we should have for the times of travelling by the shortest route

Between Baldwin and Potsdam (108° 14′)...................	1·05 minutes.
„ Greenwich and Val Joyeux (2° 1′)	0·02 „

The times actually observed are given in Table IV.

A plus sign in Table IV. indicates later apparent occurrence at the first-mentioned (most eastern) station of the pair. Supposing the shortest route taken, this would imply, on Dr. Bauer's theory, a movement towards the east. As Val Joyeux and Greenwich are intermediate geographically between Potsdam and Baldwin, on the short arc, any disturbance which took the short route between the more distant pair of stations must necessarily take the short route between the nearer pair. Also, no storm could possibly have a shorter route between Potsdam and Baldwin than between Val Joyeux and Greenwich, unless it originated between the latter two

stations, a remote contingency in view of the small difference in longitude between them. That any large proportion of the 15 storms could have originated between Val Joyeux and Greenwich is inconceivable. Thus, the phenomena exhibited in Table IV., when interpreted by Dr. Bauer's theory, would inevitably imply that the part not merely exceeds the whole, but exceeds it by a great deal.

TABLE IV.—*Time Differences between Selected Stations, in Minutes.*

Disturbance. No.	Potsdam, less Baldwin time.				Val Joyeux, less Greenwich time.			
	D.	H.	V.	Algebraic mean.	D.	H.	V.	Algebraic mean.
1	+1·9	−0·6	+0·9	+0·7	...	−1·0	−0·4	...
2	+0·4	−1·7	+0·4	−0·3	+4·2	+2·9	+5·3	+4·1
3	−0·4	−0·6	+1·4	+0·1	−1·0	0·0	+0·7	−0·1
4	−0·6	−0·6	+0·5	−0·2	−2·6	−2·6	+0·1	−1·7
5	+0·1	0·0	+1·9	+0·7	0·0	0·0	+3·6	+1·2
6	−0·7	+0·4	+0·3	0·0	−4·2	−5·4	−1·2	−3·6
7	+0·3	−0·9	−0·8	−0·5	...	−5·0	...	...
8	−0·4	−0·4	+1·6	+0·3	+1·7	+1·1	+3·7	+2·2
9	−0·9	−0·1	0·0	−0·3	0·0	0·0	+3·8	+1·3
10	+0·4	−1·5	−0·2	−0·4	+0·1	−1·1	+0·9	0·0
11	−0·6	+1·0	+0·4	+0·3	−0·3	−0·9	+3·3	+0·7
12	+0·4	+0·4	+0·9	+0·6	+0·5	+0·5	+1·9	+1·0
13	−1·2	+1·1	+0·5	+0·1	−0·2	−1·4	+3·4	+0·6
14	−0·3	+0·1	+0·1	0·0	−2·7	−2·7	...	...
15	−0·4	+0·5	−0·1	0·0	+1·1	+1·1	+2·3	+1·5
Arithmetic means	0·60	0·66	0·67	0·30	1·43	1·71	2·35	1·50

Supposing disturbances equally likely to originate in any longitude, four out of the 15 might have been expected to originate between Potsdam and Baldwin, and thus to have followed the longer route (251° 46′) between these two stations. Taking, however, the view least unfavourable to Dr. Bauer's theory, as applied to that particular pair of stations—viz., that all the disturbances described the shorter route (108°14′)—we see that the times derived from the D, H and V records separately would imply a rate of propagation nearly double that found by Mr. Faris, while the times derived from an algebraic mean of the results from the three elements would imply a rate of propagation 3½ times that found by him. It will also be seen that, so far as Potsdam and Baldwin are concerned, the signs of the time differences derived from D and H did not

agree as often as they differed, so that in the majority of instances there was not even agreement as to the direction in which the disturbance travelled. The conclusion seems inevitable that the phenomena are mainly, if not entirely, due to instrumental or observational causes, and the remainder of the Paper assumes this to be the case.

§ 6. The figures, it should be borne in mind, do not represent the highest accuracy attainable with existing magnetographs run at the ordinary rate. If the days for curve measurements had been selected in advance, some at least of the stations would probably have taken precautions not ordinarily observed. On the other hand, the measurements were probably taken with more than the usual care, and represent higher precision than is customary in purely routine work. Before analysing them it is desirable to consider how times are usually determined on

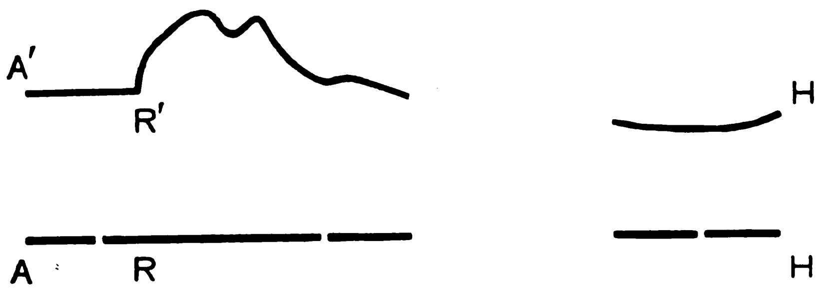


FIG. 1.—THE TWO PARTS OF THE FIGURE REPRESENT THE BEGINNING AND END OF A 24 HOUR RECORD.

magnetic curves. The nature of the photographic record is shown diagrammatically in Fig. 1. The straight line AH—known as the *base* or *time* line—represents the successive positions of a fixed spot of light reflected from a fixed mirror on to photographic paper wound round a drum turned by clockwork. The curved line A′H′ represents light from the same source reflected from a mirror attached to a magnet. In the case of the D or H instrument the drum has its axis horizontal, and if the optical arrangements are perfect, corresponding points A and A′ on the base and curve lines are on the same generator of the drum whilst the record is being obtained. This implies that after development, when the sheet is flat, AA′ is strictly perpendicular to the base line AH, which

represents the successive positions on the paper of the spot of light reflected from the fixed mirror.

The source of light is usually an illuminated slit, vertical in the case of the D and H instruments. The extremities of the image of this slit as formed by an ordinary lens placed between the slit and the two mirrors—which are closely juxtaposed—is or may be cut off by a horizontal slit between the mirrors and the drum ; but the condensation of the image to a point is due to a hemi-cylindrical lens with horizontal axis, a few centimetres in front of the photographic paper. By reducing the width of the vertical slit one may reduce the thickness of the base and curve lines to a fraction of 1 mm. ; but this does not necessarily increase the accuracy of one's time measurements. It is the shape of the spot of light and its length in the direction parallel to the time line that are of primary importance for this purpose.

In the case illustrated by Fig. 1 the time is derived from the breaks in the base line, which represent the intervention of a shutter, actuated by the clockwork, for a short time at successive regular intervals. In the Kew magnetograph the shutter comes on four minutes before each even hour, and goes off at the hour. As a check, the times of starting and stopping each record are noted.

The curves are measured in various ways, representing different degrees of accuracy. At Kew, for instance, the usual practice is to employ glass scales, an idea of which may be derived from Fig. 2. The horizontal line KLNM is intended to be placed immediately over the upper edge of the base line (AH of Fig. 1). The long lines PL, QM . . . have a millimetre scale laid off on them, intended to measure the ordinates of the magnetic curve (A′H′ of Fig. 1). LM and MN represent each an hour of time, and the hour space is subdivided into six 10-minute intervals. If we wish to measure the time of occurrence of a disturbance, say R′ in Fig. 1, we adjust the scale so that R′ lies on one of the vertical lines, say QM, and then observe where the last previous or next subsequent break in the base line comes on the time scale KN. To get times to one minute involves estimating to tenths of the 10-minute divisions shown. In the ordinary Kew pattern instruments one hour is represented by 15 mm., and so one minute by 0·25 mm.

We have thus far assumed—what is, unfortunately, often not the case—that corresponding points on the base and

magnetic traces are accurately on a perpendicular to the base line, and that the long lines PL, QM of the glass scale are truly at right angles to KN. If PL and QM are not perpendicular to KN, the error in determining the time of a disturbance increases with the length of the curve ordinate. Other possible sources of error are irregularity in the going of the clock which turns the drum and stretching of the photographic paper. Allowance can, of course, be made for some at least of the errors, but their complete eradication is hardly feasible.

Higher accuracy is obtainable in various ways. Thus, all measurements may be made with a reading microscope capable of reading to 0·01 mm. The time scale may be opened out,

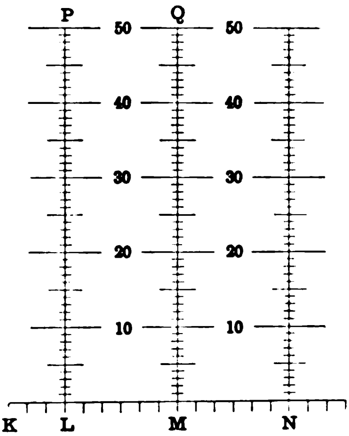


Fig. 2.

and the light interrupted at one-hour or shorter intervals. Eschenhagen magnetographs usually allow 20 mm. to the hour, and the shutter comes on hourly. In some cases the time break is applied to the magnetic curve itself.

For ordinary purposes accuracy to one minute is abundant, and this is all that has usually been aimed at at most British observatories, including Kew.

§ 7. The times for the 15 disturbances at Greenwich, Kew, Falmouth, Helwan and Zi-ka-wei were generally given only to the nearest minute. In one instance at Kew, and in a few instances at Falmouth, they were given to 0·5 minute. At the

other stations—with the probable exception of Wilhelmshaven, where the last figure is always 4 or 9—the times appear to be given to 0·1 minute; but I do not suppose that even the most optimistic would claim this order of accuracy. The degree of accuracy actually reached is probably fairly represented by the figures in the first three columns of Table V.

TABLE V.

Disturbance. No.	Average departure from the arithmetic mean time.			Mean time differences.	
	D.	H.	V.	D−H.	V−H.
1..............	0·93	0·58	1·01	+0·1	0·0
2..............	1·05	1·08	1·57	+0·9	+1·2
3..............	0·88	1·12	1·62	+0·4	+1·2
4..............	0·98	1·17	1·17	−0·3	+0·3
5..............	0·71	0·80	1·20	+0·1	+0·8
6..............	0·94	1·17	1·23	−0·2	+0·3
7..............	0·89	1·10	1·27	+0·7	+0·9
8..............	0·82	0·74	1·29	+0·1	+0·3
9..............	0·77	1·10	1·39	−0·3	+1·7
10..............	0·69	1·01	0·95	+0·1	+0·3
11..............	0·70	0·85	0·99	0·0	+1·0
12..............	1·09	1·01	1·27	0·0	+0·8
13..............	0·94	1·26	2·09	+1·2	+0·2
14..............	1·06	0·77	0·94	+0·6	+0·2
15..............	0·94	0·75	1·01	+0·2	+0·5
Mean	0·89	0·97	1·27	+0·24	+0·65

To explain the table, consider the entry 0·93 under D (declination) for disturbance No. 1. This represents the average numerical difference in minutes between the times deduced from the D curves at the several stations and the mean time of occurrence (19 h. 56·6 m.) derived from all the stations combined. The two last columns give the algebraical excesses of the mean times derived from the D and V curves respectively over those derived from the H curves. For instance, the first entry, +0·1, under D-H indicates that the time from the D curves was on the average 0·1 minute later than that given by the H curves. The absolute mean times have been already given in Tables I. and III.

The best agreement between the different stations—and so, presumably, the highest accuracy—is that shown by the H times for disturbance No. 1; but, speaking generally, the agreement is a little closer for the D than the H times. A possible explanation is that there is less variation of type

and of scale value in D than in H magnetographs. The disturbances were, as a rule, largest in H, so that, *ceteris paribus*, one would have anticipated the H times to show the best agreement. The agreement is decidedly least good in the case of V, but the inferiority is less marked than I had expected. The H mean time preceded the V mean time on all but one occasion, when they were identical, the average lag in the V time relative to the H being approximately two-thirds of a minute. The mean D time preceded the H on three occasions, but followed on 10, and on the average was almost ¼ minute later.

It has, however, to be borne in mind that only a few of the stations gave times from all three elements for all the disturbances, omissions being most common in the case of V, and that four of the stations limited their attention to D and H. Thus, the mean results in Table V are not exactly what we should have got if we had confined ourselves at each station to occasions on which all three elements were represented. The apparent lag in V owed a good deal to two or three stations, including Val Joyeux and Falmouth. It was exhibited, however, to a greater or less extent at most stations. Thus, at Potsdam, one of the few stations which contributed D, H and V data for all the disturbances, the mean apparent lag in V as compared with H was 0·89 minute. The most natural inference would seem to be that disturbances of the type considered have at the start no vertical component. This may be true, but the phenomenon is probably due at least partly to other causes. Disturbances which start from a zero value must, as a rule, take a short time to produce an effect visible on the curve. The disturbances considered were at most stations much larger in H than in V; thus a slight apparent lag in V might be expected, even if the V and H magnetographs were equally sensitive to small changes of force. As a matter of fact, the V instrument is more liable to frictional retardation than the H and D instruments. The question is one which deserves further inquiry.

§ 8. From one point of view the figures in Table V. somewhat exaggerate the inaccuracy of time determinations. The kind of disturbance most appropriate for accurate time measurements consists of a sharp movement on the trace in one direction, preceded by an absolutely quiet time of some duration. Some of the 15 disturbances by no means fulfilled these conditions, and some of the larger differences almost certainly represent misunderstandings as to the precise

movement intended to be measured. The occurrence of a few large differences was responsible in most cases for a considerable fraction of the mean differences given in Table V. It thus appeared desirable to make the analysis of the differences presented in Table VI. It shows how many of the differences from the mean at each station lay between 0 and 0·5 minute, between 0·6 and 1 minute, or exceeded 1 minute.

TABLE VI.

Station.	D.				H.				V.			
	0 to 0·5	0·6 to 1·0	> 1·0	Total.	0 to 0·5	0·6 to 1·0	> 1·0	Total.	0 to 0·5	0·6 to 1·0	> 1·0	Total.
Sitka	4	5	4	13	7	1	5	13	5	2	5	12
Ekaterinburg	3	4	5	12	4	5	4	13	2	3	7	12
Rude Skov	3	3	3	9	3	2	3	8	3	1	0	4
Stonyhurst	3	2	5	10	4	2	6	12	0	1	4	5
Wilhelmshaven	4	1	9	14	4	5	4	13	...	...	...	...
Potsdam	11	3	1	15	9	6	0	15	9	4	2	15
De Bilt	3	5	2	10	8	4	0	12	1	2	2	5
Greenwich	7	3	5	15	5	6	4	15	2	5	7	14
Kew	5	5	5	15	7	4	4	15	...	...	...	...
Uccle	4	6	3	13	5	5	5	15	1	2	3	6
Falmouth	4	6	4	14	4	3	8	15	0	3	11	14
Val Joyeux	7	1	5	13	5	2	8	15	1	2	10	13
Munich	8	5	2	15	5	2	7	14	...	...	...	...
Pola	3	0	7	10	1	2	10	13	0	1	6	7
Agincourt	7	4	4	15	7	4	4	15	...	...	...	...
Tortosa	6	0	5	11	3	2	5	10	3	1	7	11
Baldwin	11	4	0	15	11	2	2	15	7	6	2	15
Cheltenham	8	5	2	15	9	3	3	15	6	4	5	15
Zi-ka-wei	0	5	6	11	0	7	7	14	2	2	5	9
Dehra Dun	2	5	5	12	8	4	2	14	4	3	1	8
Helwan	3	2	4	9	2	2	7	11	2	2	5	9
Barrackpore	3	1	2	6	8	3	1	12	3	1	4	8
Honolulu	6	3	6	15	4	4	7	15	4	6	5	15
Toungoo	2	2	1	5	6	2	1	9	1	0	2	3
Bombay	7	3	5	15	9	2	4	15	5	5	5	15
Porto Rico	8	4	2	14	10	3	2	15	6	4	4	14
Kodaikanal	2	0	0	2	6	2	6	14	4	2	2	8
Buitenzorg	7	2	6	15	6	2	7	15	3	3	7	13
Samoa	4	1	3	8	8	2	1	11	1	1	4	6
Pilar	6	3	5	14	3	3	7	13	4	0	9	13
Totals	151	93	116	360	171	96	134	401	79	66	124	269
As percentages of totals	42	26	32	...	43	24	33	...	29	25	46	...

Goodness of agreement with the mean is not a universally satisfactory measure of merit. The winner of a newspaper prize for a list of the 100 best books in the popular estimation

may be an exceedingly poor judge of literature. Some allowance may be necessary for this aspect of the case in regard to Table VI. Also, at stations where times are given for only a fraction of the disturbances, it is not improbable that the disturbances omitted included those where identifications were most difficult and large errors most likely.

The agreement with the mean estimate is outstandingly good at Potsdam, the departure from the mean exceeding one minute in only one instance in the case of D and H and two instances in the case of V. Baldwin and Cheltenham, two of the U.S. stations whose times were revised in the light of the Potsdam curves, show also a very superior agreement.

On the whole, in the case of D and H, about two-thirds of the time estimates differed from the mean by not more than a minute, and fully 40 per cent. by not more than half a minute. Greenwich and Kew, though giving times only to the nearest minute, show practically the average closeness of agreement with the mean.

In the case of V the agreement with the mean is decidedly less close than with the other elements. It is, however, quite good at a few stations, all of which showed specially good agreement in the case of D or H.

§ 9. A large average departure from the mean may arise from large irregular errors, or from a prevailing tendency to be late or early in one's estimate. This point was investigated and the results appear in Table VII., which shows the mean numerical and the mean algebraic departure from the accepted time of the disturbance. The plus sign indicates that the estimated time was late.

To explain Table VII., take the case of D at Sitka. The estimated time of commencement on the average of the 13 disturbances measured at Sitka was 0·82 minute in "error." The error was not always in one direction, but the prevailing tendency was to be late, and on the average of the 13 occasions the time was 0·48 minute late. Some stations, including Sitka, Ekaterinburg, Stonyhurst, Greenwich, Uccle, Zi-ka-wei and Bombay, exhibit a tendency in the same direction, whether fast or slow, in all the elements. In the case at least of Greenwich this cannot well be ascribed to systematic clock errors or to any misapprehension as to longitude. Other stations are fast in the times from one element and slow in those from another. Pola and Val Joyeux are outstanding examples. At Pola the V times were all fast, on the average by 2·09

minutes, while the D times were on the average 1·47 minutes slow and were fast only once. Falmouth, again, in the case of D, showed practically no mean algebraic error, while its V times were on the average 1·60 minutes slow. This cannot, moreover, be put down to clock errors, because the D, H and V drums are turned by the same clock.

TABLE VII.—*Numerical and Algebraic Departures from Mean.*

Station.	D.			H.			V.		
	No. of observations.	Average departure from mean.		No. of observations.	Average departure from mean.		No. of observations.	Average departure from mean.	
		Numerical	Algebraic.		Numerical.	Algebraic.		Numerical.	Algebraic.
Sitka	13	0·82	+0·48	13	0·87	+0·73	12	0·77	+0·27
Ekaterinburg	12	0·98	+0·65	13	0·92	+0·78	12	1·37	+1·08
Rude Skov	9	0·74	−0·54	8	0·81	+0·41	4	0·35	+0·05
Stonyhurst	10	1·35	−0·77	12	1·39	−0·74	5	1·52	−1·00
Wilhelmshaven	14	1·36	−1·21	13	0·94	−0·34	...	...	...
Potsdam	15	0·45	−0·12	15	0·44	+0·11	15	0·63	+0·35
De Bilt	10	0·73	−0·03	12	0·42	−0·23	5	0·78	+0·50
Greenwich	15	1·03	+0·36	15	1·24	+0·47	14	1·24	+0·12
Kew	15	0·98	−0·34	15	0·92	−0·07	...	...	...
Uccle	13	0·88	+0·67	15	1·09	+0·38	6	1·02	+0·38
Falmouth	14	0·91	−0·01	15	1·31	+0·47	14	3·36	+1·60
Val Joyeux	13	0·95	+0·06	15	1·34	−0·50	13	2·28	+2·19
Munich	15	0·59	−0·12	14	1·06	+0·06	...	...	...
Pola	10	1·49	+1·47	13	1·59	−1·07	7	2·09	−2·09
Agincourt	15	0·72	+0·07	15	0·83	+0·41	...	...	...
Tortosa	11	0·93	0·00	10	1·37	−0·31	11	1·30	−0·35
Baldwin	15	0·41	+0·01	15	0·54	+0·30	15	0·62	−0·17
Cheltenham	15	0·58	+0·18	15	0·63	+0·41	15	0·88	+0·05
Zi-ka-wei	11	1·63	−0·57	14	1·32	−0·96	9	1·69	−1·11
Dehra	12	0·94	+0·27	13	0·62	+0·25	8	0·60	−0·35
Helwan	9	0·84	+0·84	11	1·39	+0·03	9	1·17	+0·59
Barrackpore	6	0·60	+0·27	12	0·56	+0·22	8	1·06	−0·21
Honolulu	15	0·89	−0·09	15	1·03	−0·68	15	0·94	−0·81
Toungoo	5	0·70	+0·26	9	0·63	+0·03	3	1·37	−1·10
Bombay	15	0·84	−0·67	15	0·60	−0·24	15	0·93	−0·93
Porto Rico	14	0·82	+0·55	15	0·62	+0·11	14	0·97	+0·13
Kodaikanal	2	0·25	−0·25	14	0·93	+0·81	8	0·71	+0·24
Buitenzorg	15	1·09	−0·55	15	1·21	−0·25	13	1·42	−0·76
Samoa	8	1·07	−0·30	11	0·67	−0·18	6	1·28	+0·85
Pilar	14	0·91	−0·11	13	1·67	−0·92	13	1·60	−1·14
Arithmetic means	...	0·88	0·39	...	0·97	0·42	...	1·23	0·71

In cases where a decided tendency was exhibited to depart from the average times in a fixed direction, it would probably be worth while to make some artificial disturbances at definite

instants and compare the known times with those derived from measurements of the curves.

The disturbances covered a period of over three years, and the idiosyncrasies presented by some of the stations seemed to persist throughout. Thus, by comparing certain selected pairs of stations, one could obtain evidence seemingly favourable to Dr. Bauer's theory, even when one included a large number of disturbances. If, for instance, we compared Sitka and Bombay times, we should get on the average, from Table VII., as the time to travel between the two stations, 1·15 minutes from the D curves, 0·97 minute from the H curves and 1·20 minutes from the V curves. So, again, for the mean time between Buitenzorg and Greenwich, we should get 0·91 from D. 0·72 from H and 0·83 from V. But in either case we could obviously derive absolutely contradictory results from pairs of intermediate stations.

At least five different patterns of magnetographs were represented: The Mascart and Eschenhagen, with very small magnets; the Watson, with composite systems formed of a number of parallel wires; the Kew, with magnets weighing about 87 grammes; and the Greenwich, which stand to the Kew somewhat as these stand to the Eschenhagen. It would appear that to the degree of accuracy customary in ordinary slow runs no very marked difference existed between the different patterns. If, however, one wished to have comparative times on quick-run curves correct to within a few seconds, it would be advisable, if not absolutely necessary, to employ instruments of the same pattern, similarly sensitive.

ABSTRACT.

The Paper is a sequel to one read before the Society in November, 1910 ("Proc.," Vol. XXIII., p. 49), dealing with the times of commencement of 15 magnetic disturbances discussed by Mr. R. L. Faris, and supposed by him to support Dr. L. A. Bauer's theory that the commencing movements of magnetic storms travel round the globe at rates of the order of 100 km. per second. In his previous Paper the author criticised Mr. Faris' Paper, and suggested that for an adequate test of Dr. Bauer's theory data could only be obtained from a number of stations encircling the earth. Shortly afterwards Dr. Bauer issued a circular requesting magnetic observatories to send him their measurements of the times of commencement of the 15 magnetic storms discussed by Mr. Faris. Upwards of 30 stations sent in data in answer to Dr. Bauer's request. A discussion of the data derived from the horizontal force curves has recently been published by Dr. Angenheister, whose conclusions are unfavourable to Dr. Bauer's theory. The present Paper deals with the data from the

declination and vertical force curves as well as those from the horizontal force curves following a somewhat different method from that adopted by Dr. Angenheister. The bearing of the data on Dr. Bauer's theory is discussed, and the peculiarities of the individual stations are considered.

DISCUSSION.

Prof. S. P. Thompson thought that Dr. Chree's conclusions pointed to the necessity of equipping all magnetic observatories with similar instruments and making them keep records to a tenth of a second.

Mr. R. S. Whipple mentioned the enormous amount of work involved in the reduction of these observations. The instrumental errors due to backlash and other causes were apt to be so great that it was really marvellous that observers went so far as to discuss times of the magnitude involved. He emphasised the need of standardised instruments.

Prof. C. H. Lees said that, like most people who had read Dr. Bauer's Paper when it appeared, he had had grave doubts of the instrumental accuracy. He thought the method of calculating the results rather biassed. He had set a student to work out the correlation number of the times of the disturbances and the longitudes of the observatories. The number was too small to justify any conclusion as to connection between the quantities.

Mr. C. W. S. Crawley thought it would be an advantage to work on a much larger time scale, say, 2 in. or 3 in. per minute, and make use of the little everyday disturbances. He had observed a large number of these in this way. Sometimes the disturbance commenced with a contrary kick and sometimes it did not. The phenomenon, when present, was quite easy to detect. He asked Dr. Chree if no connection ever existed between disturbances and volcanic action.

Dr. A. Russell asked the author if it was known whether atmospheric gales had any effect in producing magnetic storms. It was well known that in the Arctic and Antarctic regions violent magnetic storms were of frequent occurrence. These were often attributed to swarms of electrons flying past the earth. If this were so it would be possible for vortex rings of electric current to be set up which would travel comparatively slowly round the surface of the globe. These might conceivably alter the force components of terrestrial magnetism. In his opinion, the possible effects of the currents existing in the interior of the earth, or set up on its surface by celestial disturbances, ought to be considered as well as the currents in the upper atmosphere.

The Author, in reply, stated that the objection to using a wide time-scale was that an enormous amount of photographic paper would be necessary. Moreover, if the scales are too open, the curves present such a large number of features demanding attention that the staff usually at the disposal of an observatory would be unable to cope with the work involved. The objection to using the small disturbances, which were quite frequent, was the difficulty of ensuring that all the observers were dealing with the same disturbance. Dr. Moos had suggested a connection between certain magnetic disturbances and volcanic action, but he did not think the evidence was conclusive. Dr. Leyst, of Moscow, had established a relation between the amplitude of the ordinary magnetic variations and the height of the barometer.

XVII. *A Lecture Experiment on the Irrationality of Dispersion. By Prof.* S. P. THOMPSON.

NEWTON'S method of crossed prisms throws an oblique spectrum on the wall. If the prisms used are of identical kinds of glass the oblique spectrum is straight from red to violet. But if different kinds of glass are used, the spectrum is curved by reason of the irrationality of dispersion in one or both of the glasses. If a diffraction grating is used instead of one of the prisms, then the curvature which is observed is that resulting from the irrationality of dispersion of the particular prism employed; and for all known kinds of glass the refraction for blue and violet rays is disproportionately large. To exhibit these effects in the lecture theatre a diffraction grating of 12,000 lines to the inch is employed to cast on the screen a horizontal spectrum of the first order, the light from an arc lamp being sent through a small round or square hole. On interposing a prism to disperse the light vertically upwards, the resultant oblique spectrum is finely curved, being concave upwards. If in the arc lamp the carbons used are those supplied for commercial "flame arcs," the effect is more brilliant, since the spectrum contains several bright bands. If an alloy of zinc, thallium and lithium is introduced into the crater of the positive carbon, a still more striking series of bright points is produced in the curved spectrum.

XVIII. *On the Ratio of the Specific Heats of Air, Hydrogen, Carbon Dioxide and Nitrous Oxide. By* H. N. MERCER, *B.Sc., Research Student at the Imperial College of Science and Technology, London, S.W.*

COMMUNICATED BY PROF. H. L. CALLENDAR, F.R.S.

RECEIVED JANUARY 30, 1914.

THE main object of the following experiments was to test the accuracy with which the ratio of the specific heats could be measured, employing small quantities of the gas to be tested, with the ultimate view of performing experiments on the variation of the ratio at different temperatures. The method employed was to observe, with a platinum thermometer of very fine wire, the instantaneous fall of temperature corresponding to a given rapid fall of pressure. This method was first employed by Lummer and Pringsheim ("Smithsonian Contributions," 1898), using a bolometer strip 0·001 mm. thick, with a vessel of 50-100 litres capacity. It was subsequently employed by Makower ("Phil. Mag.," February, 1903) for the determination of the ratio of the specific heats of steam, with modifications introduced by Prof. Callendar, which consisted in the substitution of a platinum thermometer, compensated for conduction along the leads, in place of the bolometer strip, and in the adoption of an automatic contact for closing the galvanometer circuit at the right moment, which made it possible to work with smaller quantities of gas. The vessel employed by Makower was about 9 litres in capacity, but his experiments showed that a much smaller vessel might have been used. The apparatus employed in the present investigation was similar to that used by Makower, but it was found that with due precautions an equal degree of accuracy was obtainable with a vessel of only 300 cubic centimetres capacity.

The advantage of the method, as compared with that of Clement and Desormes, is that the adiabatic fall of temperature observed at the centre of the vessel is little affected by the walls, and that it is unnecessary to close the vessel immediately after making an expansion. As compared with methods depending on the velocity of sound there is the further advantage that whereas an error of 1 per cent. in the velocity involves

an error of 2 per cent. in the ratio of the specific heats, an error of 1 per cent. in the observed fall of temperature produces an error of less than $\frac{1}{2}$ of 1 per cent. in the ratio.

Assuming that the gas obeys the law $pv = \mathrm{RT}$, and that its specific heat S at constant pressure may be regarded as constant over the small range of temperature of the experiment, the expression for the entropy is

$$\mathrm{S} \log \mathrm{T} = \mathrm{R} \log p + \text{constant}.$$

If the gas is suddenly expanded under isentropic conditions from the state $p_1\mathrm{T}_1$ to the state $p_2\mathrm{T}_2$, we obtain the simple relation

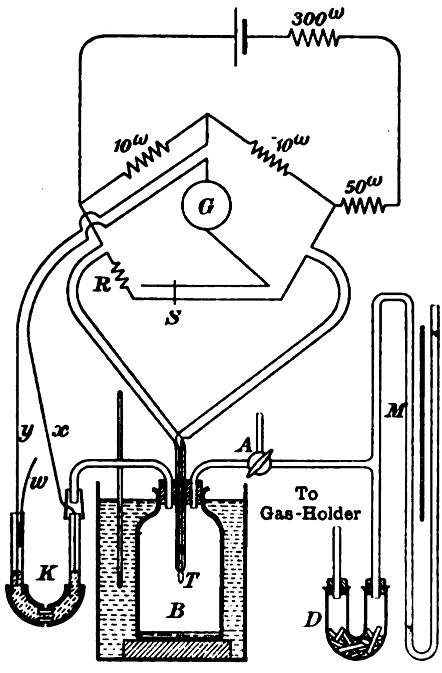
$$\mathrm{S/R} = (\log p_1/p_2)/(\log \mathrm{T}_1/\mathrm{T}_2),$$

from which the value of S may be directly deduced, since R is known, or the ratio of the specific heats γ, which is equal to $\mathrm{S/(S-R)}$. It would be in many cases easier to determine the value of S in this way than by a direct calorimetric determination, and the value found would not, as a rule, be greatly, if at all, inferior in accuracy.

If any other form of characteristic equation is assumed, the value of the ratio may be deduced in a similar manner from the expression for the entropy. But this has not been done in the present Paper, owing to the great variety of the expressions which have been proposed to represent the small deviations of gases from the ideal state. The corrections thus introduced would be small in most cases, but would differ greatly according to the type of equation assumed. It is possible, for instance, to have a considerable deviation from Boyle's law, without any variation in γ, provided that the ratio E/pv is equal to $\mathrm{(S-R)/R}$.

The apparatus consisted of a bottle, B, of some 300 cubic centimetres capacity containing the gas to be experimented upon. Near the centre of the bottle was fixed the platinum thermometer T; as this should be able to follow closely the variation of the temperature of the surrounding gas, the type used was the same as employed by Prof. Callendar in his steam-engine experiments, and consists of two loops of fine platinum wire, one being much shorter than the other. With this type the end-effect error due to conduction along the leads is eliminated. At A was a three-way tap by which gas could be admitted from the gas-holder (not shown in figure) to the bottle, or communication opened between the bottle and the outer air. Expansion was thus allowed to take place from a pressure, p_1,

some 50 mm. of mercury above atmospheric down to atmospheric pressure p_2, this latter being obtained by reading the barometer. An oil-gauge, M, was used in the pressure measurements. A drying tube, D, was inserted between the gas-holder and bottle B, and in addition the bottle contained a little strong sulphuric acid. A mercury key, K, also communicated with the bottle through a short, narrow tube, and served as an automatic contact closing the galvanometer



circuit shortly after the expansion had taken place. The wire w, together with the wires x and y, were used in the chronograph observations.

Temperature Measurements.—The point of balance on the bridge wire corresponding to the steady temperature T_1 was found, and the time noted when this was done. Communication was opened between the bottle and the gas-holder and time allowed for the temperature of the gas in the bottle and the oil in the manometer each to become steady. The sliding contact S on the bridge was adjusted by judgment from

previous trials nearly to a position where there would be no current through the galvanometer when the circuit was completed by the mercury key after the expansion. If the contact S was at the exact point of balance for the instantaneous temperature T_2, the spot of light on the galvanometer scale remained stationary for an instant and then moved off to the left as the gas heated up to regain the ordinary temperature. It was found advisable to make the adjustment of S so that a small throw to the right was obtained, the value of a small throw having been determined in terms of a shift of 1 mm. on the bridge wire. The exact point of balance could then be estimated. Having made the expansion and observed the throw, the galvanometer was brought to rest, and the point of balance for the steady temperature determined as early as possible. As the steady temperature T_1 was found to vary slightly during a series of observations, the time was noted at which the above readings were taken, so that the value of T_1 corresponding to the time at which T_2 was measured could be estimated.

Readings on the bridge wire could be taken with an accuracy to give temperature to 0·01°C.

The speed of the mercury key K was adjusted until a maximum fall of temperature was indicated for a given difference of pressure $(p_1 - p_2)$. This was done either by means of a pinch-cock on the rubber tube connecting the two limbs of the key or by inserting a glass tube with capillary bore to act as throttle.

The time which elapsed between the opening of tap A to the outer air and the closing of the galvanometer circuit by the mercury key was measured with the aid of a chronograph. For this purpose the wires w, x and y were made use of; wire w just made contact with the mercury at the pressure p_1, x at the atmospheric pressure, and y always made contact with the mercury. The galvanometer being disconnected, the key K was placed in series with one pen of the chronograph and the two storage cells, y being used as one terminal of the key and x and w together as the other. When an expansion took place the pen gave one kick when contact was broken at the end of wire w and another kick when contact was made on wire x. The other pen of the chronograph ticked off seconds.

In exactly the same way, by replacing the capillary throttle in the mercury key by an open tube, the time taken by the gas to expand from p_1 to p_2 was measured.

Pressure Measurements.—The manometer M used to measure the differences of pressure (p_1-p_2) consisted of a U-tube graduated in millimetres on both limbs and containing machine oil. The oil being very viscous, it was important that no great motion of the oil should take place. This condition was secured by the use of the gas-holder (not shown in the figure), having a volume of about 4 cubic ft., and with which the oil gauge M always communicated. Under these circumstances the motion of the oil for each experiment was only about 2 mm., so that little time was lost in waiting for the oil to become stationary.

The density of the oil was carefully determined at ordinary temperatures by comparison with a water manometer and its coefficient of expansion measured. If h_0 be the head in centimetres of mercury at 0°C., corresponding to a head h in centimetres of oil at t°C., the expression for h is

$$h_0=h(0{\cdot}0669)[1-0{\cdot}00068\,(t-18{\cdot}8)].$$

Readings of the gauge were taken immediately before making an expansion.

Preliminary Determinations.—The bridge wire was calibrated, but in no case was the correction greater than 0·01 centimetre. As 0·01 centimetre was the limit of accuracy in the observation of a bridge reading, no account was taken of the corrections. The bridge resistances R were calibrated in terms of the bridge wire.

Standardisation of the Platinum Thermometer.—For this purpose a mercury thermometer was placed near the platinum thermometer inside the bottle, and this mercury thermometer afterwards compared with a standard mercury thermometer. The resistance of the platinum thermometer is given in terms of the resistance of 1 centimetre of bridge wire as unit.

——	Mercury thermometer reading.	Resistance of platinum thermometer.
Bottle immersed in melting ice............	0·75°C.	638·44
Bottle immersed in water at room temperature	17·16°C.	655·57
Difference	16·41°C.	17·13

Hence 17·13 centimetres correspond to 16·41°C., therefore a shift on the bridge wire of 1 centimetre corresponds to a change in temperature of 0·9579 deg.

The heating effect of the current on the fine wire of the thermometer was measured and found to be 0·14 deg., but no account was taken of it because it did not vary appreciably during the tests.

The following is a typical series of observations with air:—

—	Time.	Manometer readings.	Difference p_1-p_2 cms. Oil.	Steady point of balance corresponding to T_1. Observed.	Estimated.	Point of adjustment for temperature T_2.	Throw. Scale divisions.	Estimated point of balance corresponding to T_2.
	3·2	...	...	32·89	...	...	...	...
I.	3·4	0·99 80·84	79·85	...	32·90	27·34	3	27·31
	3·5	...	...	32·90	...	...	...	...
II.	3·8	1·10 80·73	79·63	...	32·91	27·38	2	27·36
	3·10	...	...	32·92	...	...	...	...
III.	3·13	1·19 80·68	79·49	...	32·94	27·44	4	27·40
	3·14	...	...	32·94	...	...	...	...

—	p_1-p_2 mm. mercury.	p_2 mm.	p_1 mm.	Shift on bridge wire corresponding to T_1-T_2. cms.	T_1-T_2	T_1 °Abs.	T_2 °Abs.	Ratio.
I.	53·37	764·20	817·57	5·59	5·48	292·04	286·56	1·3900
II.	53·22	764·20	817·42	5·55	5·44	292·05	286·61	1·3875
III.	53·13	764·20	817·33	5·54	5·43	292·08	286·65	1·3873

About 50 observations of this kind were taken with air, 10 with hydrogen, 30 with carbon dioxide and 30 with nitrous oxide. The carbon dioxide was obtained fairly pure by using the last half of a large bottle of liquid. The residual impurity was oxygen. The gas was tested by agitation with caustic potash solution when about 98 per cent. was absorbed. With pyrogallol about 2 per cent. was absorbed. In the case of nitrous oxide when subjected to the ordinary gas analysis about 2·6 per cent. was found to be oxygen. Correction was made assuming that the oxygen was present as an impurity of air—that is, about 13 per cent. of air. The following approximate formula was used to make the correction :—

$$\gamma \text{ for mixture} = 0{\cdot}87\gamma \text{ for nitrous oxide} + 0{\cdot}13\gamma \text{ for air.}$$

The mean values of the ratio obtained in the first instance were as follow :—

Air	1·382	
Hydrogen	1·397	
Carbon dioxide.............	1·277	(corrected for an impurity of oxygen).
Nitrous oxide	1·229	(corrected for an impurity of air).

Corrections.—There are two corrections to be applied. (1) In the measurement of T_2 the thermometer will be at a higher temperature than the surrounding gas, due to radiation from the walls of the vessel. As it was thought that during the first series of experiments the platinum thermometer was not very clean it was removed and cleaned by passing it several times through a spirit flame. The thermometer was then replaced, re-standardised and a new series of values taken with air.

Value of γ for air with bright thermometer	1·390
Showing an increase of	0·008
Equivalent to an increase in T_1-T_2 of	0·079°C.

The corrections to T_1-T_2 for the other gases compared with that for air will be in the inverse ratio of the conductivities.

Gas.	Correction to T_1-T_2.	Correction to γ.	Values corrected to bright thermometer.
Air	...	...	1·390
Hydrogen	0·011	0·0011	1·398
Carbon dioxide...	0·134	0·0119	1·289
Nitrous oxide	0·118	0·0105	1·239

The error due to radiation for the bright thermometer was determined by performing experiments with the platinum thermometer coated with platinum black, other conditions being the same. Making use of a determination due to Lummer and Pringsheim—namely, that the absorption by a platinum-blacked surface is 15 times as great as that of a bright surface —the error due to radiation can be estimated. The value of γ obtained for air with the platinum-blacked thermometer was 1·364. Since the value with the bright thermometer was 1·390 the correction to be applied in the case of air is

$$\frac{1{\cdot}390-1{\cdot}364}{14}=0{\cdot}0018.$$

This is equivalent to an increase in T_1—T_2 of 0·0178. As before the correction for the other gases compared with that for air will be inversely as the conductivities.

Gas.	Correction to T_1-T_2.	Correction to γ.	Value corrected for radiation.
Air	0·0178	0·0018	1·392
Hydrogen	0·0025	0·0002	1·398
Carbon dioxide.........	0·0303	0·0027	1·292
Nitrous oxide	0·0265	0·0024	1·241

(2) An error is introduced due to the effects on the thermometer of conduction and convection in the gas during the time which elapses between the expansion taking place and the closing of the galvanometer circuit.

This correction was estimated by repeating observations with another bottle having practically twice the linear dimensions. The dimensions of the two bottles were : Small bottle, internal diameter 6·2 cm., height 10·1 cm. ; larger bottle, internal diameter 11·6 cm., height 19·5 cm. When the correction is small it may be taken to vary inversely as the linear dimensions, so that the correction for the larger bottle is approximately equal to the difference between the values obtained with the two bottles.

The values obtained and the corrected values for air and nitrous oxide are given in the table :—

Gas.	Values corrected for radiation.		Correction for larger.	Final value.
	Small bottle.	Larger bottle.		
Air	1·392	1·396	0·004	1·400
Hydrogen	1·398	...	...	...
Carbon dioxide.........	1·292	1·286	...	...
Nitrous oxide	1·241	1·251	0·010	1·261

The correction in the case of nitrous oxide is somewhat doubtful owing to impurity.

Chronograph Observations.

Gas.	Time of expansion.	Time of mercury key.
	Seconds.	Seconds.
Air, small bottle	0·16	0·23
Air, larger bottle	0·77	0·97
Carbon dioxide, larger bottle	0·78	1·00
Nitrous oxide, larger bottle	0·49	0·79

The quantity in the last column is the time which elapsed between the opening of the tap A to the outer air and the closing of the galvanometer circuit.

Conclusion.—It is interesting to compare the values thus obtained with direct measurements of the specific heats. Taking the value of R per gramme of air to be 0·06866 calorie per degree, the value of the specific heat S from the present

experiments comes out 0·2403 calorie per degree, which agrees very closely with Swann's determination at the atmospheric temperature and pressure. The values for the other gases tested are similarly compared in the following table :—

Gas.	Ratio.	R.	S.	Values found by other observers.
Air	1·400	0·06866	0·2403	Swann... 0·2417 Regnault... 0·2374 Holborn 0·2366 Witkowski 0·2372
Steam*	1·303	0·1105	0·475	Regnault 0·4805 Holborn... 0·4652
Hydrogen....	1·398	0·9880	3·4704	Regnault 3·4046 Lussana... 3·402
Carbon dioxide	1·292	0·04521	0·2000	Swann...0·2020 Regnault... 0·2164 Holborn 0·2010
Nitrous oxide	1·261	0·04523	0·2185	Regnault 0·2238 Wiedemann 0·213

Holborn's values are generally admitted to be low, but the agreement is otherwise very good, although for an accurate comparison some small corrections might be required on account of the deviations from the ideal state.

My best thanks are due to Prof. Callendar for his invaluable advice throughout the work.

ABSTRACT.

The main object of the experiments was to test the accuracy with which γ could be measured, employing small quantities of the gas, with the ultimate view of experiments on the variation of γ with temperature. The method employed was to observe with a platinum thermometer of very fine wire the instantaneous fall of temperature corresponding to a given rapid fall of pressure. The method was first employed by Lummer and Pringsheim ("Smithsonian Contributions," 1898), using a bolometer strip, with a vessel of 50-100 litres capacity. It was later employed by Makower ("Phil. Mag.," February, 1903) for the determination of γ for steam, with modifications introduced by Prof. Callendar, which consisted in the substitution of a platinum thermometer, compensated for conduction along the leads, in place of the bolometer strip, and in the adoption of an automatic contact for closing the galvanometer circuit at the right moment, which made it possible to work with smaller quantities of gas. The vessel used by Makower was about 9 litres in capacity. The apparatus employed in the present experiments was similar to that used by Makower, but it was found that with due precautions an equal degree of accuracy was obtainable with a vessel of only 300 cubic cm. capacity.

A table is given in the Paper showing the values of the specific heat at constant pressure for the various gases as calculated from the present experiments. The values show good agreement with direct calorimetric determinations.

* Makower by the same method. He finds 1·399 for air.

DISCUSSION.

Prof. C. H. Lees thought the Paper marked an advance in the subject. Hydrogen was always a severe test in such measurements, and the corrections were troublesome.

Mr. F. E. Smith asked what was the diameter of the platinum wire used, as there was probably a lag between its temperature and that of the gas. He thought more information was required on the time of making the galvanometer contact. It was possible that the gas might have commenced to cool, and in consequence to lower the temperature of the thermometer, before the correction due to the heating current had attained its final value.

The Author stated that the wire was very fine, about a fortieth of a millimetre in diameter, and he did not think there could be an appreciable lag.

XIX. *A New Type of Thermogalvanometer.* *By* F. W. Jordan, *A.R.C.S., B.Sc.*

Received January 27, 1914.

It occurred to me that the puff of air from an orifice in a chamber when the air within is suddenly heated might be utilised to deflect a small vane. A puff of air can be generated thermally by the passage of a current through a fine wire, by the incidence of radiant heat upon a thin absorbing disc and in a variety of other ways. In the following description I have confined myself chiefly to the development of a suitable thermogalvanometer.

The passage of a current through a heater of small thermal capacity generates first of all a condensational wave in the air or fluid about the wire, as in the hot-wire telephone, and a subsequent slower expansion of the fluid during the period of attainment of temperature equilibrium between the heater and its surroundings. Some idea of the magnitude of the momentum of the puff of air can be obtained from a simple calculation. The absorption of 10^{-6} calories by the air will cause an expansion of 10^{-2} cubic mm. at constant pressure, or if the air be confined at atmospheric pressure in a vessel of volume equal to 1 c.c., the change of pressure would be 10 dynes per square centimetre approximately. It is obvious that for such a small absorption of heat the impulse delivered to a vane in the region of the orifice would be exceedingly small, and in order to make it detectable the controlling torque on the vane must be made very weak. The latter is practicable owing to the fact that the vane and its attachments can be made very light and of small inertia. The whole weight of the suspended system need not exceed 10 mgm. and quartz fibres down to 0·002 mm. diameter can be safely used to control the motion of the vane. The disturbing effect of sudden small variations of atmospheric pressure have been eliminated by a compensation method. The vane is suspended symmetrically in front of two orifices connected with equal chambers, so that the pulsations of atmospheric pressure balance each other in their effects on the halves of the vane. The instrument can be shielded sufficiently from external sources of heat by (1) making the walls of the chambers thick and of good therm-

ally conducting material like copper or brass, and (2) by establishing a good thermal connection between the walls of the two chambers. Under these conditions any absorption of heat will be almost equally divided between the walls of the chambers, and, owing to their large thermal capacities, the rise of temperature of the air in each will be very slow.

Fig. 1 is a horizontal section through the chambers A, B and the vane V. The cylindrical chambers A, B were bored out of the same solid brass cylinder and hermetically sealed at one end by the cover plate C. Each chamber communicated with the half of a tube, D, which was partitioned longitudinally by a brass strip, E. The circular vane V, 2·6 mm. in diameter, was punched out of a thin sheet of aluminium and attached to

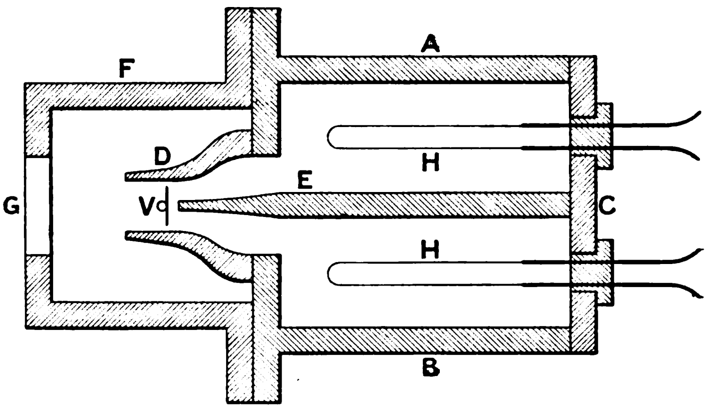


Fig. 1.

a fine glass stem. A plane mirror, 2·5 by 4·7 by 0·16 mm., was fastened to the stem beneath the vane. The stem was suspended by a fine quartz fibre, 5·5 cm. long and 0·0025 mm. in diameter, from a torsion head in the upper part of the compartment F. The free period of the suspended system was eight seconds, and the amplitude of vibration became imperceptible after the fourth vibration. The position of the vane V could be observed through the window G and was adjusted so that the vane was quite close and transverse to the partition E. The clearance between the edge of the vane and the tube D was about 0·17 mm. The vane was finally adjusted with the levelling screws so that it was insensitive to an expansion

of the air caused by touching the compartment F with the warm hand. A heater, H, consisting of a single loop of constantan wire, 0·015 mm. in diameter, was fitted axially in each chamber and their resistances were made approximately equal.

The passage of a current through a heater caused the vane to move to its extreme position in less than two seconds, followed by a return to zero in a time varying from 20 to 5 seconds, according to the clearance between the face of the vane and the edge of the partition E. The distance between the vane and the edge of the partition E was, in the latter case, 0·24 mm., and the sensibility was about 60 per cent. of its value when the vane was quite close to E. The throw of the vane is approximately doubled by diverting the current quickly from one heater to the other, since the inrush through one and the outrush of air through the other orifice both contribute to deflect the vane in the same direction. The instrument was calibrated by diverting a measured current from one heater to the other, and the following table shows how the throw depends on the current:—

Current $C \times 10^3$ amp.	Throw of vane.		Mean throw d.	$\frac{C^2}{d}$.
	Left.	Right.		
0·77	3·40 cm.	3·45 cm.	3·42 cm.	1,736
1·025	5·95 cm.	6·05 cm.	6·00 cm.	1,754
1·230	8·49 cm.	8·67 cm.	8·58 cm.	1,760
1·436	11·53 cm.	11·91 cm.	11·72 cm.	1,755
1·693	16·38 cm.	17·06 cm.	16·72 cm.	1,714

The distance of the scale from the mirror was 64 cm. and the current was measured to about 1 part in 300 with a Weston millivoltmeter. The results indicate that, over a range of 250 mm. on a scale at a distance of 1 metre from the mirror, the throw of the vane is very nearly proportional to the square of the current. The inequality of the throws to the left and right is explained by some slight want of symmetry. These measurements were made at a time when there was very little vibration in the building and the zero did not vary by more than 0·2 mm. from its mean position. The resistances of the heaters were 90·6 and 95·8 ohms, and the sensibility, as deduced from the foregoing figures, was 0·96 mm. per microwatt.

The following table shows how the sensibility varies with the clearance between the face of the vane V and the edge of

the partition E. This change of sensibility is due to a short-circuiting of the air current between the orifices :—

Clearance	0·08	0·16	0·24	0·31	0·63 mm.
Sensibility	0·89	0·72	0·53	0·40	0·13 mm. per 10^{-6} watt
Time of return to zero	17	10	5·5	(Undamped) seconds.	

These results also give some idea of how the sensibility would depend on the clearance around the edge of the vane. I find that there is not much to be gained by reducing the size of vane unless the clearance is diminished to a troublesome small value.

It was found in the preliminary experiments that the sensibility varied considerably with the distance between the wire filament and the walls of the chamber. To elucidate approximately the nature of this dependence brass hollow cylinders of various internal diameters were fitted inside one of the cylindrical chambers. The heater consisted of a narrow loop of 0·05 mm. constantan wire, and was arranged so that the loop was centrally situated within the hollow cylinder. The following results indicate how the sensibility depends on the distance between the heater and the walls of the chamber :—

Diameter of chamber	6·0	6·8	7·9	9·5	12·2 mm.
Sensibility	23·5	30·0	40·0	52·5	78

These results show that if the loop was made indefinitely narrow the sensibility would be approximately proportional to the diameter of the chamber. The change of sensibility is obviously due to loss of absorption of heat by the gas owing to its comparatively rapid conduction to the walls of the chamber.

The effect of altering the volume of the chamber was determined by attaching the heater to a closely fitting cylindrical plug within the chamber. The latter was adjusted to several positions, and no appreciable change of sensibility was detected. The available momentum of the puff of air thus seems to be independent of the volume of the chamber, and this is what is to be expected if the expansion of the air is very rapid. It is advisable to keep the diameter of the chamber within certain limits in order to minimise the effects of stray heat and pulsations of atmospheric pressure.

A modification of the above instrument, in which a pair of fine nozzles was substituted for the partitioned tube, has also

been tried. The internal diameter of the air chambers was 25 mm., or twice that in the above instrument, and the length was increased from 3 cm. to 5 cm. Each of the nozzles was 0·4 mm. in diameter and they were fixed at a distance of 2 mm. apart. The mirror attached to the suspended system was made smaller, and the free period of swing was about six seconds. A calculation showed that the controlling torque was about 0·6 of its value in the above instrument. The vane was brought close to the nozzles and adjusted in the manner described above. The vane, when very close to the nozzles, developed a troublesome instability which became accentuated by a slight vibration of the instrument. The sensibility in such a position was about 8 mm. per microwatt. When the distance between the vane and the nozzle was increased to 0·3 mm. the instability for moderate throws disappeared, the

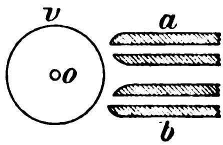


FIG. 2.

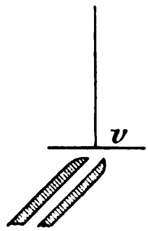


FIG. 3.

zero was much steadier and the sensibility was 4 mm. per microwatt. In this type of instrument a movement of the vane from its zero position destroys compensation for external sources of disturbance, and the effect of these becomes added to the puff of air in moving the vane. To free the vane from this objection, it can be made cylindrical and suspended so that the axis of suspension coincides as nearly as possible with the axis *o* of the vane *v* in Fig. 2. The clearance between the nozzles *a*, *b* and the vane will then remain nearly constant during the motion of the vane, and the instability can be much reduced. An alternative method is to arrange a pair of nozzles to deliver a tangential puff of air against a horizontal vane, *v*, as in Fig. 3.

The heaters R, R in the above instruments can be polarised by a continuous current C, Fig. 4, as in the Irwin hot wire oscillograph, and then the instrument can be modified for the detection of a continuous or alternate current *c*. If, for example, R=30 ohms, and the polarising current C=0·25

ampere, a continuous current $c' = 10^{-6}$ amperes will cause a change in the rate of emission of heat in each heater of $2Cc'R$, or 15 microwatts—a change that can easily be detected with a sensitive instrument.

To test the adaptability of the instrument to the detection

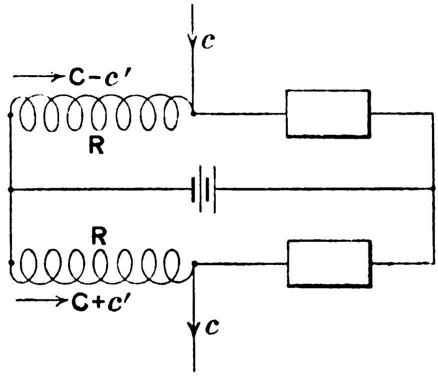


FIG. 4.

of an alternate current, a vane and mirror were fastened to a stretched strip of phosphor bronze, and the period was reduced to 0·033 second. The controlling torque per radian deflection of the vane was about 1·3 dyne cm., so that the period, if necessary, could be diminished considerably by a reduction in the size of the mirror and vane. Two fine platinum wire

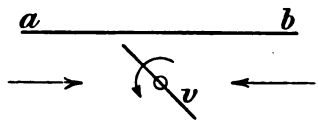


FIG. 5.

heaters were mounted in the chambers and polarised by a current of 0·25 ampere. Under these conditions the passage of a current, c', of 0·5 milliampere through the heaters produced an oscillation of 1 mm. on a scale at a distance of 1 metre from the mirror. The vane would, therefore, oscillate under the action of a superposed alternate current c of moderate frequency. I have not yet made any observations on the wave form given with this instrument during the passage of an alternate current for the exact nature of the dependence of the deflection on the current at any instant is not at all obvious.

A steady deflection of the vane can be obtained with alternate

current by making a slight modification in the adjustment of the vane. A plane vane, *v*, Fig. 5, is suspended so that it is initially inclined at an angle of 45 deg. with the axis of the channel connecting the chambers A, B. If the axis of rotation passes centrally through the vane, then a puff of air in either direction will rotate the vane in the anti-clockwise direction. Thus an alternate current through a fine wire heater in one of the chambers or through two polarised heaters will deflect the vane to a steady position provided that the heated wire is fine enough to follow the variations of the alternate current.

The results of the work with these instruments indicate that, if the use of a fine quartz fibre is not objectionable, a fairly quick and sensitive thermogalvanometer can be constructed on this simple principle. The best dimensions of the instrument can only be arrived at by a series of experiments, and I hope to be able to state more exactly, as a result of future experiments, its possibilities and limitations in its various applications.

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ABSTRACT.

The puff of air from an orifice in an air chamber when the air within is suddenly heated is utilised in this instrument to deflect a small suspended vane. The current to be measured is made or broken through a heater of small thermal capacity in the air chamber and the outrush or inrush of air through the orifice delivers an impulse to the vane. The disturbing effects of extraneous heat and pulsations of external pressure are eliminated by a compensation method. In one instrument of this type the sensibility was 4 mm. per microwatt and the extremity of the throw of the vane was attained in 2 seconds.

DISCUSSION.

Dr. W. H. Eccles said a good deal of work had been done with convection galvanometers, but Mr. Jordan was the first to measure the pulses produced by suddenly heated filaments. About nine years ago he had made some small instruments consisting of a fine filament connected to heavier leads and mounted in a small glass tube suitable for insertion in the ear. If an interrupted current passed through the filament the observer could hear every pulse produced. The instruments varied in sensitiveness. One which had a platinum filament of 12 ohms resistance could detect 0·0025 ampere, while another, with a platinised quartz filament of 900 ohms, could detect 0·00003 ampere.

Mr. W. Duddell thought the instrument was very ingenious. He asked if the sensitiveness could be increased to measure quantities of energy of the order of a microwatt.

Mr. Jordan, in reply, stated that the fibre was already extremely fine as the inertia was small. He thought the vane could be reduced considerably and the sensitiveness thereby increased without a serious increase in the time of swing.

XX. *An Instrument for Recording Pressure Variations due to Explosions in Tubes.* *By* J. D. MORGAN.

RECEIVED MARCH 10, 1914.

THE object of this Paper is to describe a mechanical oscillograph for recording the pressure variations which accompany a gas or other explosion in an open tube. Primarily the instrument was designed for the investigation of certain phenomena associated with dust explosions, the purpose being to obtain by simple means an accurate diagram of the pressure variations which occur at a given position in a tube in consequence of an explosion.

The conditions with which such an instrument must comply are : (*a*) It must be capable of operating at a high speed under small as well as under comparatively large pressures and (*b*) it must be dead-beat. Further conditions necessitated by the particular work for which it was required are (*c*) it must be robust and not liable to obscure changes and (*d*) it must be capable of producing a large number of diagrams in a short time at negligible cost.

Obviously the best known device which complies with the above conditions is a thin steel or other light elastic metal diaphragm arranged in conjunction with a style for recording the movements of the diaphragm on a revolving strip of smoked paper. Experience proves, however, that such a combination is objectionable for a variety of reasons. The amplitude of the movement of the diaphragm is not proportional to the pressure, and in consequence the diagrams are difficult to interpret. Owing to the complexity of the diagrams it is not practicable to re-draw them by hand to a scale in which the ordinates are proportional to the pressure. Further, the motion of the diaphragm is so small that a long style is required to produce the magnification necessary for a diagram of convenient size, and with such a style it is difficult to make the instrument deadbeat and to avoid defects in the diagram due to bending of the style.

Instead of a diaphragm the author has employed a light steel vane of rectangular form. This is mounted parallel to the explosion tube in a cell presenting a lateral opening to the tube interior. Along three edges the vane is free and along the fourth edge it is attached to a torsion wire. The vane is made

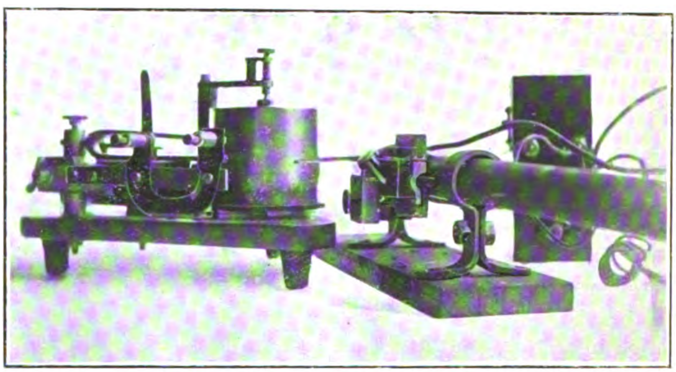


Fig. 1.

To face page 173.

to fit the cell as closely as possible around its edges without touching the sides of the cell. Friction is eliminated in this way, and the slight leak introduced makes no appreciable difference to the character of the diagrams.

Details of the instrument are shown in the illustrations, where Fig. 1 is a photograph of the whole apparatus, Fig. 2 is a plan, and Fig. 3 a cross-section of the main portion of the

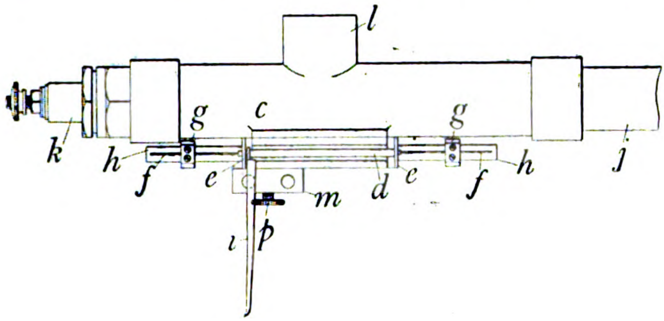


FIG 2.

instrument. The steel vane *a* is mounted in a cell, *b*, situated on one side of a breech block, *c*. Along its upper edge the vane is attached to a cylindrical bar, *d*, carried in bearings *e*. To the ends of the bar are brazed steel torsion wires *f*, which are secured in clamps *g* adjustable along supports *h*. The vertical end walls of the cell are flat; the lower wall is concave, and the

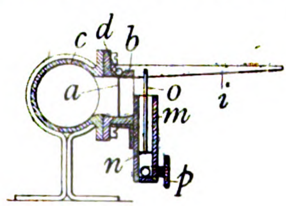


FIG 3.

upper wall is slotted to receive the bar *d*. At one end the bar *d* carries a short style *i* of ┐ section and made of an aluminium alloy which enables lightness to be combined with a high degree of mechanical strength.

The breech block, which has a bore of 1 in. diameter, is adapted to receive at one end a detachable steel experimental tube, *j*, which may be of any length, and at the other end is

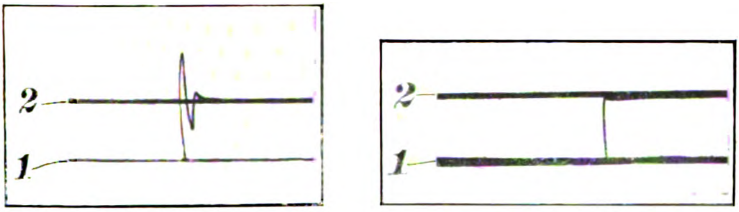
fitted with a sparking plug, *k*, when the instrument is used for gas explosions. A branch, *l*, serves for the admission of gas and air mixture, or a platinum coil for the ignition of dust clouds. Both ends of the breech block are made the same, so that an experimental tube can be inserted into each. In the latter case the gas or dust inlet and the ignition device are mounted in another fitting.

The diagram is produced by the style on a smoked paper strip wrapped around a clock-driven drum, and on the same strip is described a time curve by an electrically-driven tuning fork of known frequency.

To make the instrument dead-beat an additional feature is required. For this purpose the idea of incorporating with the cell the pole tips of an electromagnet is attractive, but owing to the mechanical simplicity of the more commonplace dash pot the latter was adopted. In the drawings it will be seen that the dashpot is mounted on the front of the vane cell. It consists of a metal block, *m*, formed with a pair of inter-communicating cylindrical passages in one of which is situated a thin metal disc or piston, *n*, attached to the style by a rod, *o*. A slight clearance is provided around the piston, and the flow of oil between the passages is controlled by a screw, *p*.

The effect of the dash pot is well illustrated at Fig. 4, where *a* is a diagram produced by allowing the vane and style to swing back suddenly from a deflected position indicated by the line 1 to the zero position indicated by the line 2, without the dash pot, and *b* is a diagram produced under similar conditions with the dash pot. Fig. 5 shows at *a* typical explosion diagrams without the dash pot, and at *b* similar diagrams with the dash pot.

Regarding the sensitiveness of the instrument, Fig. 6 gives proof of its capability of responding to slight and rapid pressure variations. If a pneumatic motor horn has its open end covered with a loose plate, leaving at its centre an aperture into which the open end of the experimental tube can fit (the other end of the tube being closed) the low fundamental note of the horn is suppressed and a note of relatively high frequency is obtained on compressing the bulb. The diagram shows the wave-line produced by the instrument in this experiment. It will be observed that the wave-line rises gradually above and descends towards the zero line. This is due to the slight increase of pressure caused by the injected air from the horn bulb.



a *b*

FIG. 4

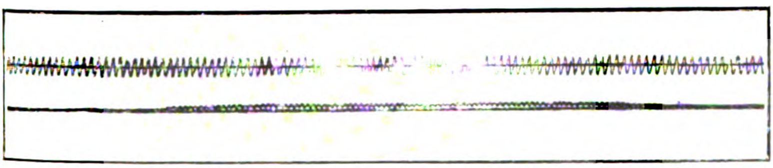


FIG. 6.

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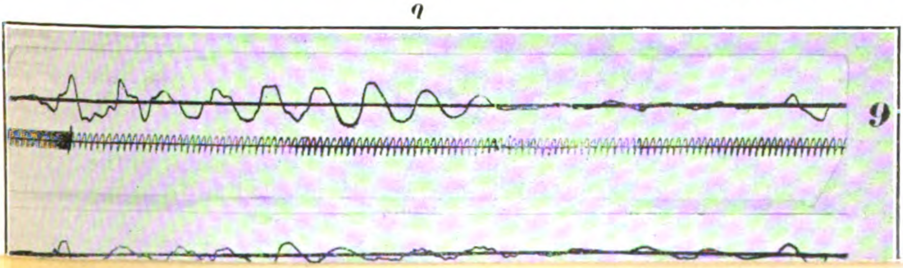
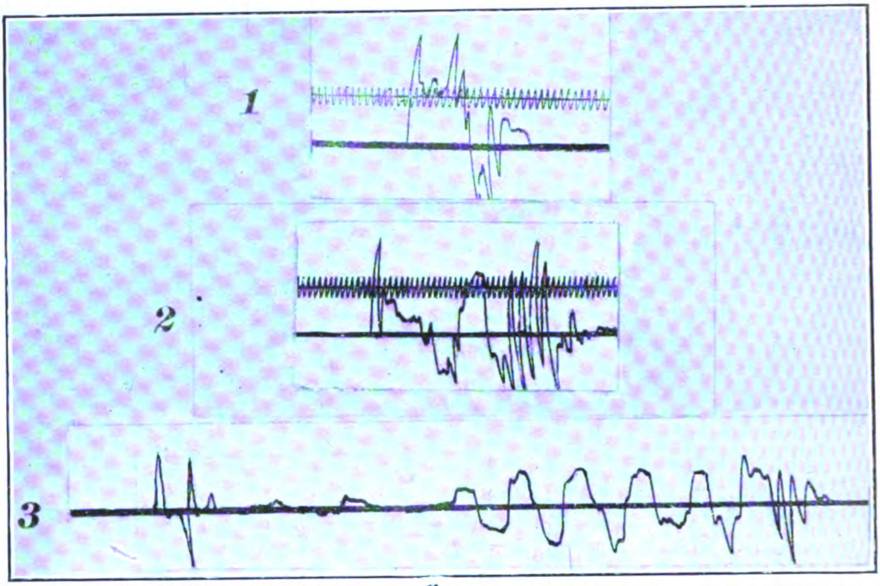
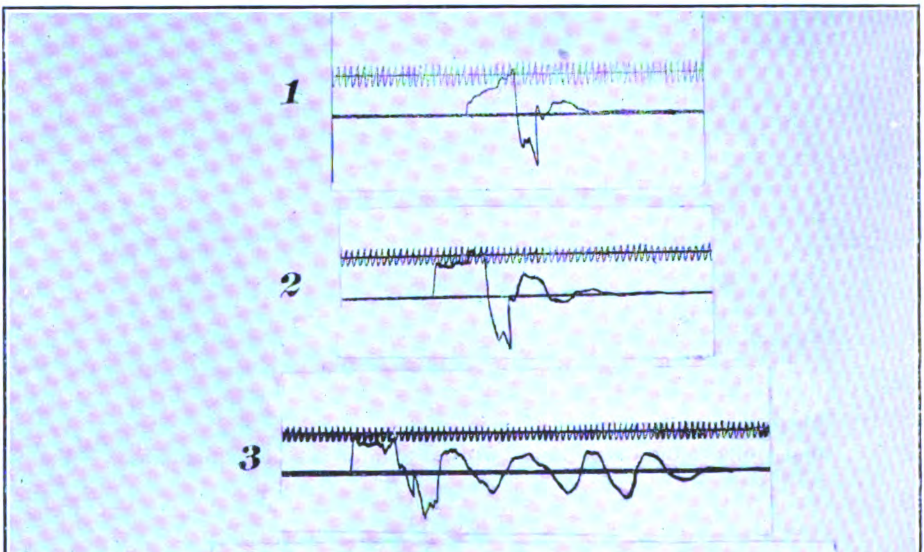
FIG. 5.

b





a



It is not contended that the instrument is capable of giving the precision obtainable by other and more elaborate means, but experience proves it to be a practical device for obtaining by a rapid and inexpensive procedure a large amount of accurate knowledge concerning the phenomena of explosions in tubes. Three defects are inherent in the design, but none of them is serious. In the first place, the ordinates of the curves are not perpendicular to the zero line, but lie along curved lines having the same radius as the style. This introduces a slight distortion, but does not interfere with the accuracy of measurements on the diagrams. In the second place, the damping effect of the dash pot appears to be disproportionately greater during the more rapid parts of the explosion than during the slower parts, and in consequence the ordinates of the diagram corresponding to high pressures rapidly generated are not so great as they ought to be when compared with those of smaller pressures persisting for a longer time. But this defect matters little or nothing when the work of investigation is concerned with the general character of the pressure variation and not the actual pressure obtained at a particular instant. In the third place, the slight leakage past the vane prevents the development of pressures which might be produced if the leakage were absent. This leakage is very slight, and comparison of the diagrams with others taken on a fixed diaphragm instrument show that it has no appreciable effect on the character of the diagrams. Otherwise the instrument appears to give results which are accurate and consistent.

In conclusion, the author desires to express his acknowledgments to E. C. R. Marks, Esq., for facilities afforded in the Marks & Clerk laboratory in Birmingham to develop the instrument, and carry on the work for which the instrument was required.

APPENDIX.

Note on Fig. 5.

The diagrams shown in Fig. 5 are obtained from the explosion of coal gas and air in a 1 in. tube of 10 ft. length. A time line (frequency 250/second) is shown in all the diagrams excepting *3a*. The diagrams begin at the left-hand end. Positive pressure is indicated above the zero line and negative pressure below that line. No. *1b* is typical of the diagrams produced when the tube is filled with an explosive mixture. The explosion is attended by a short loud report, and the dia-

gram is always characterised by its shortness, and especially by the sudden transition from positive to negative pressure. This latter characteristic is also found in all diagrams of explosions, whether short or long, when the explosion is accompanied by a sharp report. The same feature is seen in 4, 5 and 6*b*, at the ends of the diagrams. In No. 1*b* the first interval of positive pressure corresponds to the time which elapses before a compression wave is reflected from the closed end of the tube (*i.e.*, the time required for a wave motion to travel four times along the tube). The peculiarity of these short explosions appears to be due to the overtaking of the flame by a compression wave before the flame has travelled any considerable distance through the gas mixture. No. 2*b* is similar to No. 1*b*, excepting that the explosion is followed by a short damped oscillation. In No. 3*b* is shown the kind of curve obtained when the report of the explosion is of a rattling character. Nos. 4, 5 and 6*b* are typical of explosions whose report is of the nature of a rumble terminating in a sharp crack. The crack was more pronounced in 4 and 6 than in 5. The method of producing the diagrams shown in 4, 5 and 6*b* is to allow a quantity of gas to enter the tube, and to let the gas remain for a few seconds before firing. The mixture thus obtained is a poor one, and the diagrams illustrate well the effect of compression by vibration in increasing the explosibility of the mixture. It is not until some pressure has been attained by vibration that the characteristic of a short explosion is obtained, and then the vibration disappears rapidly. Each interval of positive or negative pressure in the more regular parts of the vibrations corresponds to the time required for a wave motion to travel twice the length of the tube. The presence of harmonics is shown clearly in the central portion of No. 5B. As regards Nos. 1 to 3*a*, these illustrate the irregularities introduced by the natural vibration of the vane and style. No. 1*a* corresponds to No. 1*b*, and No. 2*a* to No. 2*b*. No. 3*a* was obtained under similar conditions to Nos. 4 to 6*b*.

ABSTRACT.

The object of this Paper is to describe a mechanical oscillograph for recording the pressure variations which accompany a gas or other explosion in an open tube.

A light steel vane of rectangular form is employed and this is mounted parallel to the explosion tube in a cell presenting a lateral opening to the tube interior. Along three edges the vane is free.

and along the fourth edge it is attached to a torsion wire. The vane is made to fit the cell as closely as possible around its edges without touching the sides of the cell. The diagram is produced by a style on a smoked paper strip wrapped around a clock-driven drum, and on the same strip is described a time curve by an electrically-driven tuning fork of known frequency.

To make the instrument dead-beat a dash-pot is mounted on the front of the vane cell and attached to the style.

DISCUSSION.

The Chairman asked if the author thought all the phenomena shown in some of his curves were features of the original explosion, or if some of them were due to reflected disturbances from the walls and ends of the tube.

Mr. R. Appleyard said the different types of curve shown by the author seemed analogous to the oscillatory and aperiodic types of electric discharge. He asked if the author had tried damping with a magnet and copper plate.

Dr. W. Watson expressed his interest in the instrument. He mentioned with reference to the statement that a diaphragm cannot be made to give a uniform scale, that with a corrugated diaphragm a quite uniform scale of displacement against pressure could be obtained. He believed the author found it advantageous to have considerable inertia in the moving system, and he thought the inertia would play an even more important role in defining the extent of the vane's motion under the influence of a sudden expansion than the torsional control. He thought that the effect on the resultant curves of varying the inertia ought to be investigated. He supposed that in the curves shown the disturbance was compounded of what he might call the organ-pipe effect, and another effect due to the time taken by the combustion wave to pass along the tube.

The Author, in reply, said he had considered electromagnetic damping, but rejected it on account of relative cost. There were, however, grave objections to the use of the dash-pot, which undoubtedly modified the shape of the curves. There was no doubt that the resultant disturbances were compounded of a pressure wave along the tube and a surging of the gas as a whole. It was difficult to separate the effects. In most cases—probably in all—the flame seemed to last as long as the needle was vibrating.

XXI. *The Direct Measurement of the Napierian Base.* By ROLLO APPLEYARD, *M.Inst.C.E.*

RECEIVED MARCH 5, 1914.

THE equations of the catenary suggest that it should be possible to derive the Napierian base e from direct measurements of a suspended cord or chain. Such measurements will be found to be a useful exercise for students, especially as an introduction to the employment of hyperbolic functions in solving problems in physics. In Fig. 1, let

a=Length OV of chain, of weight equal to the tension at V, when the catenary is complete on both sides of V.

s=Length of the curved portion VP, where P is any point (x, y) on the catenary.

The familiar equations are

$$y=a\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$s=a\frac{e^{\frac{x}{a}}-e^{-\frac{x}{a}}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, at the point F on the curve, corresponding to $x=a$, $y=y_1$, we have

$$e=\frac{y_1+s_1}{a}$$

and

$$\frac{1}{e}=\frac{y_1-s_1}{a}.$$

If a is regarded as the unit of length, e is represented in Fig. 1 by FG plus the length of chain FV when straightened out; for when $x=a=1$, we have

$$e=y_1+s_1$$

and

$$\frac{1}{e}=y_1-s_1.$$

In performing the experiment, the chief difficulty is to ascertain the tension at V without altering the conditions of equilibrium. A pulley running on ball bearings was tried at V with unsuccessful results. Accuracy demands there the

equivalent of a frictionless pulley of negligible dimensions, and this requirement is fulfilled by substituting for a pulley a loop of fine thread, as shown along the line VB in Fig. 1. This loop is slipped over the end of the chain up to the point V, and the other end of it is attached to a fixed pin at B. The end D of the chain can then be moved about until VB makes an angle of 45 deg. with the vertical. Then, if V is the lowest

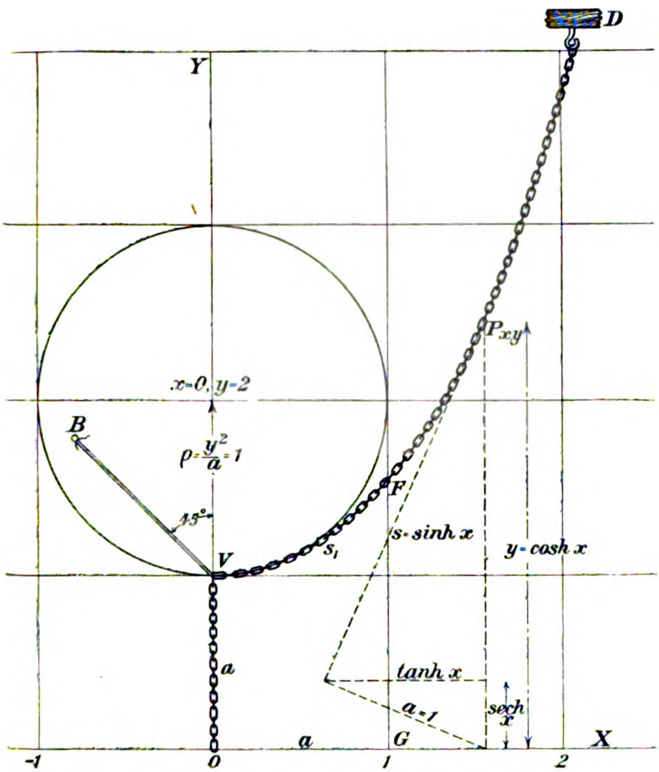


FIG. 1.

point of a catenary, the tangent at V is horizontal, and a is equal to, and at right-angles to, the tension of the catenary at V. Experiment shows however, that the end D of the chain can be moved over a wide range in various directions while retaining the thread VB unaltered in direction along the 45 deg. line, and only the roughest estimate of the value of e

can be made with the apparatus in that form. The explanation is that it is impossible to judge by eye when the tangent to the catenary at V is truly horizontal, and a small error in this estimation causes a considerable error in the measurement of y_1. To complete the method, therefore, some additional means must be found for ensuring that the curve reaches its lowest point at V, that its tangent is there horizontal, and that VB remains at 45 deg. to the vertical. This adjustment can be secured by introducing the circle of curvature, the radius of which for any value of y is

$$\rho=\frac{y^2}{a},$$

and the centre of which is always at the point $\left[\left(x-\frac{ys}{a}\right), 2y\right]$.

Hence at V, where $y=1=a$, and where $x=s=0$, the radius is 1, and the centre is at $2y=2$. After drawing the circle, the operation consists in finding the position of the end D, such that while the thread attached to the chain at V is maintained along the line VB, the curved portion of the chain, beginning at V, follows as closely as possible the circle of curvature near V. It, of course, soon departs from the circle, but these two restraints together are sufficient to allow a satisfactory estimate of e to be made. The same apparatus enables $\sinh x$, $\cosh x$, $\tanh x$, and $\operatorname{sech} x$ to be found with a piece of chain in like manner, as represented in Fig. 1, and the student is led to see that these functions, which for so long have been forced into association with the hyperbola, are far more easily exemplified by the catenary, and that x then becomes a definite length, instead of an obscure area or an elusive angle.

Illustration of Logarithmic Series.

When e has been determined as the sum of the lengths FG and FV, in terms of the length VO as unit, a helpful exercise for the student is to consider what FG and FV separately represent. By direct measurement he finds FG=1·543, which is cosh 1; and FV=1·175, which is sinh 1. Now, if he writes down the series defining e—*i.e.*,

$$e=1+1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}\;\ldots$$

and selects and adds, first the odd terms and then the even terms, he obtains 1·543 and 1·175, as given by the chain

measurements; and he recognises here the particular case when $x=1$, of the series and relations

$$\operatorname{Cosh} x=1+\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}+\frac{x^6}{\lfloor 6}+\ldots$$

$$\operatorname{Sinh} x=x+\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}+\frac{x^7}{\lfloor 7}+\ldots$$

$$e^x=\cosh x+\sinh x=1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}+\ldots$$

The point P may then be supposed to move along the chain, and the corresponding changes in the hyperbolic functions of x can be traced in an instructive manner, as represented in Fig. 1.

Logarithms from Chain Measurements.

From the fact that inverse hyperbolic functions can be put into logarithmic form, it appeared likely that chain measurements could be utilised to find logarithms; and this proved to be the case. This can be shown either from equation (1) or from equation (2). Thus, equation (1) can be written

$$e^{\frac{x}{a}}=\frac{1}{a}(y+\sqrt{y^2-a^2}),$$

which gives

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(y+\sqrt{y^2-a^2}). \quad . \quad . \quad . \quad . \quad (3)$$

Similarly, equation (2) gives

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(s+\sqrt{s^2+a^2}). \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But Fig. 1 shows that $\sqrt{y^2-a^2}=s$ and $\sqrt{s^2+a^2}=y$. Hence, from (3) or (4),

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(s+y), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $s=\sinh\frac{x}{a}$ and $y=\cosh\frac{x}{a}$.

Now, when $a=1$,

$$x=\log_e(s+y).$$

It follows that at any point along the chain suspended as in

Fig. 1 the Napierian logarithm of any number represented by a length $(s+y)$ is the corresponding x ordinate. For example, at the point $x=2, s=3{\cdot}627$ and $y=3{\cdot}762$; which means that 2 is the Napierian logarithm of $3{\cdot}627+3{\cdot}762=7{\cdot}389$. The extension to the determination of common logarithms involves the modulus, and is less simple. In conclusion, it may be pointed out that the chief object of the experiment is to direct attention to the value of the means afforded by a suspended chain for enabling a number of well-known functions to be presented in a form in which they can be co-related and remembered. A physicist cast upon a desert island and bereft of all but a chain could, to a first degree of approximation, determine the Napierian base, the hyperbolic functions, and the natural logarithms, and he could perform the operations of multiplication and division by mechanical means.

ABSTRACT.

The author described a simple apparatus intended to convey to students an idea of the way in which the base e of the Napierian logarithms enters into physical problems in a specific case of wide application. A small length of chain is allowed to hang from a loop of thread, and the remaining part of the chain is then pulled aside until the thread is at 45 deg. to the vertical. The curved portion becomes a true catenary when the angle between the vertical and curved portions of chain at the attachment of the loop is 90 deg. To ensure that this condition is reached, the circle of curvature of the catenary at that point is drawn, and this is found to have a radius equal to the vertical portion. In these circumstances, if the vertical length is taken as unity, and if its lower end is taken as origin, it is shown that e is the sum of the y-ordinate at $x=1$, and the length of curved chain between the point where that y-ordinate cuts the curve and the top of the vertical portion. The application of this result to a simple representation of the relationship and meaning of hyperbolic functions was also shown, and it was urged that such functions should be studied from consideration of the catenary rather than from the hyperbola.

DISCUSSION.

The CHAIRMAN supposed the object of the Paper was to familiarise students with the properties of the catenary rather than the determination of e, as this was so easily obtained from the formula.

Dr. W. H. ECCLES drew attention to a method of measuring e from the properties of the catenary, which the late Prof. Minchin used to set as a practical problem in the London University examinations.

XXII. *Introduction to the First Guthrie Lecture.* By *Prof.* G. Carey-Foster, F.R.S.

I have been asked, as one of the oldest members of this Society, to say a few words, by way of preface to the first "Guthrie Lecture," in order to give some idea of Frederick Guthrie's place as a scientific man and why it is appropriate that his name should be associated with the series of lectures which the Council have decided to institute.

Guthrie was born in London in 1833 and died in London in 1886. Like many men who have attained distinction in physics, as Faraday, Regnault and the present President of the Royal Society, Guthrie began his scientific career as a chemist. After a general course at University College, London, and taking his B.A. degree in 1852, he went to Heidelberg to work at chemistry under Bunsen and then to Marburg to work under Kolbe. Here he took the degree of Ph.D. After returning to England he acted as assistant, first to Frankland at Manchester, at the then recently founded Owens College, then to Lyon Playfair at Edinburgh. His first independent appointment came in 1861, when he became Professor of Physics and Chemistry at the Royal College, Mauritius.

In 1869 he received the appointment which he retained for the rest of his life, that of the Professorship of Physics in the Royal School of Mines, the institution which, eventually, gave rise to the great Imperial College of Science and Technology.

Two generations ago, when Guthrie was at the beginning of his career, physics was a much more purely experimental and descriptive science than it is now. The greater part of modern mathematical physics did not exist. It is true that Laplace, Fresnel, Fourier and Ampère had published their marvellous works, but, in order to appreciate the change that has taken place since that time, it is sufficient to call to mind that the idea of the Conservation of Energy was then only making its way and was waiting for its necessary foundation, a system of absolute measurement for thermal, magnetic and electric phenomena to be firmly laid. The first momentous step towards such a system had been taken by Gauss in the case of magnetism and was being extended by Wilhelm Weber to electricity. Joule's work was still in progress. Thomson, Clausius and Rankine were laying the foundations of thermodynamics and of the application of mathematical methods to electricity. Green's work was being re-discovered. But all these things were still matters for discussion among the leaders;

they were not yet included in the settled scientific territory to which a student had access.

The difference between the state of physics in Guthrie's early time and in the present becomes again strikingly apparent when we remember that Thomson and Tait's "Natural Philosophy" and Maxwell's "Electricity" were neither of them published till a good many years later. In those days, the best treatises (in English, at least) on Heat, Light and Electricity were the introductory parts of some of the larger books on Chemistry (*e.g.*, Vol. I. of Miller's "Chemistry"). In those days too, a chemical laboratory was the only school of scientific manipulation.

I have been led to this rather long digression by remembering that the transformation that Guthrie underwent, from chemist into physicist, was a fairly common phenomenon among his predecessors and contemporaries.

All Guthrie's work was characterised by independence and originality, and he had a remarkable gift for recognising that there was something worth studying even in the most familiar occurrences. He is probably best known by the results embodied in the series of Papers, published in the "Proceedings" of this Society, on "Salt Solutions and Attached Water," which led him to the discovery of the Cryo-hydrates and Eutectic Alloys. His experiments on the Conduction of Heat by Liquids were marked by great experimental ingenuity. Among the most remarkable of his discoveries was what he called "Approach caused by Vibration," and his experiments on the discharge of electricity by the approach of hot bodies were not recognised as being of fundamental importance only because they were made about 20 years too soon.

To this Society he stands in the relation of practically sole founder. It owes its existence to a little circular he sent out on his own responsibility in the autumn of 1873. In the spring of 1874 —just 40 years ago, therefore—the Society held its first meeting.

In addressing the Society, or reading a Paper, Guthrie's manner was sometimes almost portentously solemn, but in reality he was full of admirable humour. His strength of principle and the sacrifices he made to his sense of duty were not fully known during his lifetime, even to his most intimate friends. He died at the comparatively early age of 53, after a painful illness borne with uncomplaining fortitude.

All who knew Frederick Guthrie will agree that the Physical Society does well to keep alive his scientific reputation and to cherish his memory as a man.

XXIII. *Radiation of Gas Molecules Excited by Light.* *By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University, Baltimore.*

Being the First Guthrie Lecture, delivered February 27, 1914.

The emission and absorption of light by molecules, and the closely related phenomenon of dispersion, have led us to the conception of something within the atom which is capable of responding to light-waves, in much the same way as a tuning fork responds to sound-waves of the same frequency as its own. There seems to be no escape from this conclusion, and very elaborate mathematical treatments have been built up on this foundation, which explain in a more or less perfect manner many of the phenomena in question. If, however, we try to form some conception of just what is going on we find that we are still very much in the dark. Helmholtz explained absorption by the introduction of a frictional term into his equations of motion for the atom, and though this led at once to an expression which represented anomalous dispersion, it left us in ignorance of how the energy absorbed by the molecules was transformed into heat, or, in other words, how the mean velocity of the molecules was increased by the excitation of vibrations within them. Planck avoided this difficulty by considering that the energy abstracted from the beam of light is re-emitted, though at the time the only experimental evidence which could be cited was the phenomenon of selective reflection, which occurs only when the molecules are so densely packed together as to give us the liquid or solid state.

What became of the absorbed energy in the case of a gas? This is something that I have been looking for for many years. Personally I do not require a working model, but I never felt *completely* satisfied by an equation in which absorption is represented by a frictional term or selective reflection predicted by the occurrence of an imaginary quantity, as in Lord Kelvin's expression for sodium vapour, especially as no case of such selective reflection by a vapour was known. The subject which I have chosen for discussion this evening is the disposition made by the absorbing mechanism of the energy which it abstracts from the incident light, and the more complicated phenomena which occur when various absorbing mechanisms

of the atom are coupled together. All of this has, of course, a very direct bearing upon the structure of the atom, a subject which is receiving much attention at the present time. Our profound ignorance of the matter and inability to construct or imagine a model capable of representing the source of even the simplest emission spectra, make one almost regret the enormous amount of work which has been spent in tabulating the wave-lengths of the spectra of the elements. It is only within the last decade that the brilliant work of Zeeman, Lorentz, Sir Joseph Thomson and others have led to a definite conception of the constitution of the outer shell of the atom. What is inside of the egg can only be imagined at the present time.

The problem of the structure of matter is one which must be attacked simultaneously from many sides, for it is improbable that any single weapon will cause the surrender of the secret. The spectroscope alone proved itself powerless, and the first definite step in advance was made when Zeeman placed a source of light in a magnetic field.

One great difficulty lay in the fact that in all known methods of exciting spectra it was "the whole or nothing." Flames, arcs, sparks and vacuum-tube discharges set in operation simultaneously a host of vibrations within the atom and resulted in a complex of lines. While it is true that much has been learned from the circumstance that the spectra vary according to the method of excitation, our ignorance as to the forces in operation in the case of flames or sparks makes it difficult to interpret the phenomena.

My own line of attack has been to keep the molecules as cool and quiet as possible, and then excite them to radiation by the application of an alternating electromagnetic field of a definite frequency, which is more commonly referred to as monochromatic light. We can in this case be pretty sure of what we are doing to the atoms, if we are not too particular to ask for a specific definition of what we mean by an alternating electromagnetic field.

That this method of going at the thing has simplified matters somewhat you will see when I draw your attention to sodium vapour which emits only one of the yellow D lines. Much time and many experiments have been necessary to develop the technique of exciting luminescence in this way, for the conditions vary according to the element studied, some vapours emitting light only when reduced to a pressure of less than 0·001 mm., while others operate even at a pressure of several atmospheres.

The presence of a foreign gas is very detrimental, which is sufficient to explain the failure of my first experiment on the subject made over 15 years ago, the concentration of sunlight by means of a large condenser upon a flame very rich in sodium.

In the earlier part of the work the phenomena presented were very complicated, and it is only recently that simple types of emission, which could be studied quantitatively, and subjected to mathematical analysis, have come to light. It will, therefore, be best to review the subject in almost inverse chronological order, beginning with the simplest case of a vapour which exhibits a single absorption line and emits radiations similar in every respect to the exciting radiations when stimulated by a frequency equal to that of the line of absorption. This condition is very perfectly fulfilled by the vapour of mercury, which has an absorption line at wave-length 2,536 in the ultra-violet.*

At room temperature the vapour of mercury has a pressure of about 0·001 mm. This gives us, assuming a uniform distribution, one molecule in every cube whose sides are equal to the wave-length of the ultra-violet light employed in the experiments. If a beam of monochromatic light of wave-length equal to that of the absorption line 2,536, obtained by isolating the corresponding emission line of the silica mercury arc by means of a quartz monochromator, was focussed at the centre of an exhausted quartz bulb containing a drop of mercury (the whole at room temperature), it was found that the light was powerfully scattered by the vapour, photographs of the bulb made with a quartz lens showing the cone of rays much as if the bulb were filled with smoke. The greater part of the light, however, passed through the bulb without sensible reduction of intensity even if the diameter of the bulb was sufficient to give to the luminous cone its maximum extension, for the cone is brightest where the rays enter the bulb, the intensity diminishing rapidly as we pass along the cone, owing to the removal from the incident beam of the energy of just the right frequency for exciting resonance. For a pressure of 0·001 mm. it was found that the intensity of the effective part of the incident beam was reduced to half of its original value after traversing a distance of 5 mm. in the vapour. This determination was made by passing a beam of *parallel* rays

* "Selective Scattering, &c., by Resonating Gas Molecules," "Phil. Mag.," May, 1912.

through the vapour, and measuring the intensity of the scattered radiation along the path. If the intensity at the point of entrance is 100, the intensity after traversing a distance of 1 cm. is 25, while at a distance of 2 cm. from the point of entrance the intensity is but 6. This means that in a bulb 3 cm. in diameter the luminous cone will scarcely reach the opposite wall. Notwithstanding the astonishing stopping power of this highly attenuated metallic vapour for waves of just the right frequency, we find that a large proportion of the energy passes through the bulb without being influenced by the vapour. Experiments showed that this was due to the circumstances that the emission line had a finite width, and that its central portion only was scattered by the resonating molecules. This is made clear by Fig. 1, in which ABD represents the intensity distribution in the emission line of the arc and cBc' the central portion removed by the mercury vapour

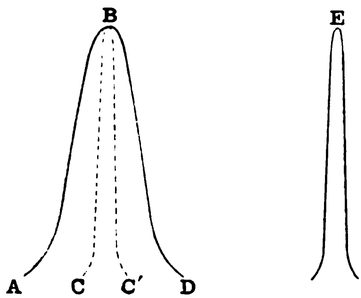


FIG. 1.

at room temperature. After passing through the bulb the emission line would appear furrowed by a fine black line of width cc' if we possessed a spectroscope of sufficient resolving power to show it. If the energy removed from the incident beam is re-emitted without change of wave-length by the vapour it is clear that the width of the spectrum line of the light given off by the cold mercury vapour in the bulb will be as shown by E in Fig. 1. In other words, we possess a method of obtaining light which is probably more homogeneous than any light obtained up to the present time. This is a matter which is being investigated quantitatively at the present time. Radiation scattered in this way without change of wave-length by resonating molecules I have named *Resonance Radiation*. We may term a bulb filled with vapour and emit-

ting light more highly monochromatic than the light which excites it a resonance lamp. With the light from a lamp of this description a photograph was made of a quartz bulb at room temperature containing a minute drop of mercury. The bulb appeared as if filled with ink, owing to the opacity of the mercury vapour for the rays. A drop of mercury placed on the top of a brass cylinder, heated to a temperature of about 5 deg. higher than that of the room, when photographed by the shadow method showed a column of vapour rising from the drop like black smoke. A lamp of this type has been used in many of the investigations of the scattering and absorbing power of mercury vapour, for the reason that it emits only waves which are in exact synchronism with the molecular resonators, whereas even the single line 2,536 isolated from the radiations of the mercury arc contains frequencies which are freely transmitted by the vapour.

Secondary Resonance Radiation.—It was found that the mercury vapour outside of the luminous cone traversed by the exciting rays also emitted light, a glow filling the entire bulb. Experiments showed that this was due chiefly to a secondary scattering of the light emitted by the directly excited molecules; in other words, the luminous cone of vapour acted as a light source which stimulated those portions of the vapour not actually traversed by the incident beam of light. The intensity of this secondary resonance radiation in comparison to that of the primary is surprisingly great, so great in fact that I was at first inclined to believe that it was due, in part at least, to a persistence in the luminosity of the rapidly moving molecules after they had passed through the region traversed by the exciting beam of light. Experiments showed, however, that the introduction of a thin quartz plate between the regions of primary and secondary resonance did not diminish the latter to any appreciable extent. This indicated that the phenomenon resulted from excitation by the light given out by the directly excited molecules, and did not result from a persistence of luminosity (phosphoresence), for the quartz plate is transparent to the ultra-violet light, but stops the moving molecules.

The comparatively great intensity of the secondary radiation results from the circumstance that at these low densities *no true absorption* exists; in other words, there is no transformation of energy. And now we come to a very important point, for we are at last in a position to measure the ratio of the scattered

to the absorbed energy, to define the conditions under which one or the other preponderates, and by varying the conditions to pass from complete scattering without absorption to complete absorption without scattering. Two methods have been found of making the measurements. We may compare the luminosity of the vapour with the luminosity of a surface of magnesium oxide when illuminated by the same beam of light. This is the direct method, and has been employed in the case of sodium vapour as I shall explain later on. Or we may measure the ratio of the intensity of the secondary resonance radiation to that of the primary. The value found for this ratio will show whether or not any true absorption has taken place, for it is possible to calculate the value of the intensity of the secondary radiation in terms of that of the primary resonance radiation if no true absorption exists. In making the experiments the conditions were made as simple as possible. A brass box was used furnished with windows of quartz, the

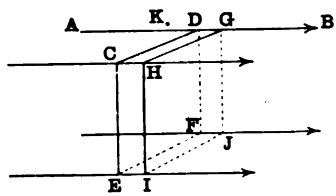


FIG. 2.

exciting beam of parallel rays of rectangular cross-section almost grazing the observation window. In this way it is possible to observe the primary and secondary resonance radiation through a minimum thickness of mercury vapour. Careful measurements showed that the intensity of the secondary radiation at a distance of about 1 mm. from the edge of the exciting beam was fully one-quarter of the intensity of the primary radiation observed within the beam of exciting rays. In some cases the value was very nearly one-third, but this was with an exciting beam of larger cross-section, and the explanation will be given presently. A rigorous calculation of the intensity of the secondary resonance under specified conditions is much to be desired, but the problem is a little more complicated than it appears to be at first, for each molecule is excited to a greater or less extent by the radiations from all of the other molecules. An approximation may perhaps be arrived at in the following way. We will consider

the incident beam AB, Fig. 2, of square cross-section (plane waves), and assume that all of the energy abstracted from it is re-emitted. We require the intensity of the secondary resonance radiation from a layer immediately above the plane CDGH of the thin cross-section of the exciting beam in comparison to the intensity of the primary resonance radiation within this same cross-section. The secondary resonance is excited by the energy poured out by the resonating molecules which lie in the path of the primary beam. The energy escaping from the rectangular cross-section figured above passes out through the four sides in equal amounts, for that which escapes through the surfaces CDEF and GHIJ remains within the beam and does not contribute to the excitation of the secondary resonance.

The excitation at a point, K, immediately above the exciting beam can, therefore, be regarded as dependent upon the energy stream passing through the side CDHG, which will be approximately one-quarter of the total energy radiated *laterally* from the cross-section figured. The total amount radiated laterally represents the energy abstracted from the primary beam by the molecules lying within the cross-section, and the intensity of the primary radiation will be proportional to this quantity. We should, therefore, expect the intensity of the secondary radiation to have about one-quarter of the value of the primary, as was found to be the case in the experiment. If true absorption exists the ratio will be very different. Suppose that, of the energy abstracted from the primary beam, one-half is re-emitted and one-half absorbed. The intensity of the primary radiation will now be only one-half as great as it was before. We can, however, raise it to its original value by doubling the intensity of the exciting beam. The amount of energy escaping from the sides of the cross-section figured is now the same as before, but only half of it is effective in producing secondary radiation, the other half being lost by absorption. The secondary radiation will now be only one-half as intense as it was before, while the primary will have the same value, the ratio being $\frac{1}{8}$ instead of $\frac{1}{4}$. We thus see that, by measuring the ratio of the intensities of the secondary and primary resonance radiation, we have a means of determining the ratio of scattering to true absorption. This makes a rigorous mathematical determination of the ratio which would be found in the case of complete scattering without any absorption much to be desired. The matter is peculiarly complicated,

for not only is a point lying outside of the primary beam illuminated by radiations coming from all points within the beam, but also by radiations from all points outside of the beam, and the same holds true for points within the beam. In other words, every molecule contributes, in a greater or less degree, to the illumination of every other molecule.

It would thus appear as if doubling the number of molecules in unit volume ought to more than double the intensity of the resonance radiation. This relation has not yet been investigated in the case of direct resonance radiation, but in the case of the fluorescence of iodine vapour, which is a more complicated phenomenon, a photometric study, made in collaboration with Mr. W. P. Speas, showed that doubling the vapour density by no means doubled the intensity. For example, we found that at a pressure of 3 the intensity was 24, while at a pressure of 6 the intensity was but 33. This is exactly the opposite of what is to be expected in the case of resonance radiation where all of the molecules are operating and there is no true absorption.

In the case of the iodine vapour, as the density of the vapour increases, the radiation from each molecule decreases as a result of the presence of its neighbours. The same effect is observed with a constant partial pressure of iodine, and an increasing pressure of some other gas, but iodine vapour is more effective in decreasing the luminosity of an iodine molecule than any of the other gases, and helium is the least detrimental, the gases arranging themselves in the order of their electro-negative character.

True Absorption.—The factor of true absorption makes itself manifest as soon as we admit air or some other foreign gas to the bulb containing the mercury drop. Even if the pressure is only a millimetre or two the effect is very marked. The intensity of the primary radiation is diminished to a certain extent, while that of the secondary radiation is reduced to a much greater degree for the reason mentioned a few moments ago. At a pressure of 6 mm. there is scarcely any trace left of the secondary resonance radiation, though the intensity of the primary is only reduced to one-third of its value in the complete absence of air, Fig. 3 (opposite page 201).

Huygens' Principle and Resonance Radiation: Selective Reflection.—Another point of considerable interest is the determination of the conditions under which it is possible to apply the principle of Huygens to the secondary waves emitted by

the molecular resonators. In certain theoretical discussions of absorption, the resonators are regarded as giving out waves which interfere destructively with the primary wave in the direction in which it is travelling, while in the opposite direction, there being no energy stream with which they can interfere, they unite into a wave which travels back towards the source, the phenomenon constituting selective reflection. While there is no doubt but that the resonators are close enough together to make the application of Huygens' principle justifiable, there are certain other factors which, it seems to me, must be taken into account.

In the first place, in a vapour at low pressure, the wave is obliged to pass by an enormous number of molecules before its intensity is much reduced. To give a numerical illustration, it was found that in the case of mercury vapour at room temperature the intensity was reduced to one-half its original value after the light had traversed a layer of the vapour 5 mm. in thickness. The pressure at room temperature is about 0·001 mm., which gives us about one molecule in every cube the sides of which are equal to the wave-length of the ultra-violet light. The light thus has to pass through 16,000 layers of molecules before losing one-half of its intensity, if we assume the molecules arranged in cubic order. If the radiations emitted by the molecules combined by Huygens' principle to form a regular wave, it would, in this case, constitute what we might term "volume reflection" as contrasted to surface reflection. It appears to me to be inconceivable that a reflection of this nature can occur, for the same reason that reflection cannot occur at the boundary between two media of different optical densities, if the transition is gradual instead of abrupt. The question may perhaps be raised as to why the molecules diffuse the light at all if there are many of them to the wave-length. We are accustomed to regard a medium in which the structure (so to speak) is small in comparison to λ as a homogeneous medium.

There is, however, in the present case another factor which doubtless has some bearing on the problem, namely, the circumstance that every molecule is excited to a greater or less degree by the radiations from its neighbours, all of which are moving at high velocities in all directions. This, it appears to me, would cause a random distribution of phase among the vibrations coming from the molecules, and would prevent

completely interference of the type considered in Huygens' principle.

Selective Reflection by a Dense Gas.—I have, however, observed that if the pressure of the mercury vapour is raised to several atmospheres regular reflection of a selective nature occurs at the inner surface of the bulb. The wave-length most strongly reflected is not quite in coincidence with the centre of the absorption line, but lies slightly on the short wave-length side of it. This is probably due to the circumstance that the refractive index of the vapour has an abnormally low value at this point, for the absorption line shows very strong anomalous dispersion. For a reflection from the inner surface of a quartz bulb we should expect a marked increase in the reflecting power for those values of λ for which the refractive index of the medium in the bulb was less than unity. The selective reflection of mercury vapour for wave-lengths in the vicinity of the 2,536 line should be investigated quantitatively. I have made only a qualitative investigation up to the present time, but as I have now learned how to control the intensity and width of the emission line of the mercury arc I feel certain that this can be done in a satisfactory manner.

I wish also to emphasise again the desirability of having a rigorous theoretical treatment of the emission of radiant energy, by resonating molecules, which give out again all of the energy which they abstract from the primary beam, both for a highly rarefied and a very dense gas. Much of the value of the experimental work will be lost if this is not done. Most important is the calculation of the relative intensity of the primary and secondary resonance radiation, under some condition verifiable by experiment. The best condition appears to me to be the case of an exciting pencil of parallel rays, of square cross-section, passing through the gas parallel to and almost grazing the window through which the observations are made.

Next in importance is the determination of what happens when the gas is dense enough to practically stop the incident rays before they have penetrated to a depth of more than one or two wave-lengths; if the intermediate condition can be examined, that too would be desirable. By this I mean the manner in which we pass from diffusion to regular or specular reflection, which I have examined experimentally. The regular reflection is not, however, nearly as powerful as we should

expect it to be if there were no absorption. As we increase the density of the vapour the intensity of the diffused radiation decreases and it is finally replaced by regular reflection. I do not, however, believe that the selective reflecting power is over 20 per cent., which means that the factor of true absorption has been introduced by the increase in the vapour density. I have not yet determined quantitatively the effect of increasing the density of the mercury vapour, as compared with the effect of raising the pressure in the same proportion by the introduction of some other gas. I am very sure, however, that true absorption is introduced to a greater degree in the latter case—that is to say, that collisions with foreign molecules are more effective in introducing absorption than collisions with mercury molecules. These questions I am now about to investigate; now that I am able to work with a source of constant intensity.

Case of Sodium Vapour.—Many of the phenomena which were discovered by photography in the case of the ultra-violet resonance of mercury vapour can be rendered visible by employing the vapour of sodium. The resonance radiation of this vapour I discovered in 1905,* heating the metal in an exhausted tube, illuminated by the rays from an oxy-hydrogen sodium flame brought to a focus by means of a large condensing lens. On gently heating the tube the path of the rays through the vapour was marked by a yellow glow, which drew back towards the side of tube through which the rays entered, as the vapour density increased, until only a thin skin of yellow light remained, which lined the inner wall of the tube, the image of the sodium flame appearing on the surface layer of vapour. Attempts to obtain regular selective reflection by increasing still further the vapour density failed, as a result of the chemical action of the vapour on the glass at high temperatures.

The experiment has been recently improved by L. Dunoyer, who employes a Bunsen flame charged with a spray of a solution of chloride of sodium, which is blown into the base of the burner by means of an atomiser. A chimney of sheet iron, with a square window, surrounds the flame, and an image of the window is projected upon the wall of the sodium bulb, by means of a quadruplet condenser, free from aberration if mono-

* Wood, "Phil. Mag.," August, 1905.

chromatic light is employed.* The bulb contains a little metallic sodium, carefully freed from hydrogen by distillation in vacuo, and is very highly exhausted. It is heated in a column of hot air rising from a tall chimney which surrounds a large Meker burner. The resonance radiation first appears at a temperature of about 125°C., a faint cone of yellow light appearing in the bulb. As the temperature is increased, it becomes brighter, and presently the secondary resonance radiation appears, a yellow glow filling the entire bulb. At a temperature of 180°C. the cone of primary resonance radiation disappears and with it the secondary radiation, and there remains only a brilliant square of yellow light on the front surface of the bulb, an image of the illuminated window thrown upon a resonating gas. It is as sharply defined as if thrown upon white paper, but has an intensity of only about one-tenth of the intensity obtained when a paper screen receives the rays coming from the condenser. In other words, the greater part of the light from the sodium flame passes through the vapour unhindered, the narrow cores only of the D lines being operative in causing resonance precisely as was the case with mercury vapour. If the bulb is allowed to cool slowly the first change observed is a fuzziness of the edges of the square image of the window, the appearance being as if the image suddenly went "out of focus." This is due to secondary radiation, and in its first stages it is confined to a shallow layer of vapour close to the wall. As the bulb cools down the cone of yellow light appears again, the phenomena previously described taking place in inverse order.

If, now, no absorption occurs in the case of sodium vapour, as appears to be the case with mercury vapour, we ought to obtain as brilliant an image on the vapour as on white paper, *provided our light source emits only light capable of exciting resonance.*

This subject has been investigated in collaboration with M. Dunoyer during the past winter. We fed the flame of the Meker burner with the spray of a solution of NaCl of varying concentration and measured the ratio of the diffusive power of a patch of MgO, formed on the outer wall of the bulb, to that of the diffusive power of the vapour within. It was found that the intensity of the resonance radiation from the sodium vapour was practically independent of the amount of

* L. Dunoyer "Sur l'Aberation de Sphéricité dans les Objectifs" ("Journal de Physique," 1913).

sodium in the flame, *i e.*, of the total intensity of the source. This means, of course, that the *cores* of the D lines, which are alone effective in exciting resonance, do not increase much in intensity as we increase the amount of sodium in the flame. The ratio above referred to varied enormously however. With a solution made by diluting a saturated solution with 30 parts of water the ratio was 15 : 1—that is, the MgO was 15 times brighter than the surface layer of resonating sodium molecules in the bulb. With the solution diluted with 1,000 parts of water the ratio was 4 : 1 and with a still more dilute solution 3 : 1. The ratio could not be made smaller than this on account of the feeble intensity of the flame. The decrease in the ratio

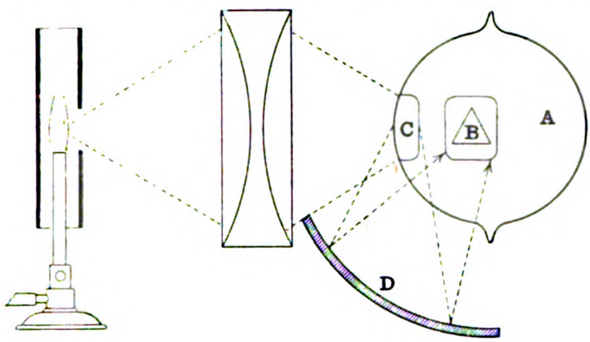


Fig. 4.

Experiment showing that the diffuse reflecting power of sodium vapour is equal to that of magnesium oxide for the highly homogeneous light of a sodium resonance lamp. A is a glass bulb, highly exhausted, containing sodium vapour. C is the image of the sodium flame thrown upon the vapour by the condensing lens. This image has an intensity of about one-fifth of the intensity shown by MgO under the same illumination. B is a triangular patch of MgO upon which an image of C is thrown by the concave mirror D. The MgO triangle cannot be distinguished from the luminous sodium vapour.

results from the circumstance that the width of the exciting lines (D lines) decreases as the amount of sodium in the flame is decreased; in other words, we are working with light more nearly in exact synchronism with the resonators. It is very important to know what the ratio would be if our exciting light were still more homogeneous. In this case we should have a ratio of 1 : 1, if the sodium vapour exhibits complete scattering with no trace of true absorption. It is impossible to accomplish anything in this direction by reducing further the amount of sodium in the flame, but by making use of the principle of the resonance lamp we can investigate the matter.

A single bulb was made to serve both as a resonance lamp and as a screen for measuring the reflecting power of the vapour, Fig. 4. An image of the sodium flame was thrown upon the side of the bulb and by means of a concave reflector, which had an effective aperture represented by F. 1 (formed by silvering one surface of double convex lens), an image of the spot of resonance radiation was thrown back upon the bulb of sodium vapour, one half of it being received by a small triangle of magnesium oxide, the other half by the vapour. Under these conditions it was found that there was absolutely no difference in the diffusive reflecting power. This means that the vapour of sodium at a pressure of probably less than 0·001 mm. has a reflecting power as great as that of the whitest paper or MgO, for light of exactly the right wavelength.

These experiments make it seem probable that absorption, as we usually understand the word, is a secondary action, resulting from reciprocal actions between the molecules. M. Dunoyer has made experiments on the effect of hydrogen on the intensity of the sodium resonance radiation and has found that the intensity is reduced practically to zero if hydrogen is present at a pressure of 10 mm. With helium at a pressure of half an atmosphere there is still considerable luminosity.

This indicates that the sodium molecule is far more sensitive to disturbances from neighbouring molecules than iodine, for which vapour the intensity of the fluorescence is reduced only from 100 to 35 by hydrogen at a pressure of 10 mm.

Now, the intensity of the iodine fluorescence decreases tremendously as the vapour density increases. From photometric observations made in collaboration with Mr. Speas I have calculated that the intensity of the radiation from an iodine molecule is reduced from 100 to 35 by a pressure increment of 0·25 mm. produced by increasing the density of the iodine vapour.*

Sodium is probably much more sensitive still, and the factor of true absorption is undoubtedly introduced by the reciprocal action between the molecules at even the very low pressure used in the experiments which have been described.

Thus far we have considered the type of resonance which results in the re-emission of radiant energy of the same type (*i.e.*, wave-length) as that of the exciting radiations. It has been studied for two cases, mercury vapour and sodium vapour,

* "A Photometric Study of the Fluorescence of Iodine Vapour." ("Phil. Mag.," March, 1914.)

the ultra-violet absorption (so-called) line of Hg at wave-length 2,536 and the D lines of sodium operating in this way. It appears probable that what is commonly spoken of as absorption results from some action upon the molecules of neighbouring molecules, for in both cases we find that an increase of vapour density or the presence of some chemically indifferent gas diminishes the resonance radiation and increases the factor of true absorption. It is to be clearly understood that by examination of the transmitted light we are powerless to discriminate between the two cases. The appearance of the spectrum is the same regardless of whether the molecules re-emit or absorb the radiations which they remove from the exciting beam.

The action of a molecule in destroying the resonance radiation of a neighbouring molecule appears to depend upon the electronegative quality of the molecule only. If the gas is strongly electronegative, resonance radiation only appears at very low pressures (bromine) or not at all (chlorine). If less strongly electronegative (iodine) we have radiation even when the pressure amounts to several millimetres, while with an electropositive molecule (mercury) resonance radiation persists even when the pressure is as high as several atmospheres.

Of all the gases, helium appears to be the least destructive in its action upon molecular radiation; at least this is true for iodine vapour, and M. Dunoyer and I have recently found that it is possible to have a fairly bright resonance radiation of sodium in helium at a pressure of halt an atmosphere.

The addition of a chemically indifferent gas not only changes the phenomenon of resonance radiation into absorption, but also increases the width of the absorption line as observed with a spectroscope. The line becomes fuzzy and is less black at the centre. I have not yet examined the action of different gases upon the absorption lines to see whether helium at, say, 50 mm. and CO_2 at 2 mm. have the same effect upon the absorption lines, which are the pressures which produce the same destructive action upon the fluorescence in the two cases. It is my belief, however, that the electronegative quality of a gas will not have much effect upon the change produced in the appearance of the absorption lines.

We now come to the second part of our subject. In many cases the molecule, when excited by monochromatic radiation, emits not only radiation of this same wave-length, but also

other wave-lengths which form what I have named Resonance Spectra. It seems as though there were numerous vibrating systems in the atom, the excitation of one system being communicated to the others in some way. Even the mercury 2,536 line and the D lines of sodium cannot be considered as due to a simple isolated vibrator, for I have found that mercury vapour can be caused to emit the 2,536 line when excited by extremely short ultra-violet waves, below wave-length 2,000 in the spectrum, and sodium vapour can be caused to emit a yellow band at wave-length 5,890 when excited by blue-green light.

We will consider first an experiment which I suggested in the Paper of 1905 already referred to, and which I have just brought to a successful conclusion in collaboration with M. Dunoyer. The question which we wished to solve was whether the mechanisms which produce the D lines are connected; in other words, if sodium vapour is excited to radiation by monochromatic light of the wave-length of D_2 only, will it emit light of the wave-length of D_1 as well as that

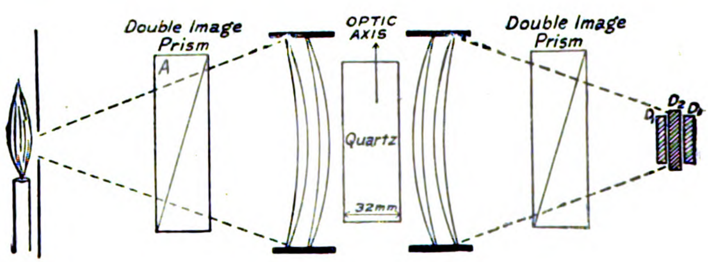


FIG 5

of the exciting radiation (D_2)? For this experiment we require some means of obtaining a powerful beam of light containing only the radiations of one of the two sodium lines. This was accomplished by means of a device which I described many years ago, depending upon the double refraction of quartz.

I have since improved this polarisation method, so that practically no light is lost (*see* Fig. 5).

Double-image prisms are used instead of nicols, and the two images which contain only D_2 light are united by a suitable rotation of the second prism, the D_1 images lying to the right and left. The separation of the two wave-lengths is effected by a block of quartz 32 mm. in thickness, cut parallel to the

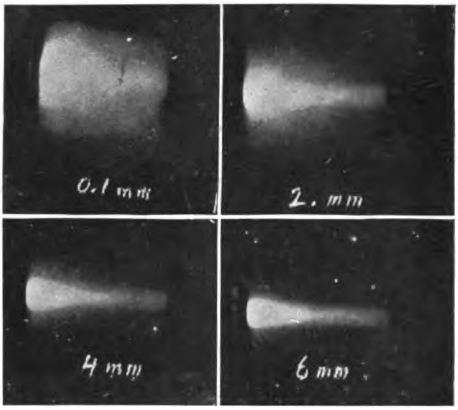


Fig. 3.

Cone of primary resonance radiation surrounded by the glow of the secondary resonance radiation. The latter has disappeared in the fourth picture as the result of admitting air at a pressure of 6 mm. "Phil. Mag.," May, 1912.

Fig. 6.

To face page 201.]

axis, which causes the two D lines to emerge polarised at right angles to each other. The monochromatic light obtained in this way is represented in Fig. 6, in which the upper portion of the slit is illuminated by D_2 light and the lower by the light of the sodium flame.*

With this apparatus we succeeded in exciting a bright resonance radiation of the sodium bulb by the light of D_2, and with an exposure of five hours obtained the spectrum of the emitted light, a single line only (D_2) with no trace of D_1. *It is thus possible to have sodium vapour emitting one only* of the D lines, and the experiment is a very good illustration of how spectra may be simplified by exciting them by means of light-waves.

We have also photographed the spectrum of the resonance radiation of sodium vapour excited by the two D lines, and have found that the emitted lines have practically the same intensity. Now, it is well known that in the sodium flame (which was used for the excitation) D_2 is much stronger than D_1, and it is very surprising to find that this same difference does not appear in the case of the resonance radiation. I have made a study of the relative intensity of the two lines in the case of flames, and have found that, with a flame very feebly coloured with sodium, D_2 is fully four times as intense as D_1. With a flame very heavily charged with the vapour D_2 is certainly not over 1·3 times as bright as D_1. Two photographs were made in coincidence simultaneously, with an exposure of half an hour, the light of the very intense flame passing through a piece of densely smoked glass before entering the instrument. The light from the feeble flame was reflected by the comparison prism. The smoked glass was so chosen that the intensity of the bright flame was reduced to almost exactly that of the feeble flame. To obtain the large ratio it is necessary to use a flame much less coloured than the flames commonly employed.

In the case of the D lines there seems to be no connection between the two vibrating systems, and I have never been able to get the D lines by ultra-violet excitation at the point of the spectrum occupied by the second members of the principal series. It may, however, be possible to do this with the improved methods.

* A full description of the apparatus will be found in the "Phil. Mag." for March, 1914, from which Figs. 3 and 6 are taken.

We will now examine some cases in which we have a connection between the vibrators.

Resonance Spectra.—It will be impossible to give anything more than a very brief outline of the extremely complicated relations which exist in the case of these remarkable spectra, which appear to become more and more involved the more we study them. A very complete account of what is known at the present time regarding the case which has received the fullest study, will be found in the "Philosophical Magazine" for November, 1913, and the "Phys. Zeit," December 1, 1913.*

The discovery of resonance spectra was made in the course of some experiments on the fluorescence of sodium vapour. It was found that if the vapour was illuminated with monochromatic light of wave-length corresponding to that of one of its many thousand absorption lines, it emitted a spectrum consisting of widely separated lines spaced with remarkable regularity along the spectrum. The phenomenon is exhibited only by sodium vapour of considerable density, *i.e.*, at a pressure of the order of magnitude of a millimetre or two. Its absence in the case of vapour at such low pressures as are used for the study of the resonance radiation (D line emission) may be due to its relative faintness, or perhaps to the circumstance that it results from molecular aggregates which do not form at the low pressures. This is a point which will probably be settled by some investigations now in progress.

The absorption spectra of iodine and bromine are very similar to that of sodium. With my plane grating spectrograph at East Hampton, the focal length of which is nearly 14 metres, I have found that there are about 35,000 absorption lines in the visible spectrum, 115 having been counted in a region no wider than the distance between the D lines of sodium.

Now, since we obtain a different emission spectrum for each absorption line which we excite by monochromatic light, we are confronted with the case of an element which can theoretically be made to emit *many thousands of spectra, no two of which are precisely alike.* The number of spectra possible is vastly in excess of the total number of absorption lines, for we may excite two or more lines simultaneously, in all possible combinations, so that *the total number of different spectra possible*

* Wood, "Resonance Spectra of Iodine under High Dispersion."

is practically infinite. In fact, it is only by taking special precautions that a single absorption line can be excited, for the lines are so close together that most of the emission lines of metallic arcs, which are used for the excitation of the vapour, have a natural width of such magnitude that they cover from two to a dozen of the absorption lines. The study of these resonance spectra has appeared to me to be of the greatest importance, for it is practically the only case in which we have any exact knowledge of the nature of the excitation. In flame, arc and spark spectra we know practically nothing about the forces which are at work upon the molecule, while in the case of resonance spectra we can be reasonably sure that we are subjecting the molecule to alternating electromagnetic forces of a definite frequency, and nothing else. Of course, these forces may bring other factors into play by dissociating the molecules or breaking up molecular aggregates, though I regard the fact that the emitted light is strongly polarised as evidence against this. Experiments made in Prof. Pupin's laboratory at Columbia University have shown that if an armature, free from current initially, is rotated in an alternating magnetic field the armature will deliver a current made up of a large number of different frequencies increasing by constant increments, and furnishing an interesting analogy with the resonance spectra.

I will now give a brief résumé of the results which have been obtained with iodine vapour, omitting most of the experimental details, which are very fully described in the Papers already referred to.

The vapour is used at room temperature in highly exhausted tubes, the excitation being by the radiations from a quartz mercury arc, sorted out by suitable ray-filters. The most careful study has been made in the case of the excitation by the green line (5,461), which can be made to cover from one to 10 absorption lines, by varying the watt consumption of the lamp. By interposing a ray-filter of bromine vapour it is even possible to remove some of the frequencies of the broadened green line, and so " throw out of action " the iodine absorption lines which happen to coincide with the bromine lines. The resonance spectra excited in this way have been photographed in the fourth order spectrum of a 6 in. plane grating used in conjunction with a collimator and a specially constructed Cooke portrait objective of about 130 cm. focus.

The vapour of iodine was contained in long glass tubes,

blown out to a small bulb at one end, an image of the mercury arc being formed along the axis by means of a large condenser.

It was found in the earlier work, that if the resonance spectrum, excited by the green mercury line emitted by the quartz arc operating under normal conditions, was photographed under high dispersion, the resonance lines were resolved into close groups. This was found to result from the circumstance that the green line was wide enough to cover seven absorption lines. By operating the lamp at a lower temperature—that is, with a small voltage drop across its terminals—the width of the green line can be reduced until it covers but a single absorption line, or at most two. The resonance spectrum is now found to be much simpler, the complicated groups being replaced by single lines or by pairs of lines. By raising the voltage of the exciting lamp, the number of lines in the groups can be gradually increased, for the green line broadens and covers other absorption lines as the watt consumption increases.

The groups are spaced at nearly equal distances along the normal spectrum, the distance between them increasing by a nearly constant amount as we pass from group to group towards the red end of the spectrum. There are slight variations, however, from the law of constant second difference, which cannot be explained by errors of measurement, for the wave-lengths have been determined to within 0·02 A.°E. in the case of the photographs made in the fourth order spectrum.

In my last Paper I gave a table of wave-lengths for the lines in all of the groups for the excitation by the broadened green line. The wave-lengths for the single lines and pairs, which replace the groups, when the excitation was by the narrowed green line (covering one absorption line only), were recorded on the photograph only, and the differences and second differences were not given. It may, therefore, be well to record them here in the form of a table.

The study of this comparatively simple spectrum is somewhat complicated by the circumstance that at two or three points we have single lines instead of pairs, and one or two of the pairs are made up of components of very unequal intensity, the components being a little closer together than is the case with the majority of the pairs. The spectrum which we are now considering is reproduced in Fig. 1, Plate XV., of the Paper referred to above. Some of the fainter lines result from excitation by the yellow mercury lines, which were not screened

off in this case. Two pairs in the series are missing altogether, and I have divided the difference in λ between the adjacent pairs by 2 in these cases.

The left-hand table gives the wave-lengths of the longer wave-length components of the doublets, the right-hand table the components of shorter λ. Absent members are indicated thus ......, and in the case of the single lines I have placed them in the left-hand table as this appears to be their proper place.

λ	*Differences.*			λ	*Differences.*	
6560·7	......	—	....	6558·4	......	—
....	......	82·0	....	....	......	82·0
6396·3	......	—	....	6394·3	......	—
—	......	79·7	....	—	......	79·9
6316·6	......	—	....	6314·4	......	—
—	......	78·6	....	—	......	78·3
6238·0	......	—	....	6236·1	......	—
—	......	77·1	....	—	......	77·0
6160·9	......	—	....	6159·1	......	—
....	......	75·0	....	....	......	75·0
6010·8	......	—	....	6009·1	......	—
—	......	73·0	....	—	......	73·9
5937·6	......	—	....	5935·2	......	—
—	......	71·7	....	—	......	70·7
5866·1	......	—	....	5864·5	......	—
—	......	70·1	....	....	......	69·1
5796·0	......	—	....	—	......	—
—	......	69·4	....	....	......	—
5726·6	......	—	....	5657·1	......	—
—	......	67·8	....	—	......	67·6
5658·8	......	—	....	5589·5	......	—
—	......	66·8	....	—	......	64·5
5592·0	......	—	....	5525·0	......	—
—	......	65·5	....	—	......	64·3
5526·5	......	—	....	5460·7	......	—
—	......	65·8	....	—	......	—
5460·7	......	—	....	—	......	—

As will be seen from the table, the second differences vary in an irregular manner. The doublets are very clearly defined in the red and orange of the spectrum, but in the region between the green and yellow mercury lines they are not so pronounced, being replaced by single lines in two cases, and by a single line with a fainter series on the short wave-length side in the case of the one at 5726·6.

Multiplex Excitation.—By raising the terminal voltage of the lamp (by gradually cutting out the resistance in series with it) other lines make their appearance to the right and left of each doublet, until we finally have groups containing as many as a dozen lines. This occurs when the green mercury line has covered eight iodine absorption lines.

The total width of each group of lines is about 30 times as great as the width of the group of absorption lines covered by the mercury line. The groups in the immediate vicinity of the exciting lines are very similar in appearance, and the regularity of their disposition along the spectrum reminds one of the diffraction spectra exhibited by a grating of small dispersion. For convenience in referring to them we may designate them as groups of the +1st, +2nd, +3rd, &c., *orders*, adopting the same nomenclature as in the case of grating spectra. Those on the short wave-length side of the exciting line we may designate −1st, −2nd, −3rd orders.

I have observed as many as 20 orders on the red side (*i.e.*, + orders) and two or at most three on the short wave-length side (*i.e.*,—orders). These latter orders constitute exceptions to what is known as Stokes's Law. In the case of the excitation by the green line the —order groups are extremely faint, while in the case of the excitation by the yellow lines they are very strong. In other words the exception to Stokes's law become more conspicuous as we excite with vibration of lower frequency. The same thing was found in the case of sodium vapour.

Origin of the Groups.—The key to the question as to how the groups originate lies in the fact that a group appears at the point of the spectrum occupied by the exciting line. This group we may call the group of zero order, and as I have already said it is about 30 times as wide as the group of eight iodine absorption lines, which we are stimulating and which lie at the centre of the zero order group.

It is a little difficult to express in words the rather complicated phenomena involved. In the Paper referred to I have attempted to account for the formation of the groups, but as explanation may be found somewhat long-drawn-out and difficult to follow, I will try to give it in a simplified form in the present report. Let us suppose that we are stimulating three iodine absorption lines instead of eight. These lines are the lines 1, 2 and 3 of Fig. 7. They are so close together that the spectroscope used for photographing the resonance spectra could not possibly resolve them. The width of the group of lines 1, 2, 3 and A, B, C, should be about $\frac{1}{30}$th of the distance between B′ and A′ to have the diagram in correct scale. The stimulation of absorption line 1 causes the vapour to emit the same wave-length which gives us line A (immediately below

line 1 in the diagram). This is what we call resonance radiation, and we may designate this line as the RR line, The vapour emits in addition the lines A of 1st order group and A of 2nd order group, &c. If this were all, we should have a resonance spectrum of the simplest type, consisting of a series of equidistant single lines.

In the earlier investigations I was of the opinion, that resonance spectra were of this nature, but it is now certain that such is not the case, for we should have, under these conditions, each of the absorption lines 1, 2 and 3 giving a series of equidistant lines, which, if the spacing were the same, would coincide, while if the spacing were different for the three series arising in this way we should have groups of three lines each, similar in appearance but with their components separated by increasing amounts as we pass from group to group. There would, in this case, be no group but only a single line (in reality three close unresolved lines at the position of zero order).

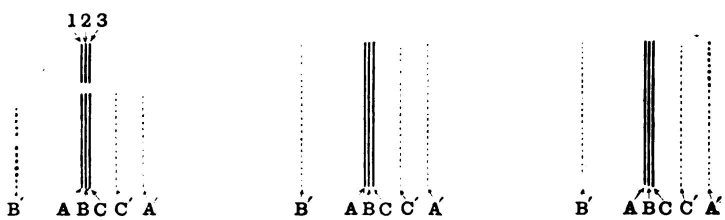


Fig. 7.

I am forced to the conclusion that absorption line 1 gives us, in addition to the lines A, the companion lines A′. Absorption line 2 gives us the lines B and the fainter companions B′, which we will assume to lie to the left of the strong lines in this case.

In the same way absorption line 3 gives us the lines C and the companions C′, the companions, in this case, lying to the right of, and nearer to, the main line. The main lines A, B and C of each group appear superposed with the resolving power obtained in the first order spectrum of the grating, and form what I termed the core of the group in the Paper referred to. In the photographs made in the 4th order spectrum they appear slightly separated, showing that the spacing of the three series is not quite the same. This explanation accounts for the similarity in the appearance of the groups, the fact that the distance between the components is essentially the same,

and the fact that we have a similar group (zero order) at the position of the exciting line.

In the communication to the "Philosophical Magazine" there will be found a diagram of the changes which occur in the groups resulting from the addition of new companion lines as the broadening mercury line covers more and more of the iodine absorption lines.

One of the most important questions, which I hope to solve next year, is whether the nature of a resonance spectrum is the same when the exciting line falls on the right-hand edge of an absorption line as when it falls on the other edge; in other words, when the frequency of the light is very slightly greater or less than the frequency of the absorption line.

Polarisation of the Light of Resonance Spectra.—The circumstance that the light emitted by the iodine vapour is strongly polarised, and that the polarisation is equally marked in all of the lines of the resonance spectrum, indicates that the emission of light is the direct result of vibrations forced in the molecule by the light waves, and does not depend upon dissociation and recombination. A short communication of this subject will be found in the "Phil. Mag." for November, 1913, illustrated by photographs of the resonance spectrum crossed by the Savart fringes, which give the measure of the polarisation.

Radiation of Gases produced by Ultra-Schumann Waves.—We come now to another side of the subject of which I should like to make brief mention.

Several years ago I made some experiments with a view of ascertaining whether the absorption of Schumann waves by air or oxygen was accompanied by an ultra-violet flourescence. A powerful condenser spark was allowed to play against the under side of a metal plate perforated with a small hole* at the point against which the spark played. It was found that if a photograph of the region above the plate was photographed in a dark room, with a quartz lens, a luminous "squirt" appeared in the pictures issuing from the hole. The spectrum of the squirt showed that the light was made up chiefly of the so-called "water band" of the oxy-hydrogen flame, together with a number of the stronger nitrogen bands, *with no trace whatever of any of the spark lines*, showing that the "squirt" was not the light of the spark diffused by dust in the air. It

* "A New Radiant Emission from the Spark." "Phil. Mag.," Oct., 1910.

was found to be much more intense in nitrogen than in air, and much less intense in oxygen. If the hole was covered with a very thin (1 mm.) plate of fluorite, however, all trace of the squirt disappeared, which made it appear as if the luminosity must be produced by some emission from the spark other than the Schumann waves, which are transmitted by lenses and prisms of fluorite.

During the past winter the subject has been investigated again in collaboration with G. Hemsalech and many new and very remarkable phenomena have come to light. In the first place we have found that even a rather feeble air blast across the "squirt" destroys its luminescence at the spot traversed by the jet, though it is quite as bright both below *and above* the jet as before. This shows that *air in motion* does not become luminous. We are unable to explain this phenomena to our complete satisfaction; at first we thought that the air might remain luminous after the passage of each spark (phosphorescence) as in the beautiful vacuum tube experiments made by Strutt, the action of the air-blast being the continuous sweeping away of the luminous gas, but long exposures, with feeble air jets, failed to show any evidence of luminosity in the air jet after its passage across the squirt.

With nitrogen, however, the reverse is true. The luminosity is much more intense (perhaps 10 times) in the jet, and the spectrum of the light from the moving jet shows the nitrogen bands, with no trace of the "water-band," which predominates in the "squirt" (in air) above and below the moving jet of nitrogen. If oxygen is added to the nitrogen the luminosity becomes much less, which agrees with the powerful influence of oxygen in destroying the fluorescence of iodine. If we place a quartz prism in front of the quartz lens, in making our photographs we find that the luminosity in the moving nitrogen jet is displaced to one side of the monochromatic image of the squirt (water-band image). This displacement results from the difference of the wave-length of the light emitted by the nitrogen (the nitrogen bands). Similar displaced images were obtained with jets of hydrogen, carbon dioxide and coal-gas, each gas giving a characteristic spectrum. The phenomena are too complicated to be given in much detail in the time at my disposal and I shall have to refer you to the Paper published by Mr. Hemsalech and myself which will appear shortly.*

* "Phil. Mag.," May, 1914.

I have still more recently endeavoured to find some vapour or gas which would emit visible radiations when passed through the squirt, as the phenomena could be much more satisfactorily investigated if we were not obliged to work always in the dark with photographic methods. The first substance which I tried was iodine, and as the detrimental effect of oxygen was well known the vapour was carried across the squirt in a jet of nitrogen. A few iodine crystals were placed in the glass tube which delivered the nitrogen jet to the "squirt." Even at room temperature a faint bluish-green fluorescence was visible, which became quite bright if the tube was slightly warmed.

If air was used instead of nitrogen, the iodine vapour refused to respond.

A fluorite plate was now placed over the hole above the spark, and the green fluorescence at once appeared, but faded away very rapidly, disappearing entirely in about 15 seconds. The fluorite plate was moved a trifle and the fluorescence at once appeared again. It was at once apparent that the spark vapours formed a deposit on the fluorite plate which made it opaque to the rays which excited the fluorescence.

It is rather remarkable to find iodine vapour fluorescing brightly when mixed with nitrogen at atmosphere pressure when we remember that the fluorescence excited by blue-green light is reduced practically to zero by nitrogen at a pressure of only 8 cm. I have not yet examined the spectrum of the iodine vapour excited by the spark rays. The colour of the light is bluish-green, in contrast to the yellowish-green fluorescence excited by visible light, and it is evidently much less affected by the presence of a foreign gas. The discovery of the deposit formed upon the fluorite plate suggested at once that the failure to observe any trace of the "squirt" in air (even with long exposures) when the fluorite was used was in all probability due to the formation of this deposit.

I accordingly repeated the experiments with air and nitrogen, cleaning the fluorite plate every 15 seconds, and succeeded in getting the "water-band" squirt, as well as the one of nitrogen. It was found, however, that it was necessary to give an exposure about 80 times as long as that necessary without the plate to obtain an image of equal intensity. This indicates that a plate of fluorite 1 mm. thick transmits only a trifle over 1 per cent. of the energy of the radiation which excites these gases to ultra-violet fluorescence. The waves are thus undoubtedly shorter than those discovered by Schumann, since these pass readily through lenses and prisms of fluorite.

Attempts are being made at the present time to develop a method of measuring, at least roughly, the wave-length of these radiations.

In conclusion, let me say that I have endeavoured to show what has been learned from the study of these vibrations, forced in the molecule by means of light waves.

It is my hope that the study of resonance spectra and related phenomena may, in time, furnish one number in the "combination" necessary to unlock the secret of molecular radiation.

XXIV.—*A Graphic Treatment of Cusped Wave-fronts and of the Rainbow.*

By William R. Bower, *B.Sc., A.R.C.S., Technical College, Huddersfield.*

CONTENTS.

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§ 1. *Introduction.*

A method of drawing the cusped wave-fronts produced by refraction and reflection at a spherical surface, and a graphic treatment of the elementary theory of the rainbow are described in the Paper. The properties of the Centre of Junction discovered by Young (Phil. "Trans.," 1801, XCI., p. 23) and independently by Cornu ("Science Abstracts," V. (1902), 1835) form the basis of the constructions.

§ 2. *Refraction by a Spherical Surface. Position of the Junction-centre.**

(I.) In Fig. 2, when BY, BY′ are the incident and refracted directions of the chief-ray of a pencil and CY, CY′ are drawn

* An account is given in a Paper by Mr. W. R. Bower on "A Graphic Method of Optical Imagery," Proc. Phys. Soc., Lond., Vol. XXV. (1913), pp. 171–176.

from the centre C perpendicular to BY, BY′ respectively, then CY$=\mu$. CY′. CKZ′Z and YY′K are at right angles; K is the junction-centre and Z, Z′ the aplanatic pair of points for the chief ray. Also CZ$=r$. μ and CZ′$=r/\mu$.

Draw KF′ parallel to BYZ. Then F′ is the second principal focus of the small pencil of meridian rays associated with the chief ray.

(II.) As B moves further from A—that is, as the angle of incidence φ increases—so does the angle of refraction φ', also $\varphi-\varphi'$, and, therefore, the angle BG′C.

Because YK and CZ are always perpendicular, K and F′ rise respectively from C and G′; finally F′, K, Y′, Z′ are coincident. Y then coincides with B and YY′ is tangential to the circle of radius r/μ.

(III.) Write y, y', f, f' for BY, BY′, BF or F′K and BF′ respectively. Then, since the triangles βZC, Y′CY are similar,

$$\beta Z/CZ=Y'Y/CY.$$

Also, since the triangles YZC, KYC are similar,

$$CZ/YZ=CY/KY.$$

$$\therefore \qquad YZ/\beta Z=KY/Y'Y=BF'/BY',$$

or

$$YZ/\beta Z=f'/y'.$$

§ 3. *Reflection by a Spherical Surface. Position of the Junction-centre.*

(I.) Let (Fig. 1) the reflected portions, BY′, B_1Y_1', of two parallel rays, BY, B_1Y_1, intersect at E′. Draw CY, CY′ perpendicular to BY, BY′ respectively. Then k, the intersection of YY′ and BC, is the junction-centre for the chief ray YBY′.

(II.) The angle BE′B_1=angle YBE′—angle Y_1B_1E′. The angle BCB_1=angle YBC—angle $Y_1B_1C=\frac{1}{2}$ angle BE′B_1. Then the angles BE′B_1, BC′B_1 are equal, and B, B_1, E′, and the mid-point, C′, of BC are concyclic.

(III.) As B_1 moves up to B, the locus of E′ is the fixed line BE′. When B_1 is very close to B, the angle BB_1C' becomes sensibly a right angle. Also the angle BE′C′.

Then E′ sensibly coincides with the mid-point, F′, of BY′.

Thus F' is the second principal focus of a small pencil of meridian rays incident at B.

∴ Also $F'k$ is parallel to BY; and F, the mid-point of BY, is the first principal focus.

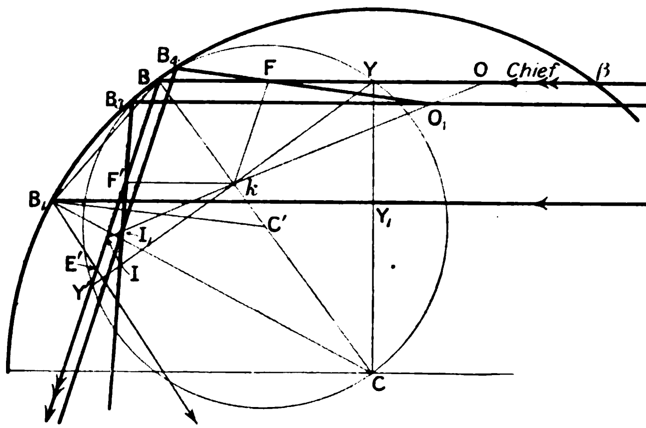


Fig. 1.—Reflection by a Spherical Surface.

(IV.) *To prove that the line joining an object-point, O, and its meridional image-point, I, passes through the junction-centre, k.*

A ray parallel and close to BY, incident at B_3, is reflected through F'. A ray, O_1F, from any object-point, O_1, on the line B_3O_1, incident at B_4, is reflected parallel to BF'. Let B_4I_1 and B_3F' intersect at I_1. I_1 is the primary or meridional image of O_1.

Join I_1O_1 and I_1k. Then sensibly

$$\frac{B_3O_1}{BF}=\frac{B_3B_4}{BB_4}=\frac{I_1B_3}{I_1F'},$$

$$\therefore \quad \frac{B_3O_1}{I_1B_3}=\frac{F'k}{I_1F'}.$$

∴ the triangles $I_1F'k$, $I_1B_3O_1$ are similar. Then k is on the line I_1O_1.

When B_3O_1 is very close to BY, O_1 coincides with O and I_1 with I. Thus the position of the meridional image-point on the reflected ray is collinear with the junction-centre, k, and the object-point, O.

(The position of the junction-centre may also be inferred from the general construction (§ 2 (I.)) by putting $\mu = -1$.)

(V.) Since $\frac{\text{BF}}{\text{BO}} = \frac{\text{F}'k}{\text{BO}} = \frac{\text{IF}'}{\text{IB}} = \frac{\text{IB}-\text{F}'\text{B}}{\text{IB}}$ $\therefore \frac{\text{BF}}{\text{BO}} + \frac{\text{BF}'}{\text{BF}} = 1$;

and since $\text{BF} = \text{BF}' = \text{BY}/2$,

$\therefore$ $1/\text{BO} + 1/\text{BI} = 1/\text{BF}' = 4/\text{B}\beta = 2/r.\cos\varphi$.

(VI.) When O is on the circumference, $\text{BI} = \text{B}\beta/3$.

§ 4. *Distribution of Foci of a Small Incident Meridian Pencil of Parallel Rays.*

(I.) In Fig. 2 let the chief ray be incident at B. Find the junction-centres, K, k', k'' . . . between the incident and

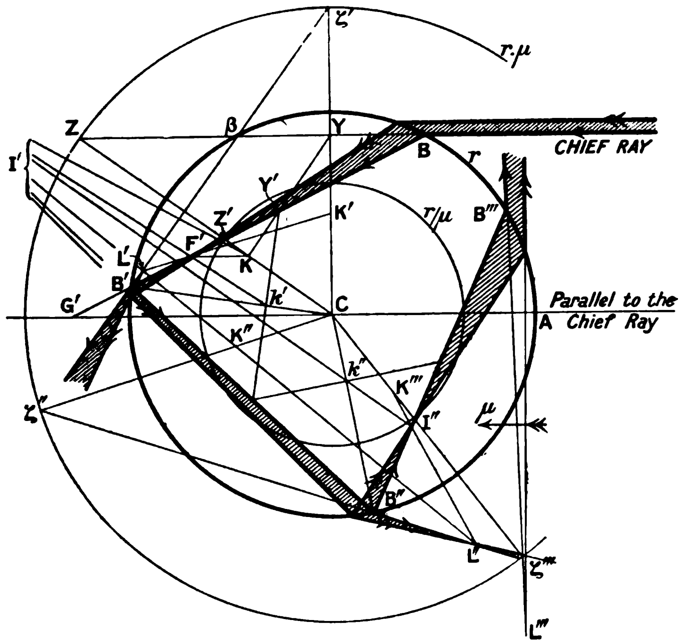


Fig. 2.—Positions of Foci.*

refracted parts, the refracted and first reflected portions, the first and second reflected portions, &c.

* In this figure some points and lines (especially CY, Cζ′) appear coincident that are not necessarily so. K‴, I″, L‴ are collinear.

Find F′ by drawing F′K parallel to the incident ray then I′, I″ . . . by drawing F′k′I′ and I′k″I″, &c.

The points, L′, L″, L‴ are the image-points on the emergent pencils at B′, B″, B‴. They are obtained from F′, I′, I″ through the junction-centres, K′, K″, K‴, &c. Make B′ζ′ =BZ, &c. The points, K′, &c., are obviously obtained on Cζ′, &c., by making CK′, &c., equal to CK.

(II.) Suppose (Fig. 3) the parallel pencil, whose chief ray is

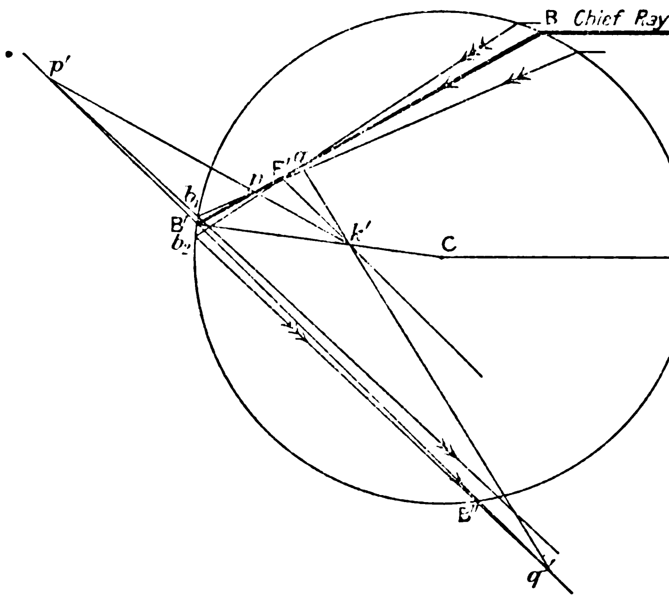


FIG. 3.—PARALLEL REFLECTED PENCIL.

incident at B, is reflected at B′ as a parallel pencil. Then F′k′ is parallel to B′B″.

A neighbouring ray incident between A and B intersects BB′ at p between F′ and B′, and the spherical surface at b_1 above B′. The line, $k'p$, intersects B″B′ at p' to the left of B′: then $p'b_1$ is the direction of the reflected ray.

A neighbouring ray incident above B intersects BB′ at q between F′ and B and the spherical surface at b_2 below B′. The line $k'q$ intersects B′B″ at q' to the right of B″: then $q'b$ is the direction of the reflected ray.

§ 5. *Cusped Wave-fronts.**

(I.) The wave-fronts after refraction or reflection at a spherical surface may be readily drawn when the rays and loci of the focal points have been obtained, as in §§ 2, 3, 4.

Referring to Fig. 4, the points 1′, 2′, 3′, &c., are the meridional foci of the refracted pencils produced from incident parallel pencils. The point of incidence of (5) is taken (§ 8), so that the refracted pencil through 3′ is reflected as a parallel pencil. Also (1) is incident tangentially at B_1 and (8) normally

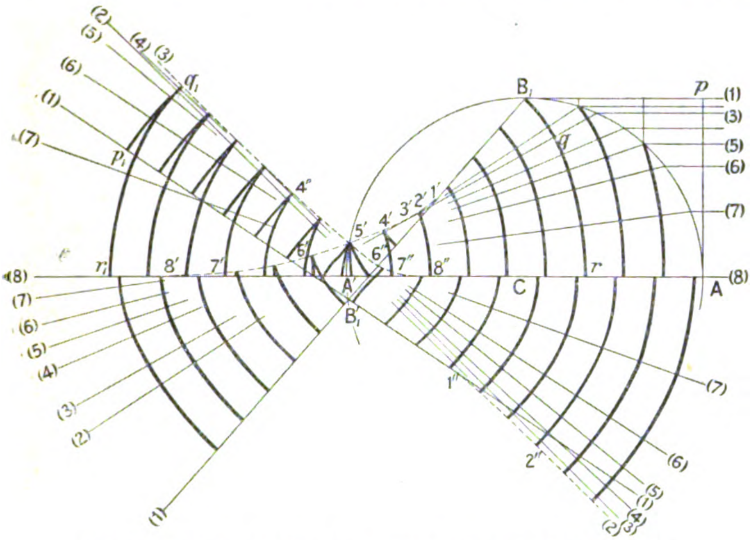


FIG. 4.—WAVE-FRONTS ON REFRACTION AND REFLECTION.†

at A. Then the caustic from 1′ to 8′ is the locus of the meridional foci by refraction of the parallel pencils incident between B_1 and A.

Consider the reflection of the pencils through 1′, 2′, &c., at

* *See* a Paper by Prof. W. B. Morton, on "Cusped Waves of Light and the Theory of the Rainbow," *Proc.* Phys. Soc., Lond., Vol. XXIII. (1910), pp. 58-64. In this other references are given.

† In Fig. 4, the wave-fronts in the first and third quadrants are those of the plane wave refracted at the upper part of the spherical surface. The wave-fronts in the fourth quadrant are those of the wave after reflection by the spherical surface. The wave fronts in the second quadrant are continuous with these. In the fourth quadrant the wave-front between the caustic through 1″, 2″ and ray (1) is practically coincident with the wave-front between the caustic and rays (4), (5), &c.

points on the spherical surface. The points 1″, 2″, &c., are conjugate foci of 1′, 2′, &c., by reflected meridional rays: 5″ coincides with 5′, 3″ is at an infinite distance. The conjugate foci of the points on the caustic by refraction between 1′ and 3′ are on the branch 1″, 2″, 3″ (at infinity) of the caustic by reflection, and of the points 3′ . . . 8′ on the branch, 3″ (at infinity), 4″ . . . 8″. The caustic by reflection between 2″, 3″ is very nearly straight and parallel to the ray (3).

The wave-fronts are obtained from the incident wave-front Ap; the initial refracted wave-front, B_1qr (for which $\mu . Ar = pB_1$, &c.) and the initial reflected wave-front, $p_1q_1r_1$ (for which $p_1B_1' = B_1'B_1$, $r_1A' = A'r$, &c.).

By the construction it is demonstrated that the caustics, which are the loci of the meridional principal focal points, are also the loci of the cusps on the wave-fronts. Also that the reflected ray (3)—the chief ray of the reflected parallel pencil—is the locus of a point of inflection on the wave-front (§ 4 (II.)).

§ 6. *Relation between the Portions of the Line of Incidence Intercepted by the Refracting Sphere of Radius r and one of Radius μr.*

(I.) Let l or $2y'$ be the length of the portion of the ray (like BB′ in Fig. 2) between two points of incidence. Then (§ 2 (III.)).

$$YZ/\beta Z = f'/y' \text{ and } 1 + y/\beta Z = f'/y'.$$

For the pth point of reflection, let u_p, v_p be the respective distances from the point of reflection of the vertices of the incident and reflected pencils. Then (§ 3 (V.))

$$1/u_p + 1/v_p = 4/l.$$

For the $(p+1)$th point of reflection, $u_{p+1} = l - v_p$.

(II.) Write $\beta Z = n . Y\beta$. Consider successive reflections. Then $1 + 1/n = 2f'/l$. $\therefore f'/l = (n+1)/2n$.

$$\therefore \quad u_1 = l - f' = l(n-1)/2n.$$

$$\therefore \quad \frac{1}{u_1} = \frac{2}{l}\left(1 + \frac{1}{n-1}\right); \quad \therefore \quad \frac{1}{v_1} = \frac{4}{l} - \frac{1}{u_1} = \frac{2}{l}\left(1 - \frac{1}{n-1}\right)$$

$$\therefore \quad \frac{1}{u_2} = \frac{1}{l - v_1} = \frac{2}{l}\left(1 + \frac{1}{n-3}\right); \quad \therefore \quad \frac{1}{v_2} = \frac{2}{l}\left(1 - \frac{1}{n-3}\right)$$

$$\frac{1}{u_p} = \frac{2}{l}\left(1 + \frac{1}{n-2p+1}\right); \quad \therefore \quad \frac{1}{v_p} = \frac{2}{l}\left(1 - \frac{1}{n-2p+1}\right).$$

(III.) When the emergent pencil is a parallel pencil after p internal reflections, $l-v_p=f=f'$.

$\therefore \quad v_p=u_1$ and $n-1=-(n-2p+1)$. $\quad \therefore \; n=p$.

When $n=2p+1$, $1/u_p=3/l$ and $1/v_p=1/l$.

Thus the image point is on the refracting surface.

When $n=2p$, $1/u_p=4/l$ and $1/v_p=0$. Thus the reflected pencil is one of parallel rays.

§ 7. *Conditions for a Rainbow.*

In order that a rainbow may be formed, the emergent, as well as the incident, portions of the effective pencil may be assumed to consist of parallel rays. Hence the image-points F′, I′, I″, &c., will be symmetrically distributed. Only certain pencils of incident rays will be so placed that their emergent portions consist of parallel rays.

When there is one internal reflection the incident pencil is in such a position that the second principal focus of its meridian rays is on the spherical surface.

When there are two internal reflections the incident pencil is in such a position that the second principal focus of its meridian rays is the first principal focus for the reflected pencil. Hence a parallel pencil passes between the two points of reflection.

When (§ 6 (III.)) there are $2p+1$ internal reflections, the $(p+1)$th point of reflection is a focus of the pencil reflected at the pth point.

When there are $2p$ internal reflections, a parallel pencil passes between the pth and the $(p+1)$th points of reflection.

Also (§ 6 (III.)) when the number of internal reflections is

one,	$f'=2y'$ and	$\beta Z=Y\beta$,
two,	$f'=3y'/2$	$\beta Z=2\,.\,Y\beta$,
three,	$f'=4y'/3$	$\beta Z=3\,.\,Y\beta$.

And for n internal reflections, $\beta Z=n\,.\,Y\beta$.

The chief rays of these pencils are at minimum deviation (§ 9).

§ 8. *Construction for Finding the Point of Incidence of the Chief Ray of a Parallel Pencil that shall Emerge as a Parallel Pencil after n Internal Reflections.*

Let (Fig. 5) circles of radii r and $r\mu$ be drawn from a centre, C. Through C draw N′MC, cutting the former circle at M. Draw NMQ perpendicular at M to N′MC, cutting the circle of radius $r\mu$ at N. Make N′M=NM.

When there are n internal reflections, mark off PM equal to n

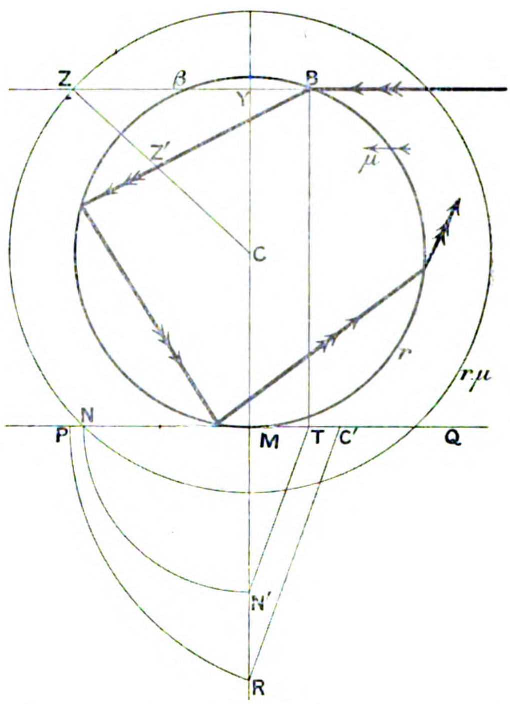


FIG. 5.—CONSTRUCTION FOR POINT OF INCIDENCE OF A RAINBOW RAY.

units of length and MC′ equal to one unit. From C′ as centre describe a circle cutting MN′ at R.

Join RC′. Draw N′T parallel to RC′. Draw TB parallel to MCY, intersecting the circle of radius r at B. B is the required point of incidence of a ray, YB, incident at right angles to CY.

Let BY intersect the circles at β, Z.

Now
$$\frac{\mathrm{MT}}{\mathrm{NM}}=\frac{\mathrm{MC'}}{\mathrm{MR}}=\frac{1}{\sqrt{n(n+2)}}.$$

Also
$$\mathrm{NM}^2=\mathrm{Z}\beta\,.\,\mathrm{ZB}=\mathrm{ZY}^2-\mathrm{YB}^2.$$
$$\therefore\quad \mathrm{ZY}^2-\mathrm{YB}^2=n(n+2)\,.\,\mathrm{MT}^2.$$

Also MT, YB, BY are all equal.
$$\therefore\quad \mathrm{ZY}^2/\beta\mathrm{Y}^2=(n+1)^2.$$
$$\therefore\quad \mathrm{Z}\beta=n\,.\,\beta\mathrm{Y}.$$

§ 9. *Deviation. Minimum Values.*

(I.) Consider the light that emerges from the spherical body after one internal reflection. The semi-deviation of any ray is (*see* Fig. 4) the angle between CB_1 and the line joining C with the point of reflection. Then the chief ray (5), whose meridional principal focus is on the spherical surface is at minimum deviation. For the caustic from 4′ to 6′ is the locus of the intersection of adjacent refracted rays between (4) and (6), and these rays gradually diminish in slope. Hence all the refracted rays except (5) strike the spherical surface at points lower than 5′.

(II.) Consider the light that emerges from the spherical body after two internal reflections. The semi-deviation of any ray is the angle between CB_1 and the perpendicular from C on to the reflected ray. Then the chief ray (3), whose reflected pencil is a parallel one, is at minimum deviation. For the caustic from 2″ to 3″ (at infinity) and 3″ to 4″ is the locus of the points of intersection of consecutive reflected rays from (2) to (3) and from (3) to (4) respectively. Thus, for a ray between (2) and (3) there is a parallel one between (3) and (4) (§ 4 (II.)), and the semi-deviation of (2) is greater than that of (3).

(III.) The same chief-ray remains at minimum deviation when the associated pencil is converging or diverging.

ABSTRACT.

A method of drawing the cusped wave-fronts produced by refraction and reflection at a spherical surface and a graphic treatment of the elementary theory of the rainbow are described in the Paper. The method is based upon the properties of the centre of junction. In the case of reflection at a spherical surface in which aplanatic points are not available the position of the junction-centre is obtained by elementary geometry.

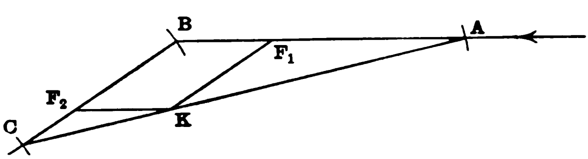
By the use of junction-centres the distribution of the successive foci obtained on refraction and reflection at a spherical surface is readily plotted. If in the case of a rainbow it is assumed that the emergent portion of the effective pencil, as well as the incident portion, is one of parallel rays, then the distribution of foci with regard to the drop is a symmetrical one.

The caustics on refraction and reflection are also readily drawn as loci of points and envelopes of rays. Hence the wave-surfaces can be obtained. These in some regions are cusped and the caustics are the loci of the cusps.

In the case of pencils that are effective in producing rainbows, the lengths of the incident portion of the chief ray intercepted by the spherical drop of radius, r, and a concentric sphere of radius, $r \,.\, \mu$, are in the ratio 1 to $n+1$, where n is the number of internal reflections. This leads to a geometrical construction for finding the points of incidence of the chief rays of the effective pencils. These rays are at minimum deviation, and the same chief ray remains at minimum deviation although the associated pencil may be converging or diverging.

DISCUSSION.

Mr. T. H. Blakesley (communicated remarks) said: In Mr. Bower's treatment of the geometrical disposition of the optical foci in the case of a refracting and internally reflecting sphere, commonly, for brevity, described as the rainbow problem, he has certainly shown an unflinching courage in loading a single figure with an immense amount of accurate matter. He is without a doubt a stickler for geometrical treatment. But when an analytical form expresses a geometrical truth it must have taken that form under strict geometrical treatment, and what is called by mathematicians analytical treatment is really only the combination of analytical forms already established by geometrical treatment. In dealing with the very valuable theorem of M. Cornu, Mr. Bower does not make it quite clear that the coordinates of the junction point, measured respectively along the two directions of the bent ray, from the point of bending as origin, are indeed equal to the two focal lengths for the light in the two compartments formed by the boundary, in the case of refraction; in the case of reflexion the simpler



result of equality takes place. If ABC represents a ray undergoing refraction at B, and BF_1, BF_2 are the focal lengths in the two media, the junction point K will be found by simply drawing F_1K, F_2K parallel to BC, BA respectively, and if any small object is placed at A its image will be at C, in AK produced when necessary to cut BC in C.

For
$$\frac{CF_2}{BF_2}=\frac{BF_1}{AF_1}.$$

Therefore, if $AF_1=m\,.\,BF_1$, it follows that $CF_2=\frac{1}{m}\,.\,BF_2$. But m or $\frac{AF_1}{BF_1}$ is the relation in linear magnification of the object to its image, and, therefore, the relation of the image to the object is $1/m$ or CF_2/BF_2—that is to say, C must be the position of the image. Viewed in this way M. Cornu's

theorem appears as a corollary of the fundamental law of conjugate foci. A word of further explanation should be added touching the *directions* of the elemental lines in the object and image, the comparison of which constitutes the relation m, or magnification ratio. In an axial pencil the incidence of the central ray is vertical, and the elemental lines are always considered at right angles to the axis. It would be more strict to describe them as parallel to the surface, for then the same description can be extended to the oblique cases. In the above figure if meridional rays are being considered the elemental lines of the object and image at A and C respectively are to be considered as parallel to the section of the surface at B, made by the plane ABC. For the rays which Mr. Bower describes as sagittal the elemental lines must be considered as parallel to the surface in a direction at right angles to the above. In dealing with the actual phenomenon of the ordinary rainbow it is not necessary, as Descartes has done, to introduce the idea of minimum deviation, and the consequent concentration of the light. That the colours are seen at all and in differing directions is sufficient to prove that the light is in a condition of sensible parallelism in its passage from the raindrop to the eye, and as it certainly is so before impact upon the drop the whole optical effect is that of a telescope in which the light is reflected between the object glass and eye-piece, as in Newton's telescope, it may be more than once. As the eye-piece has the same focal length as the object glass, the magnifying power is (-1). If there are two internal reflections there is an erecting day-glass arrangement not increasing or diminishing the magnifying power. But to be telescopic at all involves the necessity of the coincidence of two principal foci within the apparatus, and this consideration, together with that of symmetry, necessitates what Mr. Bower has pointed out, viz., the formation of a focus on the surface in the primary rainbow, and of parallel passage between the two reflections in the secondary rainbow. Abnormal rainbows are sometimes to be seen. By this expression I mean bows in which the axis does not coincide with the straight line joining the sun with the anthelial point. They are not readily explained, but are probably due to portions of ice or air in water, and the direction of gravity seems to be a factor in this formation. But, however they are constituted, the telescopic fact stated must be true of them, there must be coincidence of principal foci. Mr. Bower's methods are admirably adapted for all direct problems upon the subject, such as the impossibility of a primary rainbow for an index in excess of 2, and generally for the examination of cases where the index is taken as known ; but should the problem be to find the index for which the primary bow coincides with the secondary, it is unlikely that pure geometry could afford the answer—viz., 1·31201248.

Mr. W. R. Bower (communicated reply) writes : A graphic method cannot displace analysis when a computation is required to a high degree of accuracy. But experience is necessary when symbolical forms have to be interpreted, and, therefore, an analytical treatment of geometrical truths often hides for a time facts that should be exhibited. Educationally, graphic methods, that are not purely empirical, are more enlightening than analytical. The rainbow problem gives an example. In the usual algebraic discussion the parallelism of the emergent light is a condition in the proof of minimum deviation. We infer from the geometrical treatment—as mentioned at the end of the Paper—that the ray at minimum deviation may be the chief ray of pencils that are not necessarily parallel. In Mr. Blakesley's construction for the junction-centre, it is assumed that the positions of the principal foci are known. In the Paper in Vol. XXV. (p. 172) of the " Proceedings " the reverse process of obtaining the focal points from the junction centre is given. The junction centre is deduced and the discussion based entirely on the index of refraction, the centre and radius of curvature and the assumption that the radiant point travels along a fixed incident ray. No use is made of the sine law of refraction. The procedure follows a course nearly similar to that which would be taken if the magnification relation was assumed.

XXV. *On Gyrostatic Devices for the Control of Moving Bodies.*

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RECEIVED MAY 4, 1914.

IT is the object of the present Paper to describe a number of gyrostatic devices available for the control of moving bodies, such as torpedoes, submarine craft, airships and aeroplanes. These contrivances have suggested as a kind of by-product a variety of gyrostatic bicycles and motor-cars, both two-wheeled and four-wheeled. The stability of the gyrostatic system is, however, in all the cases considered in the present Paper, derived directly or indirectly from the propelling system. Hence these cases do not include solutions of the monorail problem; for they have not true stability when they are at rest or moving in the backward direction. Further, it will be seen that the tandem-wheeled motor-cars to be described, although they may be set to run in a perfectly straight path, will not balance on a single rail. The devices, however, have properties which are not possessed by any of the monorail devices so far evolved, properties which may possibly render them of the greatest value to a nation whose continued existence depends on its ability to retain supremacy on sea, and to obtain the supremacy of the air.

With a view to making clear the action of the devices I shall first show some old experiments and will make some deductions which are not obvious. In Fig. 1 is shown a gyrostat set up in a fork and pedestal mounting. The arrangement is substantially that made use of long ago by Léon Foucault in demonstrating the earth's rotation. The flywheel possesses three freedoms. The fork is capable of rotation, with but little friction, about a vertical axis, and the gyrostat is carried on knife-edges formed in the prongs of the fork, and is thus free to turn in the line joining the knife-edges. The axis of rotation of the flywheel XX′ and the two axes just specified—namely, ZZ′ and YY′—are mutually perpendicular when the axis of the flywheel is horizontal. Under these conditions the instrument is, of course, freely mounted.

The curved rod terminating in an arrow-head shows the direction of spin. The straight rods are intended to represent the angular momentum of the flywheel and the applied couple respectively. With the direction of rotation shown, the

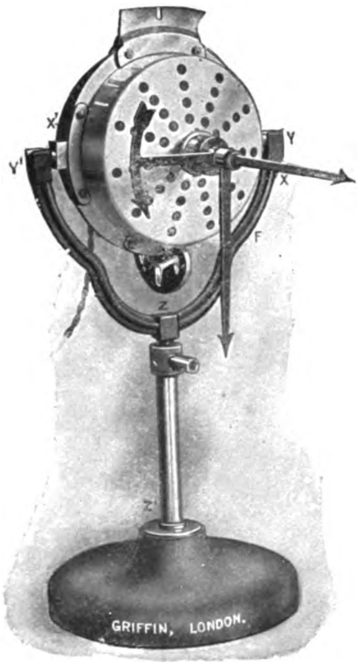

Fig. 1.—Gyrostat in "Fork and Pedestal" Mounting.

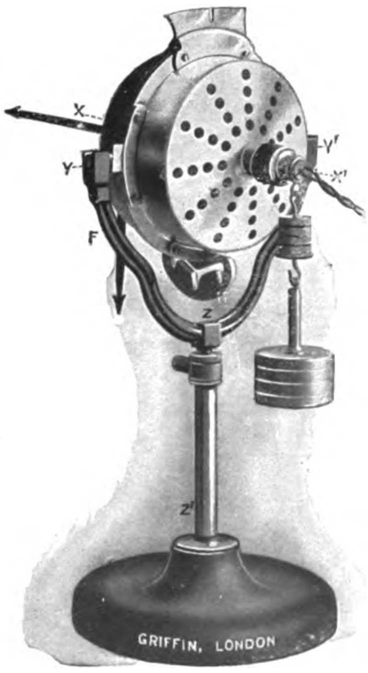

Fig. 2.—Gyrostat in Steady Precession, with its Axis Horizontal.

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angular momentum or spin is, according to the usual convention, represented completely by a straight line of proper length, drawn at right angles to the plane of the flywheel, as in the figure. The second straight rod can be moved round in a plane parallel to that of the flywheel; it is held in any position by means of a spring washer. These lines, which represent the "spin" and the applied couple, are called the spin-axis and couple-axis respectively.

The model is available for demonstrating the principal properties of the gyrostat. By its means it is easy to show that if a couple is applied to the gyrostat, so as to cause precessional motion, the motion is such that the spin-axis moves towards the instantaneous position of the couple-axis. This rule always holds. A couple tending to turn the gyrostat about the fork-axis causes precession about the pedestal axis; one tending to cause turning about the pedestal axis causes precession about the fork axis.

In Fig. 2 the gyrostat is shown in steady precession, with the axis of its flywheel horizontal, under the action of a suspended weight. If W is the weight, l the distance of its line of action from the axis of the knife-edges, I the moment of inertia of the flywheel, and ω_1 its angular velocity, the gyrostat and suspended weight turn about the vertical axis with angular velocity ω_2, where

$$Wl = I\omega_1\omega_2.$$

Providing that the vertical axis is frictionless and the spin is maintained constant the weight will be carried round at a constant rate ω_2. In practice, of course, this condition is not fulfilled; there is in general a small oscillation about the steady motion, and, in consequence of friction, there is a gradual descent of the weight with slow variation of ω_2, which (with constant spin) will be an increase or a diminution according as the couple Wl is increased or diminished by the descent. There is a loss of energy on the whole. If the weight descends through a distance d, its loss of potential energy is Wd, and, since for rapid spin ω_2 is small, it is substantially this potential energy that is converted into heat by friction at the vertical axis.

The work done in a given time, T, is $L_1\omega_2T$, where L_1 is the frictional couple. Now the greater the value of ω_1 the smaller is ω_2, and the less is the work done against friction in a given time.

Again, suppose that with the gyrostat in steady precession, with its axis horizontal, a couple of moment L_2 is applied in a horizontal plane in the direction of the precessional motion.

If L_2 is greater than L_1 the effect will be to raise the weight against gravity. The work done by this couple on the weight in time T is $(L_2—L_1)\omega_2T$, and hence the rate of working is proportional to ω_2. This is of great importance from the point of view of the application of the gyrostat as a transmitter of work and as a stabilising agent. For a given value of ω_1, ω_2 is proportional to W, and hence the greater W, the greater is the amount of work done by the net couple $L_2—L_1$ in a given time. If $L_2—L_1$ is small and W great the action is the analogue of the ascent of a heavy weight along an inclined plane of very small slope under the action of a force applied up the plane. Again it will be seen that by making ω_1 very large—that is, by rotating the flywheel at a very high speed—a large weight may be raised through a sensible distance without ω_2T being necessarily great, if L_2-L_1 be made large enough. This point is of importance.

Now let the side weight be removed, and a top weight be added to the frame of the gyrostat, so as to bring the centre of gravity of the system vertically above the line of the fork bearing when the axis of the flywheel is horizontal. When so arranged the gyrostat is unstably mounted in the fork, and if the flywheel is spinning any tendency to tilt will result in precessional motion ; tilting to one side causes precessional motion in one direction, tilting to the opposite side reverses the direction of the precessional motion. Further, if the gyrostat is precessing in one or other direction and a couple is applied to the fork in the direction of the precessional motion the gyrostat tilts on the fork bearing, so as to raise the centre of gravity of the arrangement—that is, so as to annul the tilting couple.

It should be observed that here the gyrostat acts as a transmitter. The couple applied to the fork (which may be called the stabilising couple) is applied in a horizontal plane, the gyrostat and its attachments turn instantaneously in a vertical plane. The precessional motion about the pedestal axis ceases when the centre of gravity of the system is vertically above the line of the fork bearings. If the stabilising couple goes out of existence at the instant at which the precessional motion ceases, and the motion about the horizontal arm be, as it usually is, very slow, the gyrostat is practically left in the upright position.

Precessional motion about the pedestal axis in one or other direction follows on tilting of the gyrostat on the fork bearings. If now a mechanism is devised whereby the fork is for-

cibly turned in the direction of the precessional motion immediately such precessional motion takes place, and further if the stabilising action ceases at the instant at which the necessity for its existence disappears, it is clear that the gyrostat will be maintained in the upright position.

In Fig. 3 is shown a form of stilt top devised by Prof. H. A. Wilson, and exhibited by him to the Physical Society in 1907. A gyrostat is mounted as shown in a frame, *f*. The gyrostat frame is on cross bearings carried by *f*. When *f* is upright these bearings are in a vertical line. The crank *c*, which is rigidly fixed to the frame of the gyrostat, is attached to one end of a stretched spring *s*, the other end of which is fastened to a point, *p*, in the main frame. If the flywheel of the gyrostat is set spinning and the top placed on a table with the plane of the flywheel and the main frame, *f*, in the same vertical plane, and left to itself, it will balance for a considerable time if the spin is great. Initially *f* is in the same plane as the spring *s*.

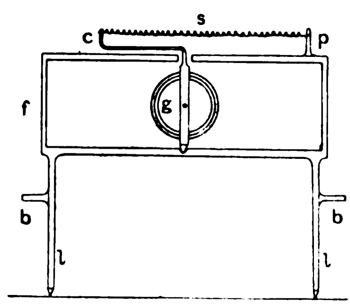

FIG. 3.—STILT TOP.

The stretching force in the latter, therefore, exerts no moment on the gyrostat about the cross bearings which attach it to *f*; but as soon as the gyrostat precesses on the latter bearings the crank gets out of line with the frame, and the spring exerts a moment in the direction of the precessional motion.

The entire top, when vertical, is unstable, without rotation of the gyrostat flywheel, about the line of contact of the feet with the table. Further, in consequence of the stretched spring, the gyrostat is unstably mounted on the frame. Thus the gyrostat is doubly unstable without rotation of its flywheel.

Starting with *f* in a vertical plane containing the crank and spring we may suppose it to tilt over on the table. As a consequence the gyrostat precesses about the cross bearings and the

precession is aided by the spring, with the result that the frame erects itself into the vertical. But at the instant at which the frame has attained the vertical the spring is out of line with the frame, and is exerting a moment on the gyrostat. Under the influence of this couple the gyrostat continues to precess about the line of contact of the feet with the table; that is, the main frame passes beyond the vertical position, after which the lateral instability of the entire structure results in the establishment of a couple tending to accelerate this precessional motion. This couple causes precession about the cross-bearing bringing the crank and spring into line with f, but when this alignment occurs, the entire top is inclined from the vertical; and so on. The amplitudes of these oscillations continually increase, and finally the top falls over.

Again, suppose that, starting as before with f and the crank in one vertical plane, the crank gets out of line with f. As a result the spring exerts a moment on the gyrostat, which, in consequence, precesses about the line of contact of the feet of the top with the table. This precessional motion is automatically accelerated and the spring is thrown into line with f, which is now inclined from the vertical, and so on.

It will thus be seen that starting with the main frame and the spring contained in one vertical plane the top balances; and if the spin is great the balancing power is very considerable. But there is not true stability. The frame oscillates to and fro on the legs, the gyrostat oscillates to and fro on the bearings which carry it in the frame. If the stability were real the top, if started in an inclined position, would erect itself into the vertical one with the spring in the plane of f.

It is interesting to consider this matter from the energy point of view. The entire structure is unstable on the legs, and thus possesses a stock of potential energy. Again, potential energy is stored in the spring. When the frame tilts on the legs and the gyrostat turns on the frame bearings energy is dissipated in friction. Consequently once the frame has become inclined to the vertical, or the crank has got out of line with the frame, the system cannot return of itself to the position of maximum potential energy, that is, to the position in which the frame and crank are in one vertical plane.

In Figs. 4 and 5 are shown two further experiments in which a gyrostat is mounted in such a manner as to possess two instabilities without rotation of its flywheel. In Fig. 4 is shown a new form of stilt top designed by the writer. A gyrostat is

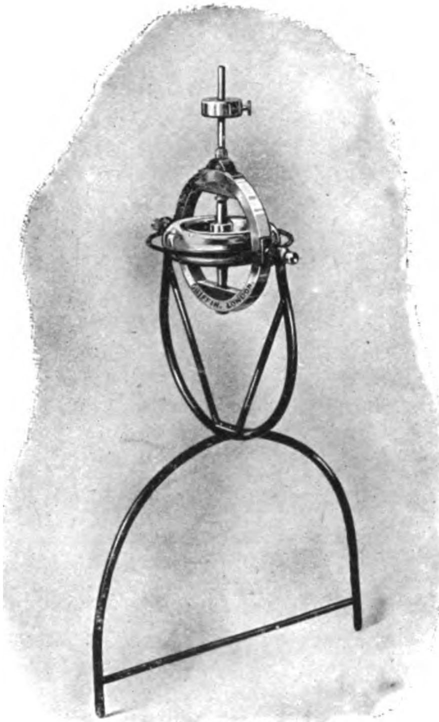

FIG. 4.—FORM OF STILT TOP.

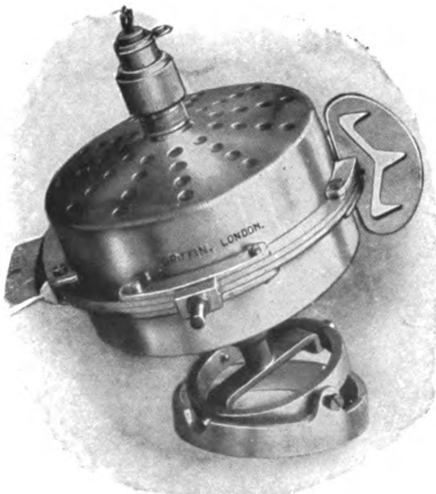

FIG. 5.—GYROSTAT ON GIMBALS.

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carried, with its axis vertical, on two horizontal bearings arranged in a frame terminating in two legs as shown. When the supporting legs and the axis of the gyrostat are vertical, with the weight in the upper position, as shown in the figure, the gyrostat is doubly unstable without rotation. If the flywheel is spun rapidly and the top set up as described, it displays considerable balancing power. But from what has been said it will be evident that this arrangement does not provide an example of true gyrostatic stability. Oscillations about the line of contact of the feet with the ground and about the horizontal bearings quickly grow up.

The top is well adapted to show the necessity for two instabilities without rotation of the flywheel. If the arrangement is set up as shown, but with the weight vertically below the gyrostat, the top exhibits no balancing power.

Fig. 5 shows Lord Kelvin's "gyrostat on gimbals" experiment. The gyrostat is supported on a universal joint, or pair of gimbal rings. When the gyrostat is in the vertical position it clearly possesses two instabilities without rotation, one about each gimbal axis. In performing the experiment the flywheel is rotated rapidly and the arrangement placed on the gimbal rings, with the axis of the gyrostat vertical, and left to itself, when it balances on the ring in contact with the table. The upright position is soon departed from, and is never regained. The axis moves round its initial position, its distance from the latter continually increasing. Energy is continually dissipated at the gimbal axes, which energy is derived from the potential energy possessed by the arrangement by virtue of the peculiar manner in which it is mounted.

Returning now to the top shown in Fig. 3 it will be seen that I have added to Mr. Wilson's arrangement the two projecting pieces, *bb*. So designed it may be set up with these projections engaging on the knife edges arranged in the fork of the apparatus shown in Fig. 1. The arrangement is shown in Fig. 6. The flywheel of the gyrostat is set into rapid rotation and the arrangement mounted on the fork with the frame *f* and the crank in one vertical plane. The fork is grasped in the hand of the experimenter. Now suppose the arrangement to tilt on the fork bearings b_1, b_2. The gyrostat precesses on the bearings that carry it in the frame, and immediately a couple, due to the spring, tending to accelerate the precessional motion, comes into existence. At the same time, the experimenter turns the fork so as to bring the frame into line with the crank.

Providing this operation is properly carried out, the frame is restored to the upright position and the crank is in line with it. The spring has supplied energy to the frame in restoring it to the vertical position, the potential energy lost by the spring has been made good by the experimenter.

Now let a weight, w, be attached to one side of the frame. This at once causes precession of the gyrostat and the establishment of a couple due to the spring. The experimenter turns the fork so as to bring the frame into line with the crank. Here energy is being transmitted from the spring to the frame by means of the gyrostat, and at the same time energy is being supplied to the spring by the experimenter. The frame turns on the fork bearings so as to raise the weight against gravity. The precessional motion continues until the centre of gravity of the entire arrangement is vertically above the line of b_1 b_2.

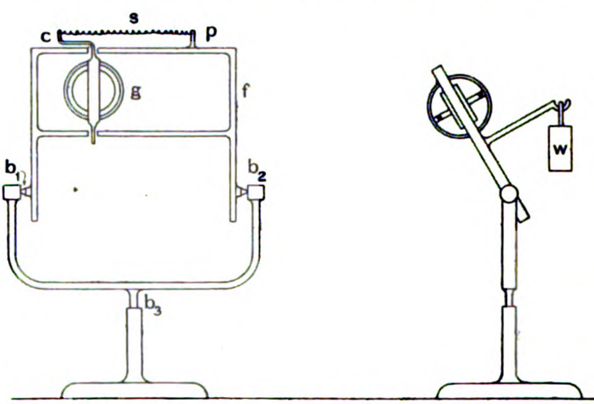

FIG. 6.—STILT TOP SET UP ON FORK AND PEDESTAL MOUNTING.

The spring is now in line with the frame, and consequently its stock of potential energy is precisely that which it possessed at the start of the experiment. The energy required to raise the top against gravity has been supplied by the experimenter.

The action of the top shown in Fig. 7 is identical with that of the one just described. The crank and spring are placed transverse to, instead of in the plane of, the main frame.

Fig. 8 illustrates the applications of the principles just described to the construction of two-wheeled and four-wheeled gyrostatic motor cars. The figure shows a car in which the wheels, of which there are two, run in tandem. The gyrostat g is mounted on top and bottom bearings provided in the main

frame F. One of the axles which carry the gyrostat is extended and terminates in a bearing for one of the wheels w_1 of the car. The construction is such that this wheel is in the plane of the flywheel of the gryostat. The back wheel of the car is geared up to driving mechanism. The gyrostat is fitted with a crank

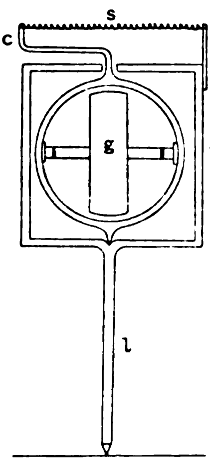

FIG. 7.—STILT TOP.

and spring device as already described. The wheel w_1 and the flywheel of the gyrostat are in the same plane. Arrows on the wheels indicate the direction of motion of the device.

Let the flywheel be set into rapid rotation and the car

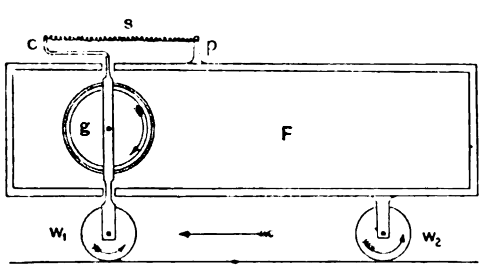

FIG. 8.—TWO-WHEELED MOTOR CAR.

placed on the floor with the main frame, the flywheel of the gyrostat, the two wheels supporting the device and the crank in one vertical plane. It will be clear from what has been said that, if left to itself, the car will balance on the wheels, but with

the accompaniment of gyrostatic oscillations. If it were allowed to remain stationary the device would eventually fall over; but when driven in the forward direction it is completely stable. This stability is obtained in the following way: The gyrostat steers the car, and when precessional motion on the cross bearings takes place this results in the main frame F, to which the end of the spring remote from the crank is attached, being brought into line with the crank. Thus the action is precisely that described above for the stilt top when mounted in the fork bearing. The stability is complete when the car is moving in the forward direction. If a weight is attached to one side the car banks up against the weight so as to bring the centre of gravity directly above the wheels, and thereafter proceeds in a straight line path.

In this device the gyrostat derives the stabilising forces from the spring, and when the stock of potential energy possessed by the spring is drawn upon an equal amount of energy is automatically supplied to it by the propelling system. The gyrostat thus detects a tendency to tilt in either direction, calls upon the spring to supply the necessary correcting forces, and upon the propeller to maintain constant the energy possessed by the spring.

The gyrostatic action of this device is illustrated graphically in Fig. 9. We suppose the car to start perfectly balanced and upright. This condition is shown in (1) of the upper diagram (A) of the figure. The arrow at the back of the car shows the direction of motion, and the curved arrow attached to the gyrostat indicates the direction of rotation of the flywheel. The angular momentum may be completely represented by a straight line drawn out from the flywheel towards the reader. Thus, a_1 is the spin-axis. Now, the car is unstable about the line of contact of the wheels with the table, and hence, after a short interval, a tendency to tilt in one or other direction will assert itself. We suppose that when the car is in the position (2) there exists a tendency of the device to tilt towards the reader. This tilting couple is completely represented, according to the usual convention, by a line a_2 (the couple axis), of proper length drawn as shown in (2) towards the back of the car. The gyrostat precesses, so that a_1 turns towards the instantaneous position of a_2, the crank comes out towards the observer, and a couple tending to turn the gyrostat counterclockwise, as viewed from above, is established. This couple is represented by a_3 in (3) above. The gyrostat now precesses

so that a_1 moves towards the instantaneous position of a_3—that is, the car erects itself against gravity; it moves away

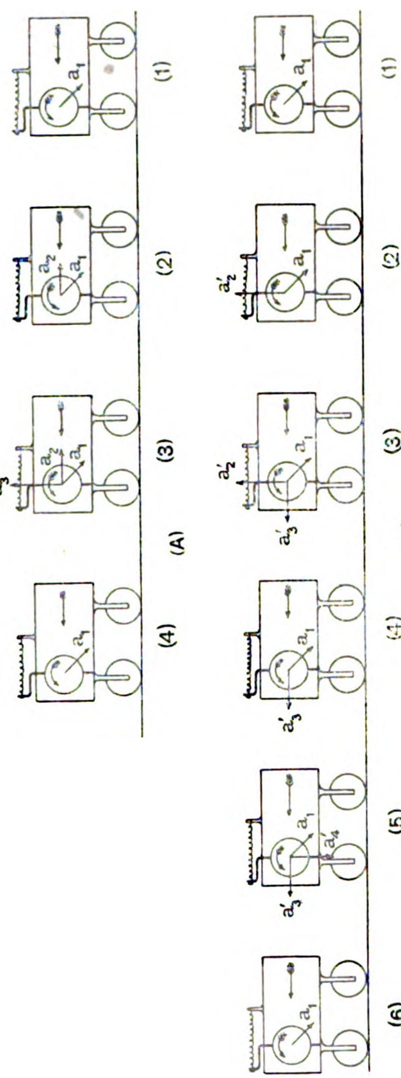

Fig. 9.—Action of Two-Wheeled Gyrostatic Motor Car.

A.—March of Vectors following on Tilting of Car towards Reader.
B.—March of Vectors following on Displacement of Spring from Central Position.

from the reader about the line of contact of the wheels with the ground. The tilting couple is thus reduced and finally an-

nulled. Further, in consequence of the fact that the gyrostat is steering the car, which is moving in the forward direction, the want of alignment of the crank and car is being reduced. Thus, a_2 and a_3 tend to diminish together, and the car is finally left as in (4) of the top diagram of the figure.

We may now investigate the annulment of the instability introduced by the crank and spring. This will be understood from the lower diagram (B) of the figure. In (1) the car is shown balanced in the upright position with the crank in line with F. We suppose it to get out of line with the car without there being any tendency of the latter to tilt (this may be due to a jolt). Let the displacement of the crank be towards the reader. The couple brought into existence by the want of alignment is represented by a_2' of (2), and the gyrostat turns so that a_1 moves towards the instantaneous position of a_2'; the car turns over from the upright position (away from the reader), and a couple represented by a_3' of (3) comes into play. The gyrostat now turns on the vertical bearings in the direction which results in a_1 moving towards the instantaneous position of a_3'—that is, in the direction which brings the crank into line with the car. The couple due to want of alignment of the crank and frame disappears, but the car is left inclined to the vertical. This stage is shown in (4). The gyrostat is now precessing on the vertical bearings, and a couple represented by a_4' (5) is introduced. The resulting precessional motion and the forward motion result in a_3' and a_4' going out of existence, as already explained, and the device is left as shown in (6). It is now upright, with the crank and frame F in line.

As we have already seen, the effect of attaching a weight to one side of F is to cause the car to bank up and then to pursue a straight path. The properties of the device are, however, greatly added to by attaching a weight, w, to the frame of the gyrostat, as shown in Fig. 10 (a) and (b). If the gyrostat is spinning in the direction in which the wheels of the car rotate, the weight should be placed as in (a); if the direction is reversed the weight should be as in (b). Consider diagram (a). Let a side weight (Fig. 6) be supposed attached. The car banks up against this weight, with the result that the line of the frame bearings becomes inclined to the vertical. A couple acting against the spring is then applied by. A steady state is now arrived at, in which the couples applied to the gyrostat by the spring and by the weight are equal. The car is not sufficiently banked

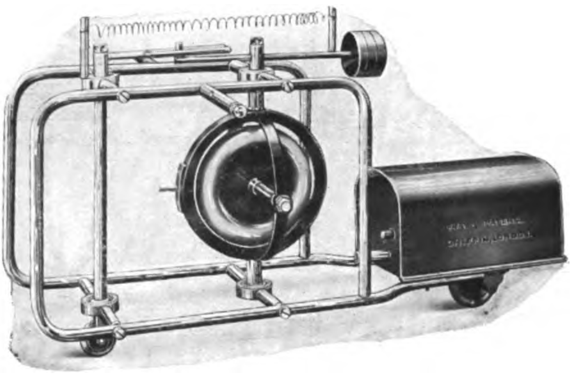

FIG. 11.—GYROSTATIC MOTOR CAR.

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up to account entirely for the side weight, and the gyrostat precesses continually. The car moves in a circular path.

When the side weight is removed the device straightens itself out and proceeds in a straight path. Attaching the side weight to the other side of the car causes the latter to move in a circular path in the opposite direction to the former one.

Fig. 11 is a photograph of an actual working model of a two-wheeled car constructed on the above principles. The front wheel is the driven one. The gyrostat is carried on the main frame on vertical bearings. It steers the back wheel through a link attachment. The car is available for demonstrating the action of a monorail car, or the gyrostat can be placed under the control of a weight as just described. It can then be set to move in a curved path in either direction. A much larger model, provided with an electromagnetic steering device

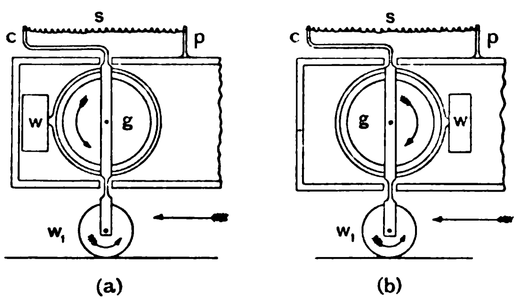

FIG. 10.—GYROSTATIC MOTOR CAR FITTED WITH CONTROLLING WEIGHT.

capable of being actuated by the wireless transmission of electrical action, is in course of construction.

Fig. 12 shows the application of the principle to the steering of a torpedo or dirigible airship. The upper diagram shows a side elevation, and the lower a plan of the arrangement. The gyrostat is mounted, with its axis horizontal, on cross bearings, b_1 b_2, carried by a frame, f. This frame is mounted on fore-and-aft bearings, b_3 b_4, attached to the moving body, and the construction is such that the system composed of gyrostat and attachments is laterally unstable on the body. The spring s_1 and the crank c render the gyrostat unstable relatively to the frame f. The moving body is supposed to be stable of itself. It will thus be seen that the gyrostat possesses two instabilities without rotation of its flywheel. By causing the gyrostat to

steer the body these two instabilities without rotation result in complete stability when the flywheel is rotating and the body moving in the forward direction.

In the construction of wheeled vehicles it has been found sufficient to connect up the gyrostat directly to the steering wheel or wheels. In the case where a steering mechanism, such as a rudder or plane, has to be operated forcibly this is not possible. Apparatus for operating a vertical rudder is shown in the lower diagram of Fig. 12. One end of a cord is attached to a point on the frame of the gyrostat. The cord is then passed once or more times round a vertical drum or pulley, d_1, and its free end attached to a point on a drum, D. A second cord is likewise attached at one end to a point on the opposite side of the frame of the gyrostat, passes once or more times round a second pulley or drum, d_2, and is attached to the

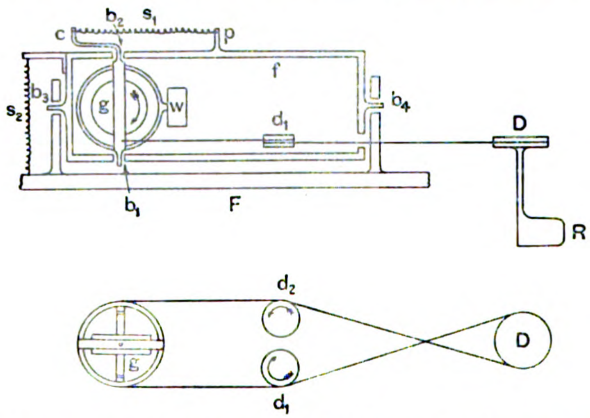

FIG. 12.—DIRIGIBLE TORPEDO, WITH STABILISED GYROSTATIC SYSTEM.

opposite side of the drum D. The two pulleys d_1, d_2, which are of equal diameter, are geared up to a small electric motor; they revolve in opposite directions with the same speed. If the gyrostat precesses one of the cords (say c_1) attached to it becomes taut. A small stretching force in the cord on the gyrostat side of d_1 gives rise to a large stretching force in the cord on the drum side of d_1. If the stretching force on the gyrostat side of d_1 is zero that on the drum side is also zero. It will thus be seen that a small couple applied by the gyrostat results in the application of a very large couple to the drum, and hence to the rudder.

The rudder is so connected up that when the gyrostat precesses the body is steered up parallel to it, that is, so as to maintain the axis of the flywheel transverse to the body.

In the form of torpedo at present in use the gyrostat is freely mounted on gimbal rings. In the absence of a disturbing couple the axis of the gyrostat will retain its direction in space unaltered. Hence when the torpedo deviates in its path a shift occurs between the direction of the axis of the gyrostat and that of the projectile. The apparatus must be made with great precision; notably the centre of gravity of the gyrostat must coincide exactly with the point of intersection of the gimbal axes. If this condition is not precisely fulfilled the torpedo will travel in a curved path.

The existing type gives very good results over the short distances; but a gyrostat freely mounted would be useless in a long-distance projectile, even if a motor gyrostat were substituted for the one now employed. This point is not as a rule understood. In a dirigible torpedo, properly so called, the gyrostatic apparatus should be such that the gyrostat is endowed with complete stability. This condition fulfilled, the gyrostat can be caused to bring about turning movements of the torpedo by the application to it of tilting couples.

The device just described is well adapted for use on small dirigible airships. It is easy to contrive apparatus on hydrostatic principles which will cause an airship to ascend to a given height and to remain at that height.

Let, now, the apparatus be supposed mounted on an aeroplane, the bearings b_3 b_4 being fore and aft. The gyrostat is balanced on b_3 b_4 and a lateral tilt of the aeroplane would bring about a shift between it and the gyrostat, which might be utilised to operate the balancing apparatus. The gyrostat, it is to be observed, is maintained automatically in the position in which its axis is horizontal and across the aeroplane. Thus this gyrostatic device, as well as a further one to be described, would appear to be available as a detector and corrector of lateral tilting.

In the device as illustrated the axis of the gyrostat is across the moving body. It is possible to mount the gyrostat, doubly unstable as before, on the body with the axis of its flywheel fore and aft. The frame of the gyrostat is suitably connected up to the steering apparatus in such a manner that the moving body is maintained with its length parallel to the axis of the

flywheel. A gyrostat so mounted would appear to be available as a detector and corrector of longitudinal tilting.

In order that a gyrostatic aviator should confer both longitudinal and lateral stability upon an aeroplane it must be mounted with complete stability on the latter with its axis vertical. This would be easy on the principles explained, if it were feasible to steer the aeroplane in a vertical plane. This, of course, is not practicable.

Attention is now directed to Fig. 13, which shows a new form of stilt top. A gyrostat is pivoted within a structure terminating in two stiff legs. When the feet of the top are supported on a table with the plane of the frame vertical the line of the pivots which carry the gyrostat is sloped to the vertical,

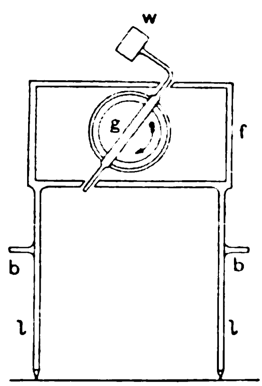

FIG. 13.—STILT TOP.

and with the direction of slope indicated it will be seen that when the plane of the flywheel coincides with that of the frame the weight w is constrained to move, relatively to f, in a circle whose highest point coincides with that occupied by w in the figure. Thus in the position shown the gyrostat, in consequence of the presence of the weight, is unstably mounted on the frame. Further, the frame is unstable about the line of contact of the feet with the table. Thus the gyrostat possesses two instabilities without rotation of its flywheel. If the flywheel is rotated rapidly in either direction and the top placed on a table as described, and left to itself, it will balance on the table. It will be readily seen, however, that the stability is not true stability; gyrostatic oscillations soon grow up.

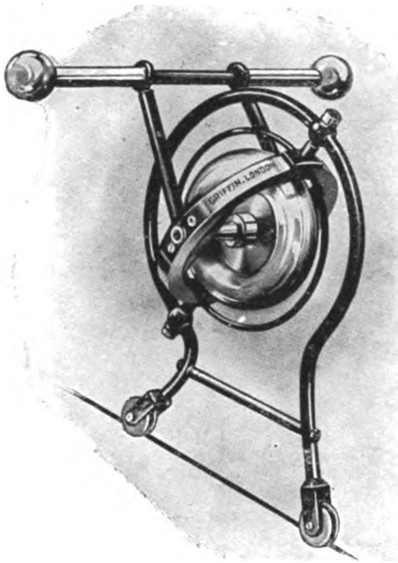

FIG. 14.—POLE-BALANCING TOP.

FIG. 15.—GYROSTATIC BICYCLE.

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Fig. 14 shows a pole-balancing top constructed on this principle. The frame terminates on two wheels adapted to run on a tight or slack wire. Attached to the gyrostat frame are two arms which support a pole weighted at both ends. This pole provides the necessary instability of the gyrostat relative to the frame. The flywheel is rotated clockwise as viewed by an observer from one side with the pole on his left, and the arrangement set up with the frame vertical and the pole horizontal. With this direction of spin the tilting of the frame to one side of the wire causes the pole to be carried over to the other side.

Returning now to the first of the tops just described, let it be spun and set up in the fork and pedestal mounting (after the manner of Fig. 6), with the frame and flywheel in the same vertical plane. As before, the experimenter operates the fork. With the direction of spin indicated in the figure tilting of the frame to one side of the fork causes the weight to be carried over to the other side. Now let a side weight w' be attached to the frame f. The gyrostat precesses on the frame bearings and w is carried (Fig. 13) over to the side of the frame remote from the attached weight; let the fork be turned by hand in the direction in which the gyrostat precesses, so that it follows up the latter. Providing that the fork is not turned too quickly the gyrostat will continue to turn on its bearings. It is to be observed that at any instant the acting tilting couple is the difference of the moments about the fork axis due to the side weight w' and w respectively. The effect of turning the fork is to diminish the moment due to w and the precessional motion is maintained. This action, it will now be shown, has resulted in apparatus of great beauty.

Fig. 15 illustrates a form of gyrostatic bicycle. A gyrostatic bicycle rider is mounted upon a safety-machine. The frame of the gyrostat is attached to the bicycle by means of a sleeved joint, and a pair of arms, carried by the frame of the gyrostat, are attached by means of pivots to the handle-bar of the machine. The construction is such that when the wheels are in one vertical plane the sleeved joint, referred to above, is considerably inclined to the vertical, and this results in the brass ball (*see* the figure) conferring considerable instability upon the gyrostat relatively to the bicycle. The entire machine is, of course, unstable on the wheels. These two instabilities, without rotation of the gyrostat flywheel, are available to annul one another with rotation.

The rider of a bicycle keeps his machine upright by operating

his handle-bar. When the machine tilts over to the left the rider instinctively turns the handle-bar to the left, and the forward momentum of the bicycle and rider aided by the gyrostatic action of the wheels (a relatively small factor), results in the erection of the machine. Similarly, if the machine tilts to the right the handle-bar is turned to the right.

The action of this gyrostatic bicycle is entirely different, and much more beautiful. The gyrostat is spun in the direction opposed to that in which the wheels of the cycle rotate when the latter is moving in the forward direction. Tilting of the machine to the right causes the gyrostat to precess, so that the brass ball and the front wheel and the brass ball move over to the left. As described above, energy is supplied from the forward motion, and the precessional motion ceases when the tilting couple is annulled. The stability, when the bicycle is moving in the forward direction, is complete. If brought to rest, following on motion in a straight path, it is left perfectly upright, and it then balances for a considerable time. Gyrostatic oscillations, however, grow up, but on the restarting of the machine these at once disappear. If the bicycle is started in an inclined position it erects itself into the upright position, and thereafter pursues a straight path.

Starting with the bicycle moving forward in the upright position, let a weight be placed on one side of the frame. The machine now proceeds in a circular path, the action being that described for my stilt top when set up in the pedestal mounting. Suppose the weight to be attached to the right-hand side of the frame as seen by an observer viewing the bicycle from behind. The front wheel and the brass ball are carried over to his left. The fact that the gyrostat steers the machine results in the latter turning continually, so as to annul the moment due to the brass weight; at the same time the precessional motion turns the ball so as to increase its tilting moment. A steady state is soon arrived at, and the bicycle moves in a circular path.

Fig. 16 is a diagrammatic representation of a large motor car constructed on the above principles. It will be seen that the gyrostat stabilises the entire structure and at the same time operates the steering wheel.

The function of the weight W_2 is to apply the necessary tilting couples. W_2 is carried on an arm which is rotated about a vertical axis by means of a small geared electric motor. When completed this car will be capable of being operated by wireless transmission of electrical action.

It should be noticed in the case of this bicycle and the motor car, that, provided the gyrostat is sufficiently powerful, the controlled device cannot possibly upset. The frame is continually following up the gyrostat, which thus cannot lose control.

Fig. 17 shows in side elevation a gyrostatic device adapted for steering a body stable of itself, such as a tricycle, four-wheeled motor-car or torpedo. The gyrostat is mounted on

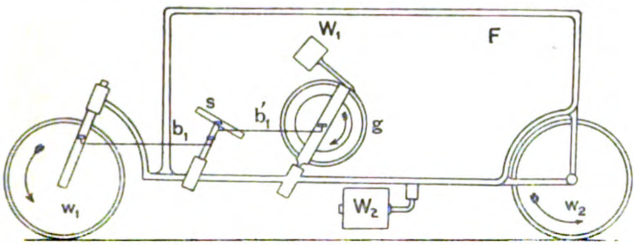


FIG. 16.—GYROSTATIC MOTOR WITH ELECTROMAGNETIC STEERING DEVICE.

bearings b_1b_1 carried by a frame, *f*; it is made azimuthally unstable by sloping the line of these bearings to the vertical, and attaching a weight, *w*, to the frame of the gyrostat. The frame is carried on horizontal bearings b_2b_2 arranged in pillars p_1p_1 attached to the moving body. The frame *f* is rendered laterally unstable on these latter bearings by attaching to it the

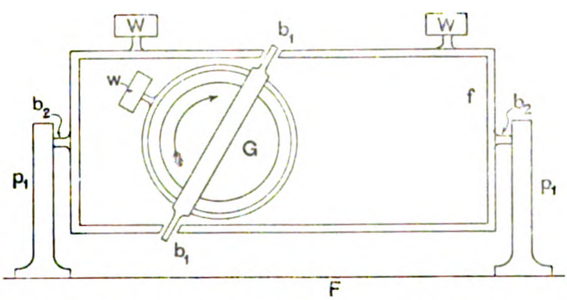


FIG. 17.—GYROSTATIC CONTROL FOR TORPEDO OR AIRSHIP.

weights WW, as shown. The gyrostat clearly possesses two instabilities without rotation of its flywheel. When the gyrostat is suitably connected up to the steering mechanism these two instabilities give rise to complete stability of the gyrostat when the body is in motion. The frame *f* may conveniently carry apparatus for applying tilting couples to the gyrostat.

In arriving at a conclusion as to the merits of the gyrostatic controls that have been described it should be remembered that the action of the gyrostatic system is a function of the angular momentum of the flywheel, of the double instability of the gyrostat, and of the forward momentum of the body. These can be varied to meet particular cases. In general, the greater the speed of the moving body the better is the behaviour of the device. The experiments exhibited show that the contrivances possess great power even when the speed of the controlled body is very small.

It may be pointed out in connection with the applications to the problem of the dirigible torpedo that by arranging that the instabilities of the gyrostat are small its properties are made those of a gyrostat freely mounted. It, however, remains correctly balanced on its mounting so long as the spin is maintained and the body is in motion. Further, if it is disturbed by any cause (for example, by tilting action to change the direction of motion of the body) it automatically restores itself to the correct position once the disturbance has ceased to exist.

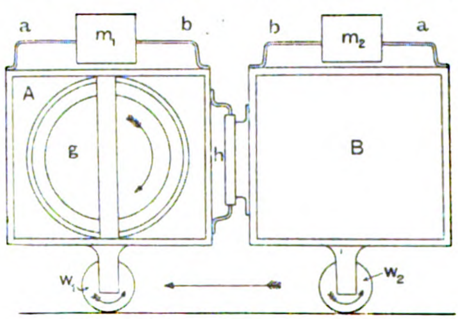


FIG. 18.—GYROSTATIC MOTOR CAR.

In Fig. 18 is shown a model illustrating a new principle. It takes the form of a two-wheeled motor-car, and has been constructed as an example of a moving body manœuvred by means of a gyrostatic "nose." The model is built up in two parts, a front one, A, and an after and propelling one, B. These are connected together by a vertical hinge, h, an arrangement which to most people will appear, on the face of it, absurd. Rigidly attached to the front part, with its axis horizontal, is a gyrostat, g. The after part carries the propelling mechanism, which is geared up to the back wheel. When the gyrostat is rotating and the car is in motion there is true stability. If the device is started in

the inclined position it erects itself into the upright one. If it is jolted any oscillations which may arise are quickly wiped out.

When the weights m_1 and m_2 shown in the figure are central the action is entirely gyrostatic. The car is, of course, unstable about the line joining the points of contact of the wheels with the ground. Further, the after part is propelling the front part, and thus an instability is introduced through the hinge. These two instabilities without rotation of the flywheel result in complete stability when the car is in motion in the forward direction.

The car is kept upright by the propeller forces. When tilting takes place the gyrostat at once sets about obtaining from the propeller the forces, *and no more*, required to erect the car. The action is shown in detail in A of Fig. 19. The view (1) represents the car when exactly vertical with the two parts in line. The direction of propulsion is indicated by the arrow placed on the after part, and the direction of rotation of the flywheel by the curved arrow. With the direction shown the spin-axis is a straight line, a_1, drawn outwards from the plane of the flywheel towards the reader. View (2) shows the car after a tendency to tilt towards the reader has asserted itself; the tilting couple is represented by a_2. The two parts now get out of line, and the after part exerts on the gyrostat a couple which is represented by a_3. The car now moves up so as to annul the tilting couple. Further, and this is very important, the forward motion of the device tends to bring about a diminution of the want of alignment of the parts. As a result diminution of the tilting couple is accompanied by diminution of the couple applied through the hinge; both couples diminish at about the same rate and finally disappear together. The car is now in the upright position with the two parts in line; its direction of motion has been slightly changed.

Again, suppose the car to be in the upright position with the two parts in line. Let a want of alignment be brought about without there being any tilting couple to account for it (this might be due to a jolt). The gyrostatic action is shown in B of Fig. 19. 1′ shows the car upright with the two parts in line; 2′ shows the device upright, but with the parts out of line; and 3′ shows the car in a tilted position brought about by the want of alignment. The vector a_2' now disappears, leaving a_3', which is accounted for in the manner already explained.

Now, let the gyrostat be rotated in the direction opposed to that in which the wheels of the car rotate, and let the weights

m_1m_2 be put in the positions *aa* of Fig. 18. If tilting takes place the parts turn on the wheels and at the hinge, with the result that the weights *ww* are carried over so as to correct the tilting couple. Further, a stabilising couple is introduced through the hinge. If the weights are placed in the positions *bb* of Fig. 18 the gyrostat should be spun in the direction in which the wheels of the car rotate.

This two-wheeled device is completely stable when being propelled, even though very slowly. When balanced it moves in a perfectly straight path. When the direction of spin is opposite to that in which the wheels of the car rotate the effect of placing a weight to one side—say, to the right, as viewed

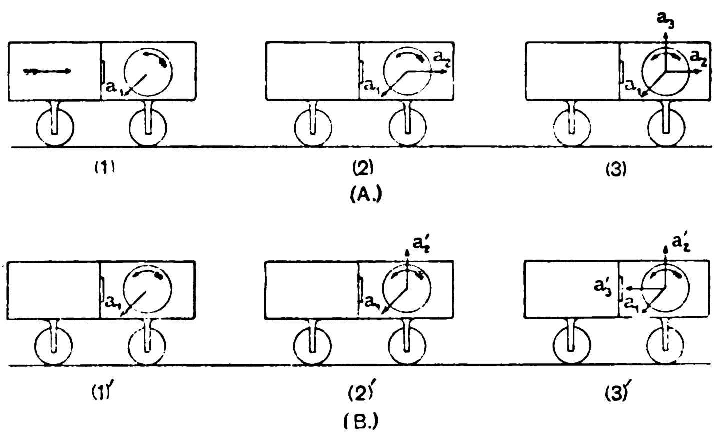


FIG. 19.—ACTION OF GYROSTATIC MOTOR CAR.

from behind—is to cause the car to move counter-clockwise in a circular path as viewed from above. Weighting the car to the left causes it to move clockwise in a circular path.

Figs. 20 and 21 show two working models of the device. One of the models, it will be seen, is provided with an electro-magnetic steering device. A very small motor, provided with worm gearing, rotates a weight carried at the end of an arm. When the motor is running the weight is carried round in a horizontal circle. When the arm is in line with the car the latter moves in a straight path; when it is to one side the car moves in a circular path in one direction; rotating the weight to the other side of the car and switching off the motor leaves the device moving in a circular path in the opposite direction.

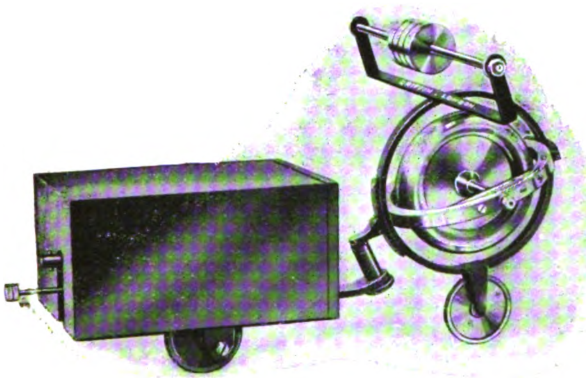


FIG. 20.—GYROSTATIC MOTOR CAR.

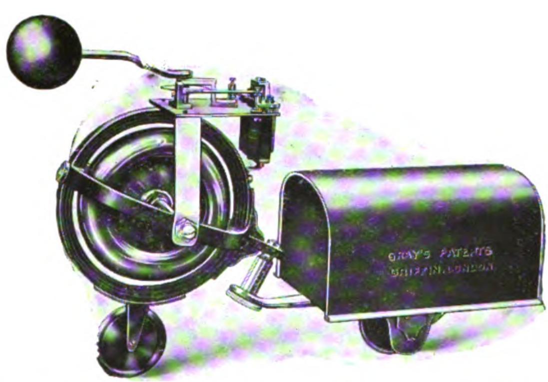


FIG. 21.—GYROSTATIC MOTOR CAR WITH ELECTROMAGNETIC STEERING DEVICE.

To face page 244.]

The principles which have just been demonstrated appear of great importance in their applications to airships. As a first example, consider an airship built in two parts, a front one and an after one, connected together by a vertical (or inclined) sleeve. On the front part let there be a gyrostat mounted so as to be laterally unstable without rotation of its flywheel. The after part carries the car, propelling mechanism and power appliances. Provided that the gyrostat is sufficiently powerful, and the construction is properly attended to, this gyrostatic arrangement is one of stability when the airship is in motion. The airship could be manœuvred forcibly in a horizontal plane; the forces required to stabilise the system and to turn the airship against the resistance of the air would be obtained from the propeller system. From the point of view of construction it would simplify matters if the entire airship were made laterally unstable.

Again, let the gyrostat be mounted on gimbal rings at the front of an airship, with its axis fore and aft, and so that the intersection of the gimbal axes passes through the centre of gravity of the gyrostat. Further, let the gyrostat be provided with a system of vanes and springs, so that it is made doubly unstable, without rotation of its flywheel, when the airship is moving in the forward direction. With the flywheel in rapid rotation the gyrostat would be completely stable, and would be of great service in manœuvring the airship in both the vertical and the horizontal directions, the couples bringing about the turning being derived directly from the propelling system, which applies a direct push. To manœuvre the airship couples are directly applied to the gyrostat.

Such gyrostatic airships on a large scale could only be brought to perfection as a result of experiment and trial. Experiments of the nature required are not posssible to a private individual. It may be said, however, that careful calculations have been made relating to the size and power of the motor-gyrostats which would be required, and these could certainly be produced. Such airships are perfectly safe if the gyrostat breaks down. With the propellers reversed and the gyrostat out of action the arrangement is one of stability. In conclusion it may be said that airships on a small scale, capable of being manœuvred by gyrostatic action, could be easily evolved. Such a contrivance could be caused to soar upwards or downwards, or to turn in a horizontal plane in either direction by means of forces derived from a propeller system exercising a direct propulsive force on the system.

ABSTRACT.

The Paper dealt with a number of new contrivances for stabilising, steering and forcibly manœuvring moving bodies, such as torpedoes and airships.

A number of old experiments were first shown. These included the "gyrostat on stilts" and "gyrostat on gimbals" experiments due to Lord Kelvin, the "crossed bifilar" experiment due to Prof. Blackburn, and a stilt top devised by Prof. Harold Wilson. It was shown that the gyrostatic system in each of these experiments, although exhibiting considerable balancing power, was not possessed of real stability. An unstable body rendered truly stable by gyrostatic action must possess the property that if displaced from the mean position it returns to, and comes to rest in, that position. The mean position is that in which the potential energy of the gyrostatic system is a maximum, and if the system is disturbed energy must be supplied to restore it to the mean, or undisturbed, position.

A number of new gyrostatic models were displayed in action. These include two-wheeled and four-wheeled gyrostatic motor cars and bicycles. These all provide examples of gyrostatic systems provided with complete or real stability, and in all the cases shown the stabilising forces are derived from the propelling system.

One of the cars shown runs on two wheels in tandem, and is stabilised by a single gyrostat. This gyrostat is mounted in the car and controls the steering mechanism; it forms, in fact, a gyrostatic chauffeur. The model illustrated a new form of torpedo and airship control.

A second form of motor car, which also runs on two wheels in tandem, consists of two parts, a front one and an after one. The front part carries a gyrostat, the back part the propelling mechanism, and the two parts are connected together by means of a vertical hinge. The front part is propelled by the back part, and the arrangement is one of complete stability. The entire system may be manœuvred by means of the gyrostat. It was pointed out that by properly fitting an airship with a gyrostatic "nose" it should be possible to manœuvre forcibly the airship by means of forces derived from the propellers.

The bicycles, which are provided with gyrostatic riders, are examples of moving bodies steered by gyrostatic action. The action is quite different from that of an ordinary bicycle. They are not "momentum" instruments.

The devices shown are at once applicable to long-distance torpedoes, both submarine and aerial. The gyrostatic system may be operated by the wireless transmission of electrical action.

At the conclusion of the Paper the author showed a new series of animated gyrostats.

DISCUSSION.

Dr. W. WATSON thought the mechanisms shown were of great theoretical importance. He gathered, however, that the author himself thought they were more of theoretical than practical interest. He concluded some time ago that a two-wheeled car would not be of much use, as, although gyrostatic control worked satisfactorily either on a straight path or on a curved path of constant curvature, any attempt to alter the curvature had to be made with great caution. Hence a train built

on this system would have to slow up on approaching either the beginning or the end of a bend. With a motor car, where one had to steer immediate courses on account of other traffic, the arrangement would be impracticable. At one time, when some cars had engines laid longitudinally and others transversely, makers of the latter type claimed that gyrostatic action came intc play and tended to prevent skidding. However, unless the gyrostat was free to move relatively to the car, one might as well have a lump of iron in its stead. He had investigated the amount of relative motion which might take place due to give in the springs or mountings, and it was quite insufficient to allow of appreciable gyrostatic action.

Mr. Duddell complimented the author on the collection of beautiful models which he had brought before the Society and the admirable way in which he had explained the principles underlying their action. He asked what speed was attained by the flywheels of the gyrostats.

Mr. R. S. Whipple also expressed his admiration of the models.

Mr. F. J. Whipple asked if the author had worked out the theory of the ordinary bicycle, and if it was his considered opinion that the rider had to perform the actions which he had described in steering. It was his opinion that when travelling rapidly this was not so, and that there was a stabilising effect due to the gyrostatic action of the front wheel. This was particularly noticeable in the way in which the wheel seemed to be pulled back into position if, when riding without the hands, the cyclist encountered a small stone.

Dr. Russell asked concerning the use of the word "gyrostat." He remembered on one occasion when Lord Kelvin was showing some of these experiments to von Helmholtz an accident occurred which resulted in one of the gyroscope wheels passing through Helmholtz's silk hat. After that it was customary to enclose the gyroscope in a brass case, and it was then usually called a gyrostat. He did not quite see why.

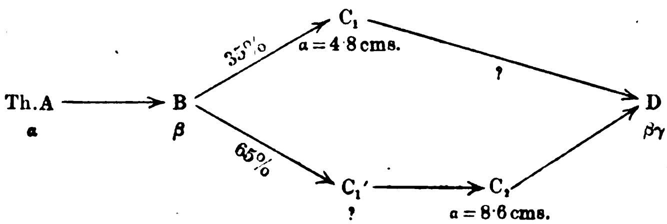
The Author, in reply, said that the larger gyrostats could be taken up to a speed of 20,000 revs. per min. in half a minute, and would run for 75 minutes. In steering a bicycle the rider turns the front wheel to the side to which the machine leans, and the forward momentum brings it up to the vertical position. The gyrostatic action helps, but only to a very slight extent. The name "gyrostat" was the one invariably used at Glasgow since Lord Kelvin's time.

XXVI. *Volatility of Thorium Active Deposit.* By T. Barratt, *A.R.C.S., B.Sc., and* A. B. Wood, *M.Sc.*

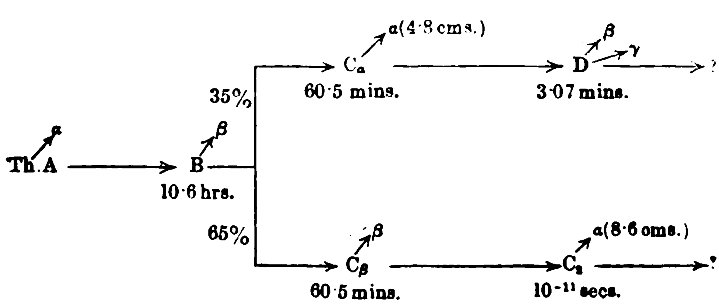
Received April 21, 1914.

I.—Introduction.

In a Paper published in 1911 by E. Marsden and one of us,* it was first proved that the two α-ray products contained in ThC are not successive, but belong to different branches of the thorium series. The following scheme of disintegration was suggested :—



The hypothetical product C_1' was included in order to account for the fact that C_1 and C_2 appeared to be always present in the same relative proportions, viz., 35 : 65. This could not possibly be the case if C_2 came directly from B, unless C_2 had the same period as C_1. This, however, is unlikely, as the range of C_2 is nearly double that of C_1, and in the case of most other α-ray products of any particular radio-active family, Geiger and Nuttall's rule that log (transformation constant)/log (range) is a constant applies pretty closely. Marsden and Darwin,† after further experiments with thorium active deposit, proposed the scheme :—



* Marsden and Barratt, "Proc." Phys. Soc., 24, p. 50, 1911.

† Marsden and Darwin, "Proc." Roy. Soc., A, LXXXVII., p. 17, 1912.

In this arrangement C_α and C_β are supposed to represent, not two different products, but the same product breaking down in two different ways, one with the expulsion of an α-particle, giving rise to D, the other emitting a β-particle, and forming C_2, which in its turn is transformed to an unknown substance on expelling an α-particle.

Miss Meitner,* however, obtained results which seemed to indicate that the hypothetical substance C_1' (or C_β) could be obtained free from C_1 (or C_α). On adding a few drops of stannous chloride to an HCl solution of the active deposit, and dipping successive nickel plates in the solution, the C_α appeared to be removed, leaving C_β and D in solution. The latter was then boiled to dryness, divided into two portions, and, the D being first allowed to decay, activities measured by α and γ-rays respectively. The γ-curve rose from zero, showing that C_α, and therefore also D (which produces the γ-rays), had been removed by the nickel plates. The α-radiation, however, arising presumably from C_2, did not commence from zero, showing that C_β (with C_2) had not been completely removed.

Marsden and Wilson† repeated Miss Meitner's experiments, and acknowledged obtaining sometimes the same results, but concluded on the whole that they were anomalous, and due mainly to the fact that, in the presence of HCl, C volatilises at a comparatively low temperature. In their experiments the active deposit was heated to temperatures not accurately measured, but estimated at 250°C. to 300°C.

In view of these conflicting results, it was thought desirable to attempt to separate the members of the active deposit by heating the latter to various accurately measured temperatures, and following the consequent changes of activity, making measurements of both α and β-radiations.

II.—Previous Work on Volatilisation.

The first attempt to separate the members of the thorium active deposit by volatilisation appears to have been made by Miss Gates‡ in 1903. She obtained apparently a partial separation of B and C (then known as A and B respectively). Two years later Miss Slater§ heated for one or two minutes a

* Lise Meitner, "Phys. Zeit.," XIII., p. 623, 1912.
† Marsden and Wilson, "Phil. Mag.," pp. 354-361, Aug., 1913.
‡ Miss Gates, "Phys. Rev.," 16, p. 300, 1903.
§ Miss Slater, "Phil. Mag.," 9, 628, 1905.

platinum wire coated with the active deposit to temperatures up to about 1,300°C. B commenced to volatilise at 630°C., but no part of C was removed till a temperature of 730 deg. was reached. At 1,280°C. the whole of B was removed, and 99 per cent. of C, the small remaining activity (presumably measured by α-radiation) diminishing with a period of about one hour. The temperatures in these experiments were measured by a thermo-junction. No β-ray measurements were made, so that the results do not throw much light on the question of the separability of the various products now known to be present in the thorium series.

III.—Apparatus and Measurements.

(a) *Active Deposit Undissolved in Acid.*

An electric furnace, consisting of a porcelain tube wound with strip platinum, was employed for heating the active deposit to any required temperature. The internal dimensions were, approximately, length 60 cm., diameter 4·5 cm., this large size ensuring that the volatilised portion of the deposit could easily escape, and would have little chance of settling once more on the plate on which it was originally deposited. The temperature was accurately measured by a platinum thermometer, contained in a tube of biscuit porcelain, in connection with a Callendar-Griffiths' "bridge," whose coils were carefully calibrated in the usual way. The "ice" and "steam" points were determined, and verified from time to time. The δ-coefficient of the platinum was taken as 1·50. In order to ensure that the temperature of the active deposit was that indicated by the thermometer, the active plate was placed in a small flat platinum "basket," which was hung on the thermometer near its extreme end. The active deposit was obtained from a preparation of mesothorium, very kindly lent by Mr. F. H. Glew. By a simple arrangement the active deposit could be obtained on one side only of a platinum foil, and by a suitable adjustment of the conditions of exposure considerable variations in its activity could be obtained. In earlier experiments the results were somewhat variable, owing to the fact that the platinum used was not perfectly clean. The slightest trace of grease, &c., is enough to spoil an experiment, as the active matter is carried away when the grease evaporates. (This may possibly account for the comparatively low temperatures of volatilisation obtained by Miss

Slater.*) Consequently the foils were strongly heated for some time in a blow-pipe flame before being exposed to the emanation. To check results, the active plates were frequently placed in the furnace in pairs. It was also found advisable not to use an electric field when exposing the plates to the emanation, as in such cases a small percentage of ThX was usually found with the active deposit. This precaution also precluded the possibility of any appreciable quantity of active deposit from radium, which is always present in mesothorium, being attracted to the plates. In addition, an interval of at least five hours was allowed to elapse—after removal from the emanation—before any measurements of activity were made. The period of decay was then governed by that of ThB (10·6 hours). Small pieces of quartz were used in some experiments instead of platinum, but no difference was observed in the temperatures of volatilisation. This result is different from that obtained by Makower† in the case of radium active deposit. In every experiment measurements of the activity of the foil were made for some time before it was placed in the furnace, an α or β-ray electroscope being used for the purpose.

(i.) *Measurements by a-rays.*

The deposit was heated to the required temperature, in most cases for 15 minutes, as the volatilisation appeared to be to some extent a time effect, and immediately after removal from the furnace the activity was again measured, and continued for several hours, until the decay was exponential, with the period of ThB.

Fig. 1 illustrates graphically the results of one of these experiments. The first part (A) gives the curve of activity before the plate was heated, the period of decay then being that of B. The active plate was in the furnace for a time indicated by *a*—*b*. After removal from the furnace, the curve of activity was much steeper, as is shown in the latter portion (B) of the curve. In order to calculate the percentage of C-activity removed by heating, the curve B was produced back to the point Q on the line P*c*, which corresponds to the time when the plate has been in the furnace four minutes. Previous observations had indicated that in this time, at a given constant temperature, practically the maximum amount of active matter has been volatilised. The percentage of C removed is then

* Miss Slater, *loc. cit.*

† Makower, "Le Radium," 6, p. 50, 1909.

given by 100 PQ/Pc. To calculate the percentage of B removed, readings were taken six or seven hours after the foil was heated. The values then observed were corrected (to the time of heating) for decay of B (period 10·6 hours), and the percentage reckoned as in the case of C.

Fig. 2 embodies the results of a great number of experiments of a similar kind at various temperatures, the measurements

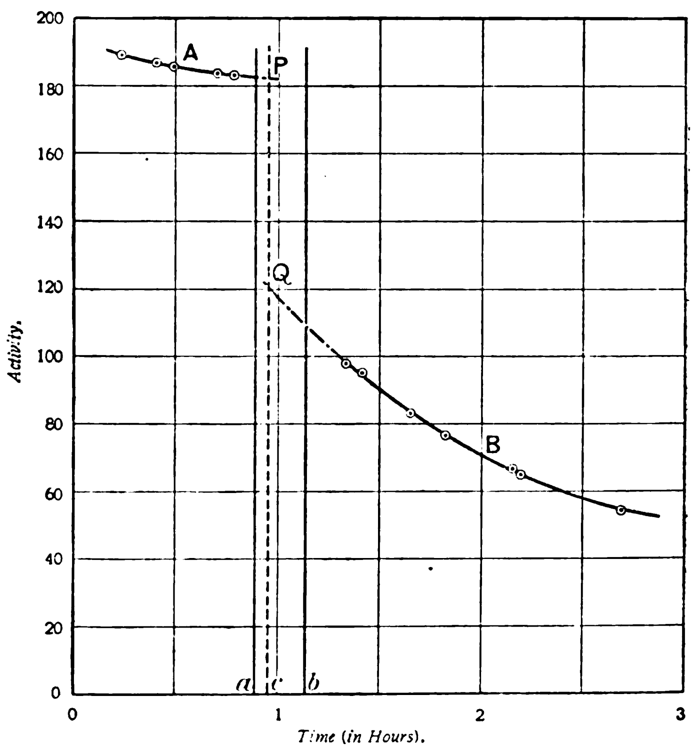


FIG. 1.

all being of the α-radiation. It can easily be deduced from the curves that :—

1. Both B and C begin to volatilise at about 750°C. —B perhaps at a temperature slightly lower than C.

2. Between 750°C. and 1,200°C. a greater percentage of B than of C is volatilised at any given temperature.

3. Volatilisation of both B and C is practically complete at 1,200°C.

4. There is an inflexion of the C curve between 750°C. and 900°C; in fact the curve is similar to two of the "B" curves placed end to end. This points to the possibility that between 750°C. and 900°C. only one α-ray product is being volatilised; after 900°C. the curve becomes very much steeper, as if the second product is also being driven off.

5. The inflexion occurs at a point where about 35 per cent. of the α-ray activity has disappeared.

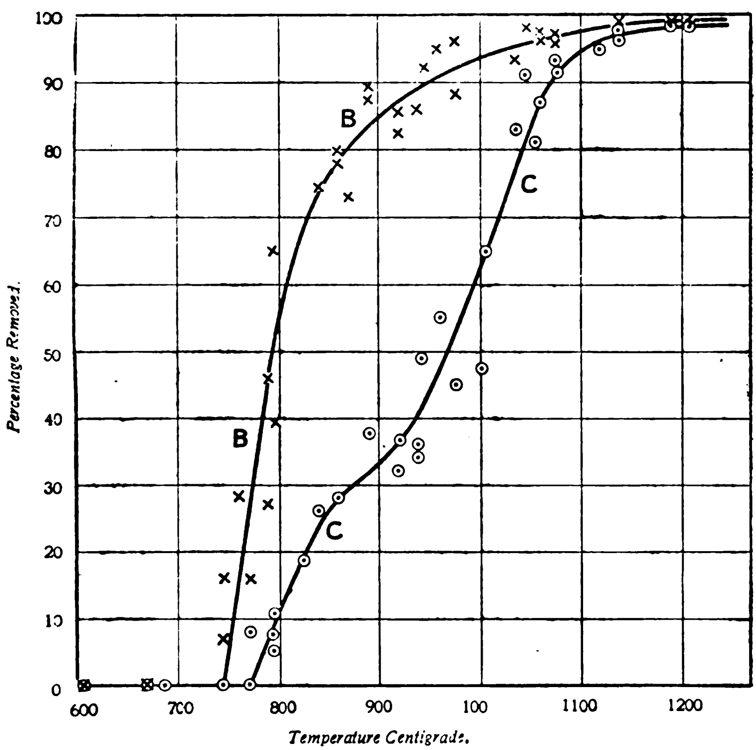


FIG. 2.

(ii.) *Measurements by β-rays.*

In view of the fact, before mentioned, that the ratio of C_α to C_β is as 35 : 65, t was recognised that the shape of the C curve was significant. In order to throw more light on the question similar experiments were conducted, employing this time the

β-radiation. The result of one of these experiments is exhibited in Fig. 3. It is noticeable that :—

1. There is at first a steep rise of the curve, undoubtedly due to the growth of D, which is known to be at any rate more volatile than B or C.*

2. The curve then descends with a period a little greater than that of C.

3. The activity after several hours decays with the period of B.

As the result of many similar experiments at various temperatures it was found that, *as measured by β-rays*, C does not begin to volatilise till a temperature of about 900°C. is reached. It

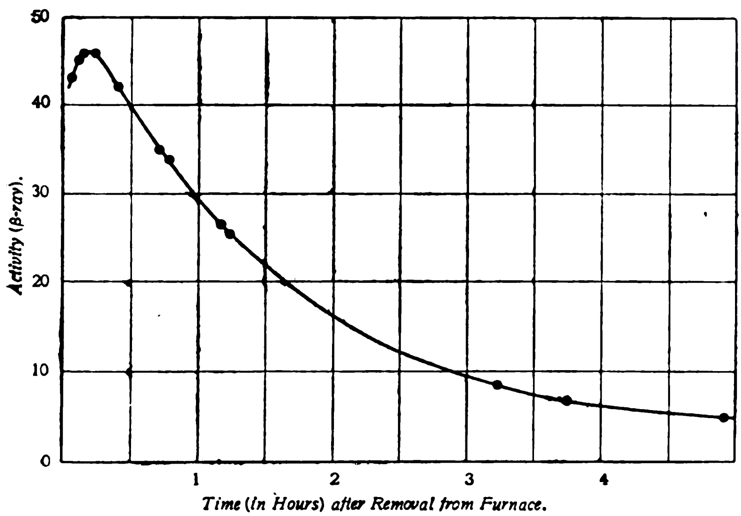


FIG. 3.

is significant that this temperature, again, is that at which the second part of the C curve, as obtained by α-ray measurements, commences. An explanation of these observations will be attempted in section IV.

(*b*) *Experiments with the Active Deposit Dissolved in Acid.*

As some of the researches quoted† were carried out with the active deposit dissolved in acid, some experiments were done with solutions of the active substance in pure conc. HCl, or

* v. Lerch and v. Wartburg, "Wien. Ber.," 118, p. 1575, 1909.

† Marsden and Wilson, Miss Meitner, *loc. cit.*

HNO_3. As a result it was found that in the case of the HNO_3 solution volatilisation occurred at practically the same temperatures as before. With HCl, however, the activity disappeared at a much lower temperature, commencing, in fact, at about 300°C. In the case of solution in HNO_3, as with the undissolved product, the deposit is probably in the form of an oxide, while the solution in HCl is no doubt a chloride. In this connection some results of Dr. Schrader's* are interesting. He found that with the active deposit of actinium, volatilisation occurred at a much lower temperature if a platinum wire coated with it were first exposed to an atmosphere of chlorine. Russell† also discovered that in hydrogen RaC was volatile at 360°C., while in oxygen the temperature had to be much higher. In Marsden and Wilson's Paper, again, it is shown that ThC volatilises at about 300°C. when dissolved in HCl.

(c) *Volatility of Thorium D.*

As the result of experiments similar to that exhibited graphically in Fig. 3 (β curve), it was concluded that D begins to volatilise at about 500°C. The active deposit, on a platinum foil, and undissolved in acid, was heated for 15 minutes in the furnace at various temperatures. Up to 500°C. there was no break at all in the curve of activity as obtained by β-ray measurements. At this temperature, however, some of the D was removed (as was indicated by the initial rise of the curve after removal from the furnace). The same phenomenon was observed at higher temperatures, up to about 900°C. Beyond this temperature, however, provided the active plate was in the furnace long enough (*e.g.*, 10 or 15 minutes) there was no perceptible rise of β activity due to growth of D. Assuming that C_α is removed almost entirely at 900°C., while C_β is not removed to any extent, this verifies the experiments of Marsden and Darwin,‡ and of one of us,§ that D arises entirely from the 35 per cent. branch, viz., C_α.

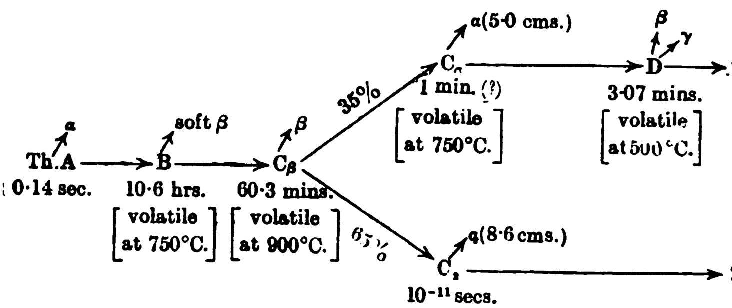
IV.—Discussion of the Results.

The fact that volatilisation of both B and C commences at 750°C., and is not complete until about 1,200°C., precludes the possibility of an absolute separation of these two products

* Schrader, "Phil. Mag.," 24, p. 125, 1912.
† Russell, "Phil. Mag.," 24, p. 134, 1912.
‡ Marsden and Darwin, *loc. cit.*
§ A. B. Wood, "Phil. Mag.," pp. 586-597, Oct., 1913.

from each other by this method. This wide range of temperature seems rather remarkable. It may be that the oxides are gradually reduced to the pure metal, which is not volatilisable except at a very high temperature. The case may be, in fact, similar to that of mercury, which, when heated in air, oxidises at a comparatively low temperature, and is again reduced to pure mercury at a considerably higher one. A temperature greater still is then required to vaporise the metal.

In order to explain the results already quoted in the present Paper it is evident that some modifications must be made in schemes of disintegration that have been proposed up to now. The following arrangement appears to us to fit the facts better than any other we can devise :—



It will be observed that C_β is assumed to be a separate product, which breaks up in two ways, 35 per cent. of the disintegration producing C_a and 65 per cent. giving C_2. As in the case of B and C (the latter taken as a whole), there is only a partial separation of the two products, but evidently C_a is more readily volatile than C_β.

The scheme accounts for :—

1. The branching of C_a and C_2.

2. The differences in volatility obtained in α and β-ray measurements respectively. Neglecting the small percentage of β-radiation from D, which can easily be identified and allowed for, all the β-activity measured in the present experiments comes from C_β, as the base of the electroscope employed was too thick to be penetrated by the soft β-rays from B. The experiments show then that C_β is not removed to any perceptible extent until a temperature of 900°C. is reached, while C_a begins to volatilise at 750°C.

3. The periods of B and C (10·6 hours and 60·3 minutes

respectively). On the above scheme the period of C is governed by that of C_β. This assumes, of course, that the period of C_α is comparatively small (perhaps only a minute or so). This will be discussed later.

4. The inflexion in the curve of volatilisation of C. This has been already mentioned. (*See* Section III.)

5. The constant ratio of the activities of C_α and C_2. Several experiments were tried in the present research in order to test the constancy of this ratio. Measurements were taken of the activity of the plates before and after heating :—

(*a*) With active plate uncovered;

(*b*) With plate covered with just sufficient aluminium foil to stop the α-rays of range 4·8 cm.

The ratio of activities (before and after heating) was about the same in both cases (*a*) and (*b*). This result, however, is easily explained if we make the assumption suggested in (3) above, *i.e.*, that the period of C_α is no more than a minute or so. It must be remembered that by far the greater proportion of the α-activity is produced by C_2 in virtue of—

(*a*) Its longer range (8·6 cm. to 4·8 cm.).

(*b*) The fact that it gives twice as many α-particles.

(*c*) It produces its effect in both cases, whether the foil is covered or uncovered.

It would therefore be difficult to prove the existence or otherwise of a small amount of C_α. In any case, assuming that C_α has a short period, the above scheme shows that C_α and C_2 would each be in equilibrium (in the ratio of 35 to 65) with the common parent product C_β in a very short time—almost, in fact, before any reliable measurements of the relative α-activities could be made.

As a great deal in the explanation given depends on the life of C_α being considerably shorter than that usually attributed to C as a whole (viz., 60·3 minutes), it is of importance to note that, in order to fit the relation given by Geiger and Nuttall,* the period of C_α (with its range 4·8 cm.) should be only two or three minutes. The range usually attributed to C_α (4·8 cm.), was determined by Marsden and Barratt† by a scintillation method, which is well known to give lower results than the usual ionisation determinations.‡

* Geiger and Nuttall, "Phil. Mag.," 22, p. 613, 1911; 23, p. 439, 1912.

† Marsden and Barratt, *loc. cit.*; Barratt, "Proc." Phys. Soc., 24, p. 112 1912.

‡ Rutherford, "Radioactive Substances and their Radiations," p 161, 1913.

If the range were taken as 5·0 cm. (as given by some experimenters),* then Geiger and Nuttall's relation would assign a half-period of less than a minute to C_{α}.

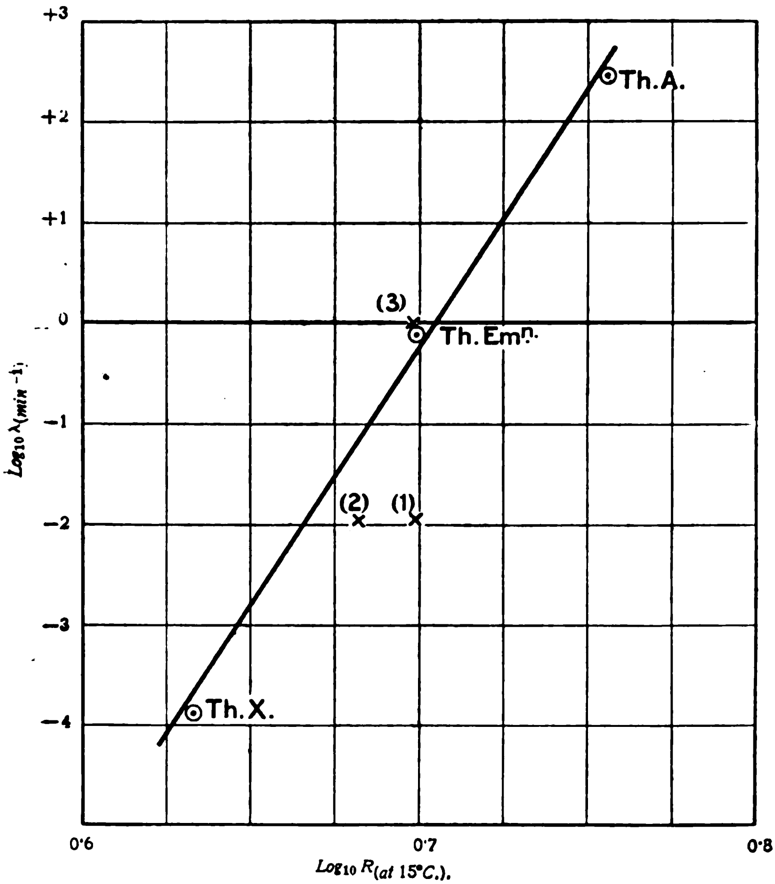


FIG. 4.

This is clearly shown in Fig. 4, the crosses indicating the positions that would be occupied by C_{α} if its half-period and range were respectively—

1. 60·3 mins.; 5·0 cm.
2. 60·3 mins.; 4·8 cm.
3. 40 secs.; 5·0 cm.

* Hahn, "Phys. Zeit.," 7, p. 456, 1906.

Experiments are in progress with the object of further elucidating the points raised above. The radium and actinium series are also being tested in a similar way; and by heating these products in atmospheres of different gases it is hoped that further knowledge may be gained of the separability, &c., of the members of these series.

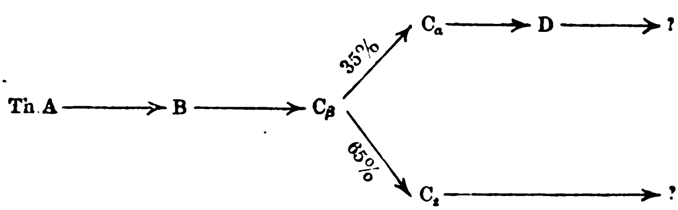
Summary.

1. By heating thorium active deposit to various temperatures up to 1,250°C., it is found that B and C each begin to volatilise at 750°C. (when measured by α-radiation), but volatilisation is not complete until a temperature of about 1,200°C. is reached.

2. The curve of activity of C is similar to two "B-curves" in series, the inflexion occurring at about 900°C., where about 35 per cent. of the α-activity has been removed.

3. When measured by its β-activity, C volatilises at 900°C. and D at 500°C.

4. In explanation of these results, it is assumed that C_β is a separate product, which breaks up in two ways, each with the expulsion of an α-particle, producing C_α and C_2 as shown in the following scheme :—



5. Temperature of volatilisation "in HNO_3" same as ordinary active deposit. Temperature of volatilisation in HCl, 300°C. for Th.B and C.

The experiments have been carried out at the Woolwich Polytechnic, and our best thanks are accorded to the Principal and Governors of that institution; also to Mr. F. H. Glew, who very kindly lent us, for a considerable period, the mesothorium used in the research.

ABSTRACT.

On heating thorium active deposit to various accurately measured temperatures up to about 1,250°C., it is found that B and C each commence to volatilise at 750°C., but the volatilisation is not complete until 1,200°C. is reached, the measurements being made by an α-ray electroscope. The C curve is peculiar, being similar to two of the

B curves placed end to end, the inflexion occurring between 750°C. and 900°C., where about 35 per cent. of the α-activity has been removed. When measured by β-radiation, C is not volatile until a temperature of 900°C. is reached. D commences to volatilise at 500°C.

To explain these results, it is assumed that the part of C which produces the β-rays—viz., C_β—is a separate product, which is not so readily volatile as C_α. A suitable scheme of disintegration is suggested.

DISCUSSION.

Dr. R. S. Willows congratulated the authors on their able treatment of the subject. The suggested scheme met most of the requirements of the case and was not in contradiction with any of the other known properties of the thorium series. He hoped the authors would extend their work to other radio-active series.

Dr. S. Russ drew attention to the apparent lack of parallelism between the characteristics of the thorium and radium series. The volatility of thorium B appeared to be not very different from that of thorium C, whereas in the radium series, radium B is the most, and radium C the least, volatile of the series. Again, the authors concluded that there was no appreciable difference in the volatility of the thorium active deposit from a quartz or a platinum surface, whereas there was an appreciable difference in the case of the radium active deposit as Dr. Makower had shown.

Mr. D. Owen observed that one of the products was stated to have a period of 10^{-11} second. He thought it hardly possible to catch a product which only lived for that time.

Mr. Barratt said that only two surfaces were used, quartz and platinum, on account of the high temperatures to which they had to be subjected. In these cases there had certainly been no difference. When different acids were used, considerable differences were observed in the temperature of volatilisation. The period, 10^{-11} second, assigned to Th C_2 was simply in order to fit the Geiger-Nuttall relation. The curve connecting log (λ) and log (Range) is a straight line, and the period 10^{-11} second was necessary to make the product fit the curve.

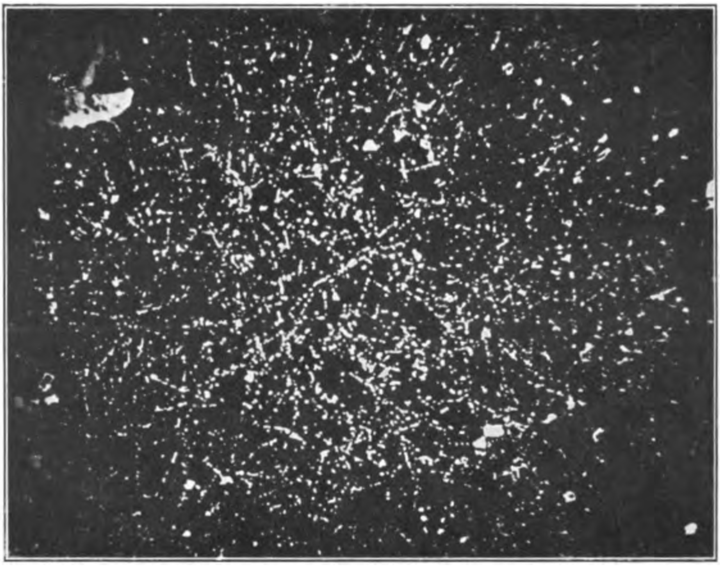


Fig. 1.

Fig. 2.

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XXVII. *The Passage of α-Particles through Photographic Films.*

By H. P. WALMSLEY, *M.Sc.*, *and* W. MAKOWER, *M.A.*, *D.Sc.*

RECEIVED APRIL 17, 1914

THE photographic action of α-particles was first studied in detail by Kinoshita,* who showed that, whenever an α-particle, strikes a grain of silver halide in a photographic plate, that grain is subsequently capable of development. It was also shown that this was the case throughout the range of the α-particle in the gelatin. It therefore seemed probable that the path of an α-particle, projected tangentially to a photographic film, would, after developing the plate, be seen under a microscope as a trail of grains. This effect has been observed by Reinganum† and the method has been employed by Michl‡ and Mayer§ to detect α-particles ; but microphotographs demonstrating the phenomenon do not seem yet to have been obtained. In some experiments upon which we were engaged, it became necessary to examine microscopically photographs taken with very weak sources of α-radiation and it occurred to us that it might be of interest to obtain microphotographs of the trails which could be clearly seen in the films.

As a nearly pure source of α-radiation of fairly long range, a flat metal plate was exposed for a few seconds to a fraction of a millicurie of radium emanation and then immediately placed in contact with an Ilford process plate. The α-rays from the radium A on the metal surface produced a photographic impression on the plate which was just visible to the naked eye after development. Under the microscope this faint impression was seen to consist of a multitude of definite tracks passing through the film at all angles. A microphotograph of the film taken with a $\frac{1}{3}$ in. objective is reproduced in Fig. 1. It was found best to use this relatively low-power objective for, as the grains of silver affected by the α-particles were at different depths in the film, it was impossible to focus all the grains simultaneously when using an objective of higher power. The photograph confirms in a striking manner the conclusions

* Kinoshita. "Proc." Roy. Soc., A83, 432, 1910.
† Reinganum, "Phys. Zeits.," 12, 1076, 1911.
‡ Michl, "Wien. Ber.," 2A, 1431, 1912.
§ Mayer, "Ann. d. Phys.," 41, 5, 931, 1913.

drawn by Kinoshita which have been mentioned above. Careful examination of Fig. 1 shows that many of the α-particles, after traversing the film in straight lines for some distance, suffer sudden deflections, thus illustrating the well-known effect of scattering; but the great number of the tracks on this photograph and the smallness of the magnification make it difficult to follow out these details easily. Moreover, the α-particles penetrate the film in all directions, so that only a relatively small number leave tracks sufficiently nearly parallel to the plane of the film to be examined throughout their lengths. On this account the photographic impression produced just outside the edge of the active plate was examined; for in this region the density of the tracks is much smaller and a greater proportion of the α-particles producing them were travelling parallel to the plane of the film. A microphotograph similar to that shown in Fig. 1 was taken of this region of the photographic plate and a few isolated tracks were again magnified by photography with an ordinary camera. Three such tracks are shown in Figs. 2, 3 and 4, which show the phenomena in greater detail. In the cases of the tracks shown in Figs. 2 and 3 the magnification was measured by determining the lengths of the paths on the original photographs with a graduated scale in the eyepiece of a microscope. The magnification in Fig. 2 was found to be 1,650 diameters; that in Fig. 3 was 1,800. It will be seen that the magnification in Fig. 4 is slightly less. The large spots seen on Figs. 3 and 4 near the tracks are due to minute particles of dust on the original photograph.

Some difficulty was experienced in making sure that the tracks showing deflections were due to single α-particles and not to two particles passing through the film in different directions. There is no means of deciding this point from a study of the microphotographs, but it is an easy matter to trace out the complete path of an α-particle throughout its length by visual observations through a microscope. For as a rule the grains in any track are not precisely at the same depth in the film and it is possible by careful focussing to ascertain whether an α-particle has been steadily penetrating into the film or emerging from it. In cases in which two tracks crossed each other, at the point of apparent intersection the grains due to the two α-particles were in general seen to be at quite different depths in the film.

This method of studying the projection of α-particles through matter shows many of the characteristics demon-

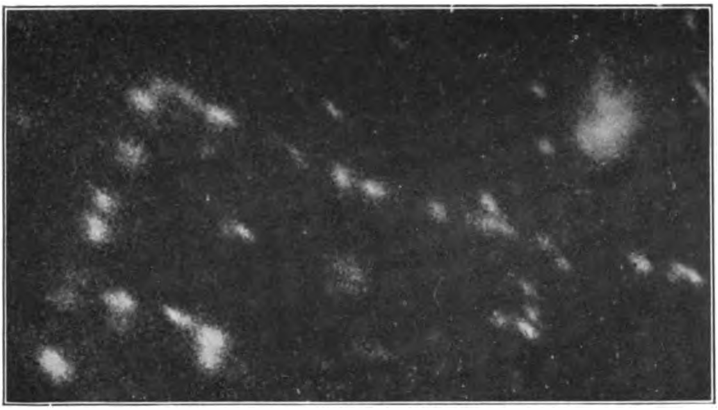

Fig. 3.

Fig. 4.

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strated by C. T. R. Wilson* in his beautiful photographs of the tracks of α-particles through air. The method just described possesses, however, the advantage of great simplicity, and may prove of use in studying the scattering of α-particles produced by heavy substances, such as silver.

ABSTRACT.

It has been shown by Kinoshita that when an α-particle strikes a grain of silver halide, that grain is subsequently capable of photographic development. It therefore seemed probable that the path of an α-particle projected tangentially to a photographic film should, after development, be visible under a microscope. This was shown to be the case, and micro-photographs showing the tracks of α-particles through a photographic plate have been obtained. The effect of "scattering" of α-particles can also be seen in the photographs and this method may prove of use in studying the scattering of α-particles by heavy atoms such as silver. This method of studying the path of an α-particle possesses the advantage of great simplicity.

* C. T. R. Wilson, "Proc." Roy. Soc., A87, 277, 1912

XXVIII. *On a Null Method of Testing Vibration Galvanometers.* *By* S. BUTTERWORTH, *M.Sc., Lecturer in Physics, School of Technology, Manchester.*

RECEIVED APRIL 11, 1914.

1. In a recent Paper A. Campbell* has given the constants of a number of vibration galvanometers. In obtaining these constants the measurements made were the alternating-current and voltage sensitivities, the direct-current sensitivity and the resonating frequency. Of these, the alternating-current and voltage sensitivities are the most difficult to measure, since, because of the sharpness of resonance, there is a considerable fluctuation in the amplitude of vibration for very small variations in the frequency of the source, while the presence of any harmonics in the source will introduce a definite error in the results.

In the course of an investigation on electrically-maintained vibrations, the author has succeeded in developing a null method whereby the vibration constants of a galvanometer may be determined and in which the harmonics in the source have no effect. Measurements by this method, together with a knowledge of the direct-current sensitivity, are sufficient to determine the galvanometer constants.

The principle of the method depends on an extension of the theory of the vibration galvanometer. It is shown that, as far as the electrical behaviour of the instrument is concerned, we may regard it as built up of a parallel combination of a capacity, an inductance and a resistance, in series with a resistance.

At resonance, the capacity and inductance neutralise each other and the galvanometer then behaves as a pure resistance. The value of this resistance is the effective resistance as measured by Campbell.

A direct measurement of the effective resistance at resonance by a null method, is, however, practically impossible, since fluctuations in the frequency of the source have their maximum effect on the impedance of the galvanometer in this state.

It will be seen, however, that by a suitable arrangement of inductances and capacities a complete balance can be obtained

* "Proc." Phys. Soc., Feb., 1914.

for any form of current, and that the method will be applicable, with certain limitations, to all the galvanometers tested by Campbell.

The units employed will be those of the C.G.S. electromagnetic system, unless where otherwise stated. For the physical interpretation of the symbols reference should be made to Campbell's Paper. The present notation as compared with his is as follows :—

a	β	γ	A	r_v	r	p	p_0	k
mk^2	b	c	$10g$	$R'-R$	R	ω	ω_1	$\frac{2}{h}\times 10^{-4}$

The symbols in the second line are those used by Campbell.

2. Using the notation employed in a previous Paper,* the equation of motion of the coil is

$$a\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}+\gamma y=\mathrm{A}i, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where y and i are the instantaneous values of the deflection and current respectively. If the galvanometer has ohmic resistance r and inductance l, the E.M.F. across the galvanometer terminals is

$$e=ri+l\frac{di}{dt}+\mathrm{A}\frac{dy}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the last term being the back E.M.F. due to the motion of the coil.

When the current is alternating with a frequency $p/2\pi$, equations (1) and (2) transform into the vector equations

$$\left.\begin{array}{r}(\gamma-ap^2+jp\beta)\mathrm{Y}=\mathrm{AI}\\ \mathrm{E}=(r+jpl)\mathrm{I}+jp\mathrm{AY}\end{array}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Y, I, E are the vectors representing the vibration current and E.M.F. respectively. Eliminating Y

$$\mathrm{E}=\left(r+jpl+\frac{jp\mathrm{A}^2}{\gamma-ap^2+jp\beta}\right)\mathrm{I}. \quad . \quad . \quad . \quad . \quad (4)$$

The quantity in ( ) is the vector impedance of the vibration

* Butterworth, "Proc." Phys. Soc., Vol. 24, p. 75, 1912.

galvanometer, the last term being that contributed by the vibration. If we put

$$\beta/A^2=S_v=\frac{1}{r_v},\quad \alpha/A^2=C_v,\quad A^2/\gamma=L_v,\quad . \quad . \quad . \quad (5)$$

then the vector impedance becomes

$$G=r+jpl+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}.\quad . \quad . \quad . \quad (6)$$

The inductance (l) of the galvanometer can be neglected in practice, so that with this assumption we see that the galvanometer is equivalent to a parallel combination of a conductance (S_v), a capacity (C_v) and an inductance (L_v) in series with the ohmic resistance (r).

If
$$p^2=\frac{1}{L_vC_v}\equiv\frac{\gamma}{\alpha}=p_0^2\text{ (say)},\quad . \quad . \quad . \quad . \quad . \quad (7)$$

then
$$G=r+\frac{1}{S_v}=r+r_v.\quad . \quad . \quad . \quad . \quad . \quad . \quad (6\text{A})$$

This holds when the galvanometer is in resonance with the source, so that (6A) gives the effective resistance at resonance.

If $p\neq p_0$, then for good galvanometers the damping (β) may be neglected, so that, putting $S_v=0$, we have

$$p>p_0\qquad G=r-j/pC_v\left(1-\frac{1}{n^2}\right),\quad . \quad . \quad . \quad . \quad (6\text{B})$$

$$p<p_0\qquad G=r+jpL_v/(1-n^2),\quad . \quad . \quad . \quad . \quad . \quad (6\text{C})$$

where $n=p/p_0$.

Hence, for frequencies of the source above resonance the galvanometer may be treated as a condenser with a series resistance, and for frequencies of the source below resonance as an inductive resistance.

The quantities r_v, C_v, L_v will be referred to as the *vibration resistance*, *vibration capacity* and *vibration inductance* respectively, and they will be called generally the *vibration constants* to distinguish them from the intrinsic constants α, β, γ, A.

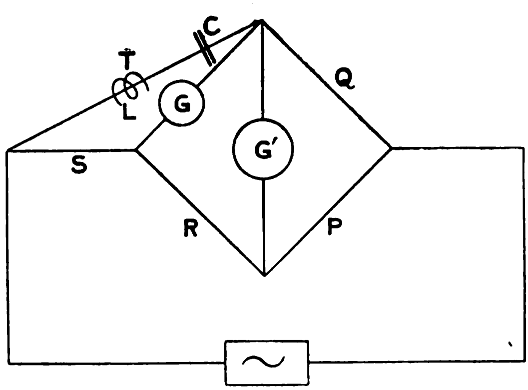
3. *Determination of the Vibration Constants.*

In order to obtain approximate values of L_v and C_v all that is necessary is to measure the apparent inductance or capacity of the galvanometer at two known source frequencies. Then, by means of equations (7), (6B), (6C), the values of L_v, C_v and p_0 can be found. The quantities thus measured depend on the

frequency, so that the source should be free from harmonics and the fluctuations of frequency should be small. If the source frequencies chosen are well removed from the resonating frequency, however, the effect of fluctuations may be neglected.

In order to obtain the vibration resistance, it will be necessary, however, to work with the galvanometer almost in resonance with the source. The effect of the fluctuations in frequency is now very large, so that although equation (6A) holds at resonance no balance will be possible on a direct resistance bridge unless the source is almost perfectly steady. By using the bridge below, however, a balance can be got even with a very impure form of alternating current.

In the figure G is the galvanometer under test, G′ is the detector, P, Q, R, S are non-inductive resistances, and the arm



T is made up of a non-inductive resistance, ρ, an inductive resistance (U, L), and a condenser (C) all in series.

If T, G are the vector impedances of the corresponding arms, the condition of balance is

$$T(GP-QR)=Q\{G(R+S)+RS\}. \quad . \quad . \quad . \quad (8)$$

In this equation T is given by

$$T=\rho+U+j(pL-1/pC)$$

and

$$G=r+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}.$$

Now, let the two following balances be made with *direct* currents :—

(*a*) With the arm T open, adjust P or Q for balance. Then, since $G=r$,

$$rP=RQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

(*b*) With the condenser (C) short-circuited, $\rho=0$, and the arm G open, adjust S for balance. Then

$$UP=(R+S)Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When these conditions are satisfied equation (8) reduces to

$$(T-U)(G-r)=r(U+S), \quad . \quad . \quad . \quad . \quad (11)$$

so that, putting in the values of T and G, the further conditions of balance *for all frequencies* are

$$\frac{\rho}{S_v}=\frac{L}{C_v}=\frac{L_v}{C}=r(U+S). \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In attaining these conditions in practice the values of L_v and C_v are already known approximately, so that the values of L and C are thus roughly fixed. Also L_v depends on γ (*see* equation 5), which in turn is altered by the tuning of the galvanometer. Hence, by using a variable self-induction for L, all the adjustments may be made without disturbing the direct-current balances (9) and (10).

Hence, complete balance for alternating currents can be got by successive adjustments of L_v (by tuning), of L, and of ρ.

4. *Limitations of the Method.*

The most important limitation of the method lies in the fact that the time constant (L/U) of the inductive coil in the arm T cannot readily be made greater than about 0·01. Hence, by (12), putting S=0, the maximum vibration capacity measurable is given by $0{\cdot}01/r$. An examination of the table of vibration constants for Campbell's galvanometers shows that, with the exception of galvanometers H, J_1, J_2, all the vibration capacities are greater than this, so that before the method will apply to these galvanometers the vibration capacities must be reduced.

Since the vibration capacities are inversely proportional to A^2 (*see* equation 5), and A is proportional to the flux density (B) of the air-gap of the galvanometer magnet, the required reduction may be brought about by increasing this flux density. The flux density necessary is given in the last column of the table. It will be seen that these values may be readily obtained by an electromagnet.

5. *Experimental Illustration of Method.*

As a suitable vibration galvanometer was not available, the following arrangement was employed in order to test the method :—

A loop of No. 32 phosphor bronze wire (diameter=0·27 mm.) was stretched between two ivory stops at a distance of 10 cm. apart. Its tension could be adjusted by means of a spring attached to an ivory pulley over which the wire passed. The arrangement was placed between the poles of an electromagnet, the width of the air-gap being about 3 mm.

As source, a small alternator, whose normal frequency was 100 ∿ per second, was employed. An oscillogram of the wave-form of this machine showed that it possessed pronounced third and fifth harmonics.

The detector was a Duddell vibration galvanometer. This instrument could not be brought to resonance with the fundamental frequency of the machine, so that in all measurements the galvanometer was tuned to the third harmonic.

The frequency of the fundamental of the source was measured by a reed frequency meter.

Tests were carried out as follows :—

(*a*) With the fundamental frequency of the machine at 100 ∿ per second, a current of about $\frac{1}{3}$ ampere was passed along the wire, and its tension adjusted until it was in resonance with the third harmonic. This adjustment could be made with some precision, as the wire emitted a distinct note at resonance.

(*b*) The source frequency was lowered first to 90 ∿ per second and then to 80 ∿ per second, and the apparent inductance of the wire was measured in each case by Anderson's method.

In this test it was found necessary to bring the third harmonic to resonance and to eliminate the fifth harmonic by means of a Campbell wave-sifter.*

The results obtained were :—

At a frequency of 270 ∿ per second, inductance=0·260 mh.
,, ,, 240 ∿ ,, ,, =0·140 mh.

Hence, by (6c) and (7),

Resonating frequency$\doteqdot$300, L_v=0·05 mh., C_v=6 millifarads.

* A. Campbell, "Proc." Phys. Soc., Vol. 24, p. 107, 1912.

(c) Using the approximate values of L_v and C_v as a guide, the bridge of section 3 was built up as follows :—

The arm T consisted of an Ayrton and Perry variable standard of self-induction of range 4 to 33 millihenries, in series with a condenser of capacity 12·6 mfd. and an adjustable resistance ranging from 0·1 to 10 ohms.

The resistances P and R were fixed at 2·00 and 0·100 ohms respectively. The values of Q and S were obtained by the direct-current adjustments of equations (9) and (10), the fractions of an ohm being obtained by using parallel combinations of resistance boxes. These gave Q=8·75 ohms, S=2·083 ohms. Hence, from (9) and (10),

$$r=0{\cdot}438 \text{ ohm}, \quad U=9{\cdot}55 \text{ ohms}.$$

The alternating-current balance was obtained by successive adjustments of L, the tension of the wire, and ρ; these gave

$$L=30{\cdot}05 \text{ millihenries}, \quad \rho=0{\cdot}90 \text{ ohm}.$$

Hence, from equation (12),

$$r_v=5{\cdot}7 \text{ ohms}, \quad L_v=0{\cdot}0641 \text{ mh.}, \quad C_v=5{\cdot}90 \text{ millifarads}.$$

The resonating frequency of the wire follows from equation (7) as 259 ∿ per second. In order to test whether fluctuations of frequency disturbed the balance, the frequency of the source was varied from 70 ∿ per second to 90 ∿ per second. It was found that, unless the frequency was varied abruptly (thus causing unsteady motion of the wire), the balance remained perfect.

Further, it was found to be quite unnecessary to eliminate either the fundamental or the fifth harmonic of the source.

6. *Values of the Vibration Constants.*

In order to show the magnitude of the vibration constants for actual galvanometers the following table has been prepared from Campbell's data. The resonating frequency is taken in every case as 100 ∿ per second. For any other frequency only the vibration inductance will be affected. Its value at any frequency (f) may readily be obtained from the relation $L_v f^2$=constant. Also a variation of the flux density (B) of the air-gap will cause a variation of all the vibration constants, the variation being such that r_v/B^2, L_v/B^2 and $C_v B^2$ are constant.

Galvanometer A of Campbell's table is omitted. The

vibration resistance of this galvanometer is less than its ohmic resistance, and this condition serves no useful purpose, since, for the most sensitive conditions of the working of a vibration galvanometer, the vibration resistance of the instrument must be at least equal to the ohmic resistance of the working circuit.*

Table of Vibration Constants.

Galvanometer.	r ohms.	r_c ohms.	C_r millifarads.	L_r millihenries.	B	B′.
B_1	6·0	14·1	21·2	0·118	2,500	8,900
B_2	4·6	11·5	16·9	0·148	2,500	7,100
C	10·5	37·5	5·02	0·500	2,500	5,800
D_1	5·7	8·4	43·0	0·058	1,650	8,300
D_2	5·7	25·3	15·6	0·160	2,500	7,500
E_1	6·0	68	3·75	0·667	2,500	3,800
E_2	6·0	94	2·70	0·926	2,500	3,200
F	7·0	168	2·08	1·20	2,500	3,100
G	8·8	342	1·21	2·07	2,500	2,600
H	14·2	2,290	0·251	10·0	2,500	1,500
J_1	14·5	765	0·921	2·72	(1,650)	2,000
J_2	14·5	1,525	0·349	7·16	(2,700)	2,000

r = ohmic resistance of galvanometer.
r_c = vibration resistance.
C_r = vibration capacity.
L_r = vibration inductance.
B = actual flux density.
B′ = flux density required for measurement.

7. *Determination of the Intrinsic Constants.*

By equation (5) a knowledge of r_v, L_v, C_v enables us to determine A^2/β, A^2/γ and α/A^2. In order to obtain α, β, γ, A separately we require another relation. This is supplied by the steady current constant k (defined as the current to produce unit deflection of the coil), which is equivalent to γ/A.

We then have

$$A=L_t k,\quad \alpha=C_v A^2,\quad \beta=S_r A^2,\quad \gamma=kA.$$

Further, the steady current constant will generally give the most rapid method of comparing the flux densities in different air-gaps by means of the relation kB=constant, so that if it is necessary to test a vibration galvanometer suspension in an air-gap other than the one in which it is to be used, this relation, together with the relations in section 6, will be sufficient to determine the vibration constants for any air-gap.

* Butterworth, *loc. cit.*

8. *Conclusion.*

The method given in section 3 has a number of possible uses. The theory, with slight modifications, applies to the motion of any electrically-maintained vibrations—*e.g.*, the motion of the diaphragm of a telephone receiver. Hence the method may be employed to determine the energy losses in such systems. Again, if the galvanometer under test is replaced by a leaky condenser with a series resistance and the condenser in the arm T is removed, the balance obtained in this case will give the values of the leakage, capacity and series resistance.

Finally, the fact that the vibration constants depend on the square of the flux density suggests a method of comparing intense magnetic fields, such as exist in the air-gaps of dynamos, by a null method.

I wish to thank Prof. Gee, Mr. J. Hollingworth, and Mr. A. Campbell for valuable criticism and advice, and Mr. White for the construction of the vibrating system.

ABSTRACT.

The methods usually employed in the determination of the constants of a vibration galvanometer involve the measurement of a deflection under three different conditions. Two of these deflections can only be obtained very approximately.

By extending the theory of the vibration galvanometer it is shown how the constants may be determined by methods which involve only the measurement of one deflection. The remaining measurements are carried out on an alternating-current bridge, and the results obtained are practically independent of the wave-form of the source.

The principle of the method depends on the fact that a vibration galvanometer behaves as a parallel combination of a conductance, a capacity and an inductance, in series with a resistance. It is shown how to balance such a combination, and the method is illustrated experimentally. The constants of various galvanometers are quoted in order to show the applicability of the method. Other uses of the bridge are suggested.

DISCUSSION.

Mr. A. Campbell remarked that it was most interesting to find that the electrical behaviour of a circuit capable of dynamical resonance could be imitated exactly by putting in parallel a resistance, a condenser and an inductance. It was a pity that this combination could not be realised in practice since the inductance must have zero resistance. However, Mr. Butterworth got over the difficulty by his special form of bridge. The limitations of this bridge somewhat lessened the range of application to practical cases, and he hoped that the author would be able to modify the bridge so as to remove these limitations.

Mr. D Owen stated that he had found no difficulty in maintaining the frequency of the source sufficiently steady to maintain the voltage sensitivity of a vibration galvanometer constant within one or two per

cent. for a considerable time. The author's analysis of the vibrating coil was very ingenious, but one would like to know whether sensitivities calculated by his rather complex bridge agreed with those obtained by the usual direct method.

Dr. R. S. WILLOWS asked whether the method could be used to find the dielectric constant of a slightly conducting liquid; if so, was it sensitive enough to be practically useful and did the largeness of the required inductance again limit its applicability.

Mr. BUTTERWORTH, in reply, stated that the method would apply to any vibrating system provided that the conditions mentioned in the Paper could be satisfied. It was true, as Mr. Owen had pointed out, that the *voltage* sensitivity of certain galvanometers could be determined very precisely. This held in the case of instruments capable of developing a high back E.M.F. The reduction of current at resonance would then prevent any considerable rise in the vibration in spite of the large increase in *current* sensitivity. The application of the method to the measurement of small capacities required investigation.

XXIX. *Note on the Connexion between the Method of Least Squares and the Fourier Method of Calculating the Coefficients of a Trigonometrical Series to represent a given Series of Observations of a Periodic Quantity.* *By* CHARLES H. LEES, *D.Sc., F.R.S.*

RECEIVED MAY 6, 1914.

1. In view of the fact that several methods of representing a number of observations of a variable quantity by means of a trigonometrical series of the Fourier type have been proposed which differ from that adopted by Fourier himself, it seems advisable to call attention to a characteristic of the Fourier method of determining the coefficients of the terms of such a series, which gives them a greater degree of reliability than the coefficients determined by any other method. This property they owe to the close connexion between the method of their calculation and the method of least squares as used in the solution of a number of simultaneous equations which exceeds the number of independent variables to be determined.

Although this connexion is referred to by Kelvin and Tait * it is seldom mentioned in recent books† on the subject of Fourier series, which concentrate attention mainly on the validity of the expansion when the number of terms is infinite and there are no errors of observation.

The connexion ought not to be lost sight of, as it serves not only as a good introduction to the Fourier method of expansion, but also exhibits very clearly how the expressions used in calculating the coefficients of the terms of the series arise.

2. Let it be required to determine the coefficients in the expansion of a periodic function $f(x)$ of x of period l in a finite series of the form

$$\begin{aligned} &A_1 \sin 2\pi x/l + \ldots + A_g \sin 2g\pi x/l + \ldots + A_n \sin 2n\pi x/l \\ &+ B_0/2 + B_1 \cos 2\pi x/l + \ldots + B_g \cos 2g\pi x/l + \ldots + B_n \cos 2n\pi x/l \end{aligned} \quad (1)$$

and let the values of $f(x)$ be known for q values of x—*i.e.*, x_1, $x_2 \ldots x_p \ldots x_q$—in the interval $0<x<l$, where $p<q$ and q is considerably greater than $2n+1$.

* "Treatise on Natural Philosophy," Vol. 1, art. 398, p. 456.

† *See* Hobson, "Theory of Functions"; Bromwich, "Theory of Infinite Series"; Thomae, "Bestimmte Integrale und Fouriersche Reihen"; Carslaw, "Fourier Series and Integrals"; Byerly, "Fourier Series."

Then, if the errors in the values of $f(x)$ follow the normal law of errors, we get the most probable values of the coefficients by the method of least squares*—that is, we take those values of the coefficients which make the sum of the squares of the q differences between $f(x)$ and the series at the points $x_1, x_2 \ldots x_p \ldots x_q$ a minimum.

In symbols,

$$\sum_{p=1}^{q} \{f(x_p) - \sum_{g=0}^{n}(A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\}^2$$

is to be made a minimum by a proper choice of the coefficients A and B.

Differentiating with respect to each of the coefficients A_k, B_k, where $k \ngtr n$, we have for a minimum or a maximum

$$\sum_{p=1}^{q} \{f(x_p) - \sum_{g=0}^{n}(A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\} \sin 2k\pi x_p/l = 0$$

and

$$\sum_{p=1}^{q} \{f(x_p) - \sum_{g=0}^{n}(A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\} \cos 2k\pi x_p/l = 0. \quad (2)$$

Differentiating again we see that the preceding equations give the conditions to be fulfilled for a minimum, and the $2n+1$ equations of type (2) determine the $2n+1$ coefficients required.

3. If the q points $x_1, x_2 \ldots x_p \ldots x_q$ are uniformly distributed over the range 0 to l, we may write $x_p = lp/q = hp$, say, where $l/q = h$, and the equations determining the coefficients become

$$\sum_{p=1}^{q} \{f(ph) - \sum_{g=0}^{n}(A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)\} \sin 2k\pi ph/l = 0$$

and

$$\sum_{p=1}^{q} \{f(ph) - \sum_{g=0}^{n}(B_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)\} \cos 2k\pi ph/l = 0, \quad (3)$$

or

$$\sum_{p=1}^{q} f(ph) \sin 2k\pi ph/l = \sum_{p=1}^{q} \sin 2k\pi ph/l \sum_{g=0}^{n}(A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)$$

and

$$\sum_{p=1}^{q} f(ph) \cos 2k\pi ph/l = \sum_{p=1}^{q} \cos 2k\pi ph/l \sum_{g=0}(A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l). \quad (4)$$

* For laws of error for which this would not be the case *see* Edgeworth's article on "Probability" in the "Encyclopædia Britannica," Vol. 22, p. 397.

On interchanging the order of summation the two right members become

$$\sum_{g=0}^{n}\{A_g\sum_{p=1}^{q}\sin 2k\pi ph/l \,.\, \sin 2g\pi ph/l+B_g\sum_{p=1}^{q}\sin 2k\pi ph/l \,.\, \cos 2g\pi ph/l\}.$$

and

$$\sum_{g=0}^{n}\{A_g\sum_{p=1}^{q}\cos 2k\pi ph/l \,.\, \sin 2g\pi ph/l+B_g\sum_{p=1}^{q}\cos 2k\pi ph/l \,.\, \cos 2g\pi ph/l\}. \quad (5)$$

On expressing each product as a sum or difference of the sines or cosines of the sum and difference of the angles, it is seen that the sums with regard to p are zero except in the case $g=k$, when terms $\frac{1}{2}\cos 0°$ lead to the sum $q/2$.* Hence we have

$$\sum_{p=1}^{q} f(ph)\,.\,\sin 2k\pi ph/l=A_k q/2$$

and

$$\sum_{p=1}^{q} f(ph)\,.\,\cos 2k\pi ph/l=B_k q/2.$$

That is,

$$A_k=(2/q)\sum_{p=1}^{q} f(ph)\,.\,\sin 2k\pi ph/l$$

and

$$B_k=(2/q)\sum_{p=1}^{q} f(ph)\,.\,\cos 2k\pi ph/l\,. \quad . \quad . \quad . \quad (6)$$

4. If the number q of observations is very large h becomes very small, and on writing x for ph and dx for h the equations become

$$A_k=(2dx/l)\sum_{p=1}^{q} f(x)\,.\,\sin 2k\pi x/l,$$

$$\text{and } B_k=(2dx/l)\sum_{p=1}^{q} f(x)\,.\,\cos 2k\pi x/l$$

or, in the notation of the calculus,

$$A_k=(2/1)\int_0^l f(x)\,.\,\sin 2k\pi x/l\,.\,dx,$$

$$\text{and } B_k=(2/l)\int_0^l f(x)\,.\,\cos 2k\pi x/l\,.\,dx. \quad (7)$$

It is thus seen that the method of least squares leads to the usual Fourier expressions (6) and (7) for the coefficients of the series (1), and shows very clearly how the sine and cosine terms come into them. By the theory of probability if the errors of observation follow the normal law the coefficients determined by (6) or (7) are the most probable.

* *See* Fourier, "Théorie analytique de la Chaleur," Art. 269.

ABSTRACT.

In view of the number of alternative methods which have been suggested for calculating the coefficients of the terms of a trigonometrical series to represent a number of observations of a periodic quantity, the author points out that the Fourier method gives the most probable values of the coefficients, since it makes the sum of the squares of the errors at the points of observation a minimum.

DISCUSSION.

Dr. C. CHREE mentioned that the method was dealt with in Tait's "Natural Philosophy," but no proof was given. Prof. Lees had supplied the proof, and the Paper was of interest on that account.

Dr. W. WILSON (communicated remarks) said: Prof. Lees deals with a function $f(x)$ of period l, whose values are given in the whole interval from 0 to l. In this case the validity and *uniqueness* of the Fourier expansion (provided $f(x)$ is subject to certain restrictions) have been demonstrated with complete rigour (Dirichlet, "Collected Works," Vol. I., pp. 133–160; and G. Cantor, "Journal für Mathematik," LXXII.). It seems to me, therefore, that there is no question of the reliability of the Fourier coefficients, and the fact that the method of least squares leads to the usual Fourier expressions for the coefficients confirms the reliability of this method rather than that of the Fourier coefficients themselves.

Prof. LEES (communicated): I think the reference mentioned by Dr. Chree is the one cited in my Paper, but since the Paper was read Sir Joseph Larmor has told me he believes Sir John Herschel dealt with the subject in one of his Papers, which I have not yet succeeded in finding. I am sorry anything in my Paper should have led Dr. Wilson to think I was dealing with the infinite series of Dirichlet and others whose work is embodied in the text-books cited. n is restricted to finite values, and in nearly all actual cases is a very small integer. Thus the independent simultaneous equations whose number q exceeds that of the $2n+1$ variables do not admit of a *unique*, but only of a *most probable*, solution. Hence the alternatives to the Fourier solution suggested at various times by *e.g.*, Wedmore, Lyle, Fischer-Hinnen and Thompson.

XXX. *A Magnetograph for Measuring Variations in the Horizontal Intensity of the Earth's Magnetic Field.* By F. E. SMITH, *A.R.C.Sc.* (*From the National Physical Laboratory.*)

IN March of this year Dr. C. Chree read a Paper before this Society on "Time Measurements of Magnetic Disturbances"* in which he discussed the question of the simultaneity of occurrence of magnetic disturbances over the whole earth. The records examined by Dr. Chree resulted from the use of at least five different patterns of magnetographs: the Mascart and Eschenhagen, with very small magnets; the Watson, with composite systems formed of a number of parallel magnets; the Kew, with magnets weighing about 87 grammes; and the Greenwich, which stand to the Kew somewhat as these stand to the Eschenhagen. If one wished for comparative times on quick-run curves correct to within a few seconds, Dr. Chree said it would be advisable, if not absolutely necessary, to employ instruments of the same pattern, similarly sensitive. This highly desirable condition of affairs may not result in the immediate future, but if the movement is helped forward by this description of the magnetograph in use at the National Physical Laboratory our main object will be attained.

In all instruments known to the author for recording the variations in the horizontal intensity of the earth's magnetic field, a magnet, or system of magnets, is suspended by a fibre, or fibres, the torsion on which is sufficient to deflect the magnetic system to a position nearly at right angles to the meridian.

In the case of unifilar instruments, if θ is the angle which the magnetic system makes with the meridian, M the moment of the magnet, and H the horizontal intensity of the earth's field, the couple MH sin θ is balanced by the torsional couple of the fibre. If T is the torsional couple for 1° of twist and φ is the angle through which the upper end of the fibre is turned with reference to the lower end, we have

$$MH \sin \theta = T\varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Normally, declination and horizontal intensity are changing simultaneously so that θ changes wholly irrespective of

* "Proc." Phys. Soc., Vol. XXVI., p. 137, 1914.

changes in H or φ. For the purpose of comparing sensitivities of instruments with various values of θ and φ we may, however, suppose the declination to remain constant. The manner of calculating the changes in H when the declination varies is considered later.

When the declination is constant and H changes to $H-\partial H$ with a corresponding deflection $\partial\varphi$ of the magnetic system, we have

$$M(H-\partial H)\sin(\theta+\partial\varphi)=T(\varphi-\partial\varphi) \quad . \quad . \quad . \quad (2)$$

and

$$\frac{\partial H}{H}=\cot\theta d\varphi+\frac{\partial\varphi}{\varphi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For purposes of calculation it is convenient to regard the angles as measured in degrees. We then have

$$\frac{\partial H}{H}=0{\cdot}0175\,\partial\varphi\cot\theta+\frac{\partial\varphi}{\varphi} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\frac{\partial\varphi}{\partial H}=\frac{\varphi}{H[1+0{\cdot}0175\,\varphi\cot\theta]}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$\cot\theta$ is necessarily small, and the term including it is not of great importance unless φ is very great. In most instruments used in observatories, φ is about 60°, and in such cases sufficient precision is often obtained by neglecting the $\cot\theta$ term and writing (5) in the form

$$\partial\varphi=\varphi\partial H/H. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The same relation is obtained by letting $\theta=90°$ and differentiating equation (1). The sensitiveness of such instruments is directly proportional to the torsion on the fibre.

In the instruments we have experimented with φ has been made very great. In the general case it was of the order of 5,000°, but in one case it exceeded 100,000°. Since in practice θ is never 90° (except by accident, for the system is unstable when θ has this value) the term involving $\cot\theta$ becomes of considerable importance.

The following table shows the variation of the sensitiveness with variation of φ and θ. The angle of deflection $\partial\varphi$ of the magnetic system is given when $\partial H/H=1/18{,}000$, *i.e.*, for a change in the horizontal intensity from $H-0{\cdot}5\gamma$ to $H+0{\cdot}5\gamma$, the total change in H being 1γ.* For convenience we give also

* $1\gamma=0{\cdot}00001$ of the C.G.S. unit of intensity of magnetic field.

the displacement d in millimetres at a distance of 2 metres of a reflected ray of light.

θ.	$\phi=60°$.		$\phi=100°$.		$\phi=1{,}000°$.		$\phi=5{,}000°$.		$\phi=10{,}000°$.	
	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.
89°	0·20′	$0{\cdot}2_3$	0·32′	$0{\cdot}3_8$	2·6′	3·0	6·6′	7·7	8·2′	9·6
88°	0·19′	$0{\cdot}2_3$	0·31′	$0{\cdot}3_7$	2·1′	2·4	4·1′	4·8	4·7′	5·5
87°	0·19′	$0{\cdot}2_2$	0·31′	$0{\cdot}3_6$	1·7′	2·0	3·0′	3·5	3·3′	3·8
86°	0·19′	$0{\cdot}2_2$	0·30′	$0{\cdot}3_5$	1·5′	1·7	2·4′	2·7	2·5′	2·9
85°	0·18′	$0{\cdot}2_1$	0·29′	$0{\cdot}3_4$	1·3′	1·5	1·9′	2·3	2·1′	2·4
84°	0·18′	$0{\cdot}2_1$	0·28′	$0{\cdot}3_3$	1·2	1·4	1·7′	1·9	1·7′	2·0
83°	0·18′	$0{\cdot}2_1$	0·28′	$0{\cdot}3_2$	1·1′	1·2	1·4′	1·7	1·5′	1·8
82°	0·17′	$0{\cdot}2_0$	0·27′	$0{\cdot}3_1$	1·0′	1·1	1·3′	1·5	1·3′	1·5

The particular object of the table is to show how the sensitiveness varies with change of θ and φ. *The values tabulated must not be used to convert measured ordinates into changes of force.* For such conversion the ordinates are measured from a fixed base line representing a known angular position from the magnetic meridian, and the corresponding changes in H are deduced from Fig. 1. This method will now be considered (1) when H varies but the declination is constant, and (2) when H and D vary simultaneously.

Subject to the condition that the declination is constant, the variation of θ and of φ with change in H is shown graphically in Fig. 1. If we suppose that initially θ is exactly 90°, we have

$$\mathrm{MH}=\mathrm{T}\varphi.$$

If H changes to $\mathrm{H}+h$ and a is the corresponding deflection, we have as an exact relation

$$\frac{h}{\mathrm{H}}=\left(\frac{1}{\cos a}-1\right)+\frac{a}{\varphi\cos a}.$$

As ordinates we have plotted $\left(\dfrac{1}{\cos a}-1\right)$ and $a/\varphi\cos a$ for values of a between 0° and 8°, corresponding to a change in θ from 90° to 82°. The ordinates to the curve OA from the base OP represent the values of $\left(\dfrac{1}{\cos a}-1\right)$; they are to be considered as positive. The ordinates to the curves OB, OC, OD, OE and OF from the same base OP represent the corresponding values of $a/\varphi\cos a$ for initial values of the torsion of 60°, 100°, 1,000°, 5,000° and 10,000° respectively. The chart enables one to find the change in H necessary to deflect the magnetic

needle from any one position to any second position conditionally that both positions are between 82° and 90° from the magnetic meridian. Thus for an instrument with 1,000° of torsion on the fibre an increase in H of 20 parts in 10,000, *i.e.*, 36γ, is required to deflect the magnet from a position 87° from the meridian to a second position 86° from the meridian. The increase in H is given by the difference between the ordinates KL and GH. The figure shows that even for

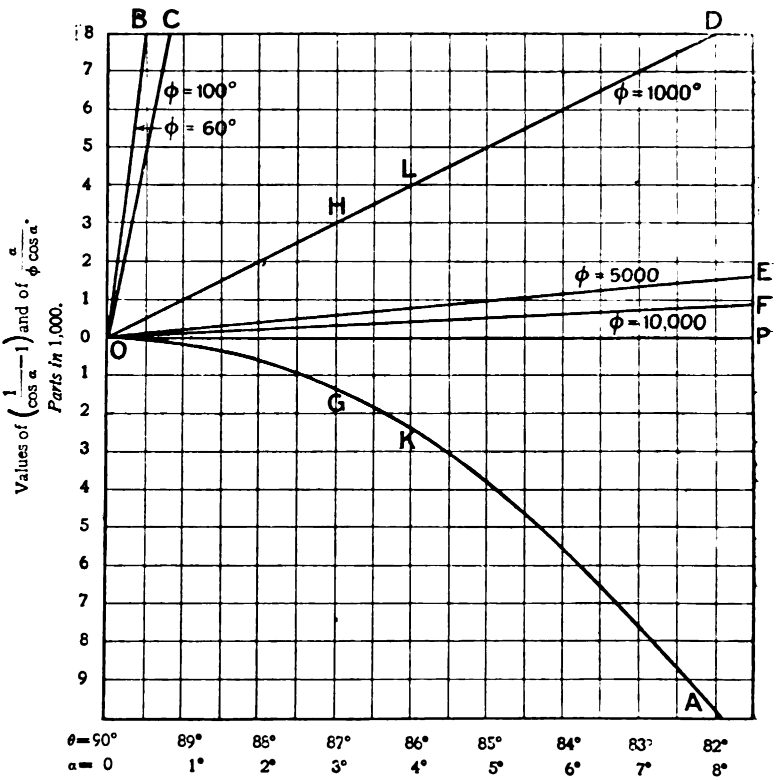


Fig. 1.

measurements of very small variations no great advantage is gained by increasing the torsion beyond 5,000°. With the latter value for φ a decrease in H of four parts in 10,000 (*i.e.*, about 7γ) suffices to deflect the magnet from 89° to 90°, and so disarrange the system. While, therefore, such an instrument is admirable for measuring very small and sudden variations

in the horizontal intensity, it cannot be used for measuring very large fluctuations unless the sensitiveness is reduced by reducing θ.

As previously stated, declination (D) and horizontal intensity (H) usually change simultaneously, so that θ changes independently of variations in H or φ. To determine the real changes in H, it is necessary to measure the changes in D, and it is desirable to have the D and H variations recorded on one sheet, the scale values (in degrees) of the D and H records being identical or nearly so (*i.e.*, the D and H instruments should be equally distant from the recording drum).

Initially we may suppose the base line record of the H instrument to coincide with the H record when θ is 90°. In practice there are several ways of doing this. One of the simplest is to, at first, convert the H instrument into a declination instrument by freeing the fibre from torsion. The magnetic system is then deflected through an angle θ (say 89°) by producing torsion on the fibre equal to $\varphi-\beta$ where β is the extra torsion required to make $\theta=90°$ and $\varphi=(\varphi-\beta)/\sin\theta$. The mirror used for the base line record is now adjusted to produce a record in the position corresponding to $\theta=90°$, and the position of the record of the second instrument (the declination instrument) is noted at the same time. When thus set up, the records obtained enable one to read off d (the change in declination from the initial declination and which may be plus or minus), and a the angular displacement of the magnetic system from $\theta=90°$.

Let θ and D be regarded as measured from geographical north. Then, when the magnetic system is at right angles to the magnetic meridian $(\theta-D)=90°$ and

$$MH\sin(\theta-D)=MH=T\varphi.$$

When H [the value of the horizontal intensity when $(\theta-D)=90°$] changes to $H+h$ and the declination increases by an amount d, we have

$$M(H+h)\sin[(\theta-D)-(a+d)]=T(\varphi+a)$$

where a is the angular displacement of the magnetic system and is given by the distance of the H record from the base line.

The above equation reduces to

$$\frac{h}{H}=\left[\frac{1}{\cos(a+d)}-1\right]+\frac{a}{a+d}\cdot\frac{a+d}{\varphi\cos(a+d)},$$

and we are enabled readily to determine the value of $\frac{h}{\mathrm{H}}$ from Fig. 1. Thus suppose $a=3°$, $d=+1°$, and $\varphi=5{,}000°$.

Then

$$\frac{h}{\mathrm{H}}=\left[\frac{1}{\cos 4°}-1\right]+\frac{3}{4}\left[\frac{4°}{5{,}000° \cos 4°}\right].$$

The values of the quantities in brackets are given in Fig. 1. We have

$$\frac{h}{\mathrm{H}}=[0{\cdot}00244]+\frac{3}{4}[0{\cdot}00080],$$

i.e., $\frac{h}{\mathrm{H}}=0{\cdot}00304$, or h=about 55γ if $\mathrm{H}=0{\cdot}18$.

As a summary, we may say "Whatever may be the angular deflection (a) of the magnetic system from the initial position corresponding to $\theta-\mathrm{D}=90°$, we add (or subtract) the change in declination, d, from it. To the ordinate of the OA curve (Fig. 1) corresponding to $a\mp d$ we add $\frac{d}{a\mp d}$ times the ordinate of the φ curve. This gives the change in the horizontal intensity.

I believe that most recording instruments in constant use have a sensitiveness such that a change of 5γ in H produces a deflection of the magnetic system of about 1 minute of arc. The corresponding displacement of a spot of light at a distance of 2 metres is about 1 millimetre. It is therefore difficult, and in some cases impossible, to determine even from high-speed records, the magnitude of very small and sudden changes.

The design of a magnetograph for detecting such changes engaged my attention some time ago. My main object was to detect with certainty sudden changes in the horizontal intensity and to measure these changes within $0{\cdot}5\gamma$.

The form of instrument which I have used is shown in Fig. 2. It differs from other instruments in general use inasmuch as the swinging system is critically aperiodic and the damping is effected by air. It may be used for detecting changes as small as $0{\cdot}1\gamma$, or it may be employed as a less sensitive instrument for producing records of changes extending over many years. The weight of the suspended system is usually less than 1 gram. The instrument is cheap to construct, is portable, and is easy to set up.

The details of construction are as follows. A fine quartz

fibre F is suspended from a torsion head H, and supports two aluminium vanes v and V, a concave mirror m and a magnet M. The magnetic system is attached to the back of the mirror and may consist of one or several small magnets. The lightest magnetic system I have used consisted of one magnet 2 mm. long and about 0·5 mm. in diameter. The weight of the magnet was a little less than 0·01 gram; together with the mirror and aluminium vane the weight was 0·1 gram. The heaviest system consisted of 6 magnets, each of which was 2·5 cm.

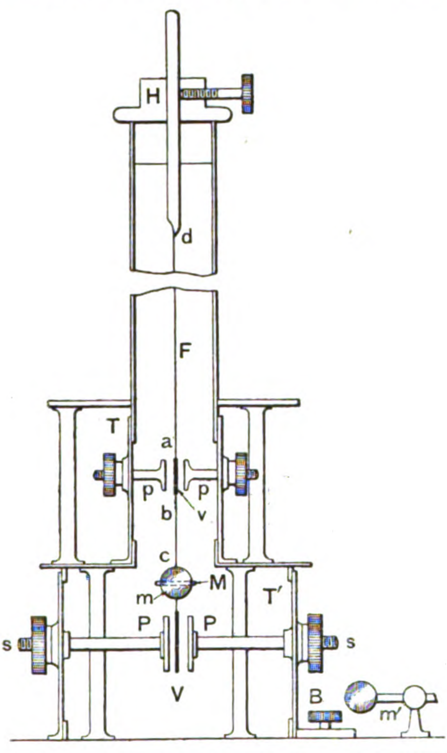


FIG. 2

long and about half a millimetre in diameter. This system was at times loaded with lead to increase the moment of inertia, and the total weight was then about 50 grams. In general the magnetic system was chosen to make the value of φ between 3,000° and 5,000°.

PP are damping plates of aluminium, their distance apart

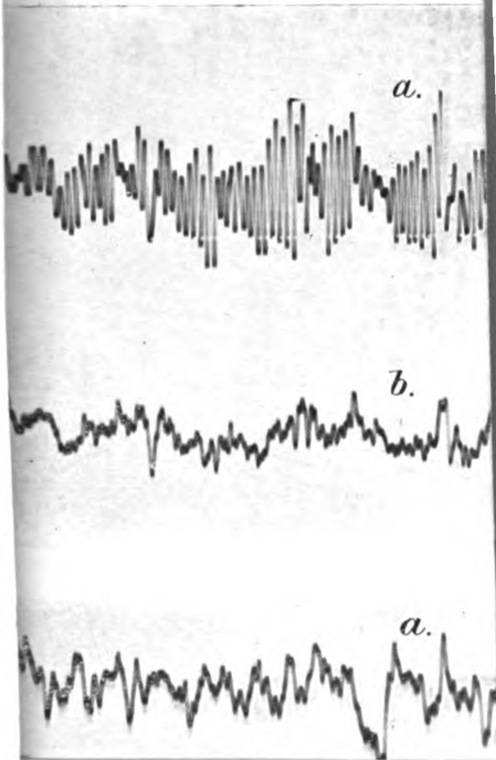
being controlled by the screws *ss*. *pp* are a similar but smaller pair of plates.

The damping vanes *v* and V are of thin aluminium foil, *v* being of circular shape with two projecting ears, thus 

; the diameter of the vane is greater than the diameter of one of the plates *pp*. The quartz fibre is laid across the vane in the direction *ab*, and secured by shellac to the projecting ears. Afterwards the short length of fibre between *a* and *b* is removed.

The tube T supporting the plates *pp* is free to rotate and is provided with two small circular windows to enable the vane and plates to be viewed. When the vane is clamped by screwing the plates into firm contact with it the length of fibre controlling the magnet is that between the points *b* and *c*. When the vane is free to move the effective length of the fibre is increased to *bc* plus *ad*. The relative sensitivities of the two arrangements depends on the ratio of *bc* to $bc+ad$. In practice, suppose the more sensitive arrangement to be in use and that it is desired to make the instrument less sensitive. The position of the spot on the recording drum having been observed, the tube T is rotated until the plates *pp* are approximately parallel to *v*; *v* is then clamped. Should the position of the spot have been disturbed the tube T may be rotated until the position is the same as before. φ has now been reduced to $\varphi bc/(bc+ad)$ (this supposes the fibre to be uniform in thickness) and θ remains constant. The change occupies but one or two minutes. Of course if an instrument of high sensitiveness only is desired the plates *pp* and the vane *v* are omitted.

In our instruments the vane V was in general about 1 cm. long and 1 cm. broad, but in special experiments the length of the vane was increased to 5 cm. When the axis of the magnetic system is displaced by torsion so as to lie in a plane nearly 90° from the meridian, the vane V swings approximately in the meridian. The tube T′ which supports the damping plates PP may be rotated through about 20° in order to ensure that the surfaces of the plates are approximately parallel to the vane. The tube is then clamped in position by the screw B. There are two circular windows which enable the vane and plates to be viewed, and through one of these the light is transmitted to the mirror *m*. A fixed mirror, *m′*, is supported by a stiff ball and socket joint and reflects the light for a base line record.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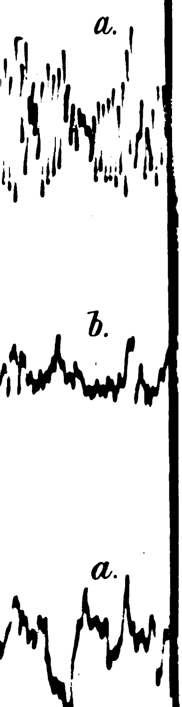
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To determine the sensitiveness of the instrument* a current of 0·002 ampere is passed through two coaxial circles of wire (not shown in the figure), the circles being 35 cm. in diameter and 3 cm. apart. The planes of the circles are at right angles to the meridian, and the centre of the magnet M is on the line joining the centres of the circles, or nearly so. The two circles of wire are supported by the base of the instrument. When making a determination of the sensitiveness the current is passed in the same direction through the two circles of wire and the deflection of the spot is recorded. The current is then reversed for a record of the deflection in the opposite direction. The sensitiveness is thus determined with ease and precision. It must, however, be remembered that the scale value varies across the recording sheet.

In our experiments two horizontal intensity instruments have been used. One has been erected in a permanent position and has been but little interfered with; the other is a rough model built for experimental work. Both instruments were constructed in the laboratory workshops.

The general usefulness of the instruments is best illustrated by some of the records which have been obtained.

Record 1.—This record shows the normal behaviour of the permanent instrument when the damping is critically aperiodic. The value of ϕ is nearly 5,000° and the value of θ is about 86°. The large displacements are the result of passing and reversing a current of 0·002 ampere through the two circles of wire. The mean sensitiveness across the portion of the sheet traversed by the record is 2·3 mm. for a change in H of 1γ. The time scale (abscissa) is such that 430 mm. represent one hour (7 mm. represent 1 minute). The sudden changes of the order of 4γ, which are depicted, are largely due to the earth currents produced by the London United Electric Tramway system. These changes very rarely exceed 5γ

Record 2.—This shows the usual magnitude of the disturbances when no trams are running, the sensitiveness being the same as before. The record was taken between 3 a.m. and 4 a.m.

Record 3.—It follows from the Table (p. 281) that if θ be increased to 89° a much greater sensitiveness should be obtained. The result of such a change is shown in Record 3. The mean sensitiveness across the portion of the sheet traversed

* It is not necessary to determine the sensitiveness experimentally if ϕ and θ are known.

is such that a change in H of 1γ is represented by a displacement of 8 mm. Several such records, with different values for θ, were taken to check equation (5). The sudden changes shown in Record 3 in no instance exceed 4γ.

Record 4.—This illustrates the change from a sensitiveness of 2·8 mm. displacement for 1γ to 0·31 mm. for 1γ, and *vice-versa.* The time occupied in changing from one system to the other was from 1 to $1\frac{1}{2}$ minutes.

Records 5 *to* 8.—These records are in pairs and illustrate the importance of correctly damping the system. Two instruments were used to produce a pair of records, the records being simultaneously taken and the sensitivities of the instruments being identical within 5 per cent. The *b* records resulted from an instrument the damping of which was always critically aperiodic.

Records 5*a and* 5*b.*—5*a* shows the result of under damping, the damping plates being withdrawn to their maximum extent. The mean sensitiveness of each instrument was 3·0 mm. for a change of 1γ. It will be observed the maximum displacement in 5*a* occurs when there are a series of small impulses ; these are shown in 5*b*. If we attempt to measure the magnitude of the sudden changes in the horizontal intensity from the displacements recorded in 5*a* errors of several hundreds per cent. occur.

Records 6*a and* 6*b.*—The two instruments were made equally sensitive and both were critically damped. The system for 6*a* consisted of six small magnets, a comparatively coarse fibre, and a damping vane about 4 sq. cm. in area. That for 6*b* consisted of one small magnet, a fine fibre and damping vane 2 sq. cm. in area. The value of φ was about 4,000° for 6*a* and 3,500° for 6*b*, and the periods of the undamped systems were eight and seven seconds respectively. The records, together with others, were taken to detect any instrumental peculiarities and to gauge the precision of our measurements. The results are remarkably good and show the instrumental peculiarities to be practically negligible. When these results are considered in conjunction with the data establishing the relationship between the magnitude and duration of a sudden change and the deflection, we conclude that the measurement of a sudden disturbance of 5γ lasting for at least three seconds may be relied on within $0{\cdot}1\gamma$. The error involved in measuring a disturbance lasting for more than three seconds is even less.

The time occupied in making the suspended system and

setting up the second instrument (for 6*a* records) was about one hour. Clearly it is not difficult to construct instruments to give practically identical results.

Records 7a, 7b and 8a, 8b.—The two instruments were equally sensitive, but one (giving the *a* records) was over-damped. Our main object was to alter the period and damping of one system in order that it would respond to slow variations in the horizontal intensity, but not to sudden oscillatory ones such as those produced by an electric traction system. The mean value of H over a period of five minutes or longer is thus more readily deduced. Such an instrument is of considerable value when absolute measurements are being made (as with the Kew magnetometer) for the record shows the slow natural changes but masks the local disturbances. In an attempt to obtain the records 7*a* and 8*a* I at first made the moment of inertia of the suspended system very great to obtain a long time of swing, and I increased the damping by employing a large vane. However, the results were disappointing as the mass of the system was too great for the air damping to be sufficiently effective. A very light suspended system (0·1 gram) was therefore tried with a fine fibre, and a vane of 2 sq. cm. area, and with this excellent results were obtained. The record 7*a* shows very few of the rapid disturbances indicated in 7*b* notwithstanding that the sensitivities of the two instruments were identical. For the record 8*a* the damping was still further increased, and it will be seen from the smooth curve that small disturbances of short duration have practically no effect on the instrument.

For the permanent instrument (when the damping was critically aperiodic) photographs of the damping curve were taken to determine the relationship between the magnitude and duration of a sudden change, and the deflection produced. In addition, artificial disturbances of known magnitude were produced for from ½ to 20 seconds. If *d* is the deflection produced by a disturbance lasting so long that the effect of damping may be neglected, the deflection resulting when the disturbance lasts from ½ second upwards is as follows :—

0·5	second deflection	=	about	32	per cent.	of *d*.
1·0	,, ,,	=	,,	55	,,	,,
1·5	,, ,,	=	,,	72	,,	,,
2·0	,, ,,	=	,,	85	,,	,,
2·5	,, ,,	=	,,	95	,,	,,
3·0	,, ,,	=	greater than	95	,,	,,

The magnitude of a disturbance lasting for more than 3 seconds may, therefore, be measured with considerable accuracy. If necessary, the suspended system may be modified in order that a disturbance lasting for 1 second may be accurately measured.

As with other forms of magnetograph, temperature variations produce deflections of the magnetic system, and it is necessary therefore to know the temperature coefficient of the instrument or to keep the temperature constant. A considerable reduction of the temperature coefficient (possibly its elimination) would result by the action of a bimetallic strip on the torsion head. With increase of temperature, φ would be diminished, and with decrease of temperature it would be increased. In the near future we hope to experiment with such a compensating device.

ABSTRACT.

In the case of unifilar instruments for recording variations in H, if θ is the angle which the magnetic system makes with the magnetic meridian, M the moment of the magnet, and H the horizontal intensity of the earth's field, equilibrium results when $MH \sin \theta = T\phi$, where ϕ is the torsion on the fibre and T is a constant. In the instrument described ϕ may be made great or small, but high sensitiveness is secured by making ϕ great. The magnet system is supported by a quartz fibre, and critically aperiodic damping is obtained by means of an aluminium vane and two parallel damping plates. To diminish the sensitiveness the effective length of the fibre may be reduced. The general usefulness of the instrument is illustrated by photographic records, which show the instrumental peculiarities to be very small, and indicate that unless the system is aperiodic increased difficulty must result in the interpretation of the records. An over-damped system responds but slightly to rapid pulsations in H, but follows the slow changes which are common all over the world. The general sensitiveness of the records is about 3 mm. for a change in H of 0·00001 C.G.S. unit, but one record shows a displacement of 8 mm. for such a change.

DISCUSSION.

Dr. CHREE thought the instrument ingenious and likely to be very useful for the purpose for which he understood it was primarily intended, viz., the observation of the disturbances produced at the National Physical Laboratory by existing and prospective electrical tramways and railways. There were various features in the existing disturbances whose investigation seemed likely to be of interest and to the public advantage. There were also certain natural phenomena, for example, "pulsations," or small oscillations of magnetic force, for whose investigation the instrument from its great sensitiveness seemed well adapted, only for such purposes it would have to be set up at some station, such as Eskdalemuir, remote from London or any other large centre of electrical industry. There were, however, two features, viz., the somewhat rapid variation of scale value across the sheet, and the fact that the instrument

responded sensibly to changes of declination as well as horizontal force, which, he thought, stood somewhat in the way of its employment for ordinary *observatory* purposes.

Mr. R. S. WHIPPLE asked whether the author had considered the advisability of flashing a spot of light across the sheet to give the time scale, as was done at Potsdam.

Mr. C. W. S. CRAWLEY considered the tramways were a great nuisance to magnetic observers. He had gone a good deal further in sensitiveness than the author, and even when situated 16 miles from the nearest tramways he had found them very troublesome. The little kick in the middle of the disturbance which Mr. Smith thought might be an instrumental error had often been noticed by him, and was a definite phenomenon in no way due to the instrument.

Prof. T. MATHER asked if the author had tried using the finest quartz fibre which he could handle. It appeared that the instrument took two or three seconds to attain its final position. If a finer fibre were used, a lighter and quicker magnetic system could be employed.

The AUTHOR, in reply, agreed with the remarks of Dr. Chree. He had thought it would be interesting to set up an instrument to detect very small pulsations, such as might occur during solar eclipses, while neglecting the larger ordinary disturbances. He admitted that the variation of sensitiveness across the sheet would be a disadvantage to an ordinary observatory assistant, but it gave him no trouble whatever. In reply to Mr. Whipple, he had already started to employ the flash of light method for the time scale. In reply to Prof. Mather, there was no doubt that the system could be made a good deal lighter if it were desired to measure very sudden disturbances lasting only a few seconds.

XXXI. *The Atomic Weight of Copper by Electrolysis.* By

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RECEIVED FEBRUARY 27, 1914.

I. INTRODUCTION.

The following is an account of the determination of the relative weights of copper and silver deposited when the same electric current passes through copper and silver voltameters. For the successful determination of the relative weights by the electrical method several important effects have to be borne in mind :—

1. *The solution of the deposited metal* in the liquid of the cell. In a preliminary investigation it was found that if the solution

was assumed to be proportional to the area of the cathode very concordant results were obtained for the corrected values of the ratio of the deposited weights of copper to that of silver. In this way the mean error in the value of the ratio for nine early experiments was altered from 1 part in 500 to 1 part in 6,000. In order to apply the solution correction four copper cells in series, having different cathode areas, were used in the final experiments. By a graphical construction the amount which would have been weighed on a cathode, free from loss by solution, was always taken as the corrected weight of the copper deposited in the cell. In the case of silver this correction was found to be unnecessary, and therefore in the final work the silver was determined by taking the mean of the weights of the deposits of two silver cells, these cells being separated by four others in which copper was deposited.

2. The cathode current density must be kept *below a certain limiting value*, depending upon the concentration of the electrolyte, the amount of acid present and the geometry of the cell. If this value is exceeded the deposit is non-coherent, powdery in consistency and apparently amorphous. Experiment shows that such deposits do not yield concordant results on weighing. In order to determine the limiting current-density a series of experiments in which the conditions were varied systematically was carried out. The chief result obtained was that in all cases the deposition of unweighable metal was heralded by a rise in the reading of a voltmeter placed across the cell. Under the heading of "Limiting Current Density" (Section II.) the method of predetermining the safe current for any cell is explained. In the final determinations the information thus gained is utilised so as to permit the employment of widely varying current densities, all, however, well under the limiting values.

3. A deposit may, however, be unsatisfactory in other ways. Visible crystalline growths, liable to be lost in washing or weighing, may present themselves. Coarse-grained deposits of this type may also do harm by imprisoning some of the electrolyte. The allowable current density in this case depends upon the weight to be deposited—the greater the amount to be deposited the lower must be the current density. By rotating the cathodes, or by using porous pots, both the weight deposited and the current density may be increased. These points are further considered under the heading "Crystalline Growths" (Section III.).

4. Before the final determinations were made the effects of impurities in the cell were investigated. These effects were either nil or were eliminated by using suitable current densities, by rotating the cathodes, or by using porous pots. Details are discussed under the heading "Impurities" (Section IV.).

5. It was found to be essential that the changes in the weights of the deposits, due to exposure in drying and weighing, should be eliminated by equal treatment of all the deposits in each experiment. The graphical method of correcting for loss by solution will then at the same time correct for oxidation, &c., if the washing and drying be done simultaneously for all the deposits.

II. The Limiting Cathode Current Density.

1. *The Cylindrical Cell.*

The deposits obtained in the common types of cells are far from being uniform, especially with parallel plate electrodes, where the deposit on the edges is often granular and non-coherent. The cylindrical cell was finally adopted because it gives uniform deposits and allows, other things being equal, a higher current density to be used. A cell of this type, a copper voltameter, is shown in the accompanying diagram (Fig. 1). The anode consists of a copper tube. For other metals the anode foil is kept in position by xylonite tubes, which drop into circular slots cut in the base, and are fixed with paraffin wax. The cathode consists of a brass rod. The brass cap, shown in position, need only be used when it is desired to rotate the cathode, the cap and the V-shaped slot then being amalgamated. Sometimes it is necessary to introduce porous partitions between the electrodes. These are the ordinary battery porous pots, the top being cut off to the required length. After being cleansed by boiling successively with aqua regia, potassium cyanide, nitric acid and water, they, too, are dropped into circular grooves and fixed in position with paraffin wax. It is important that there should be no leakage under the pot.

2. *Deposition of Hydrogen.*

Suppose we take the copper cell just described, fix the cathode and add a solution of copper sulphate. Let the current be gradually increased and the deposit examined from time to time. At first the deposit is bright, metallic, coherent and uniform throughout. After a time the deposit immediately below the surface of the electrolyte becomes tarnished

and then quickly develops into a chocolate coloured non-coherent mass. As time proceeds, or the current is raised further still, this non-coherent mass extends from the upper

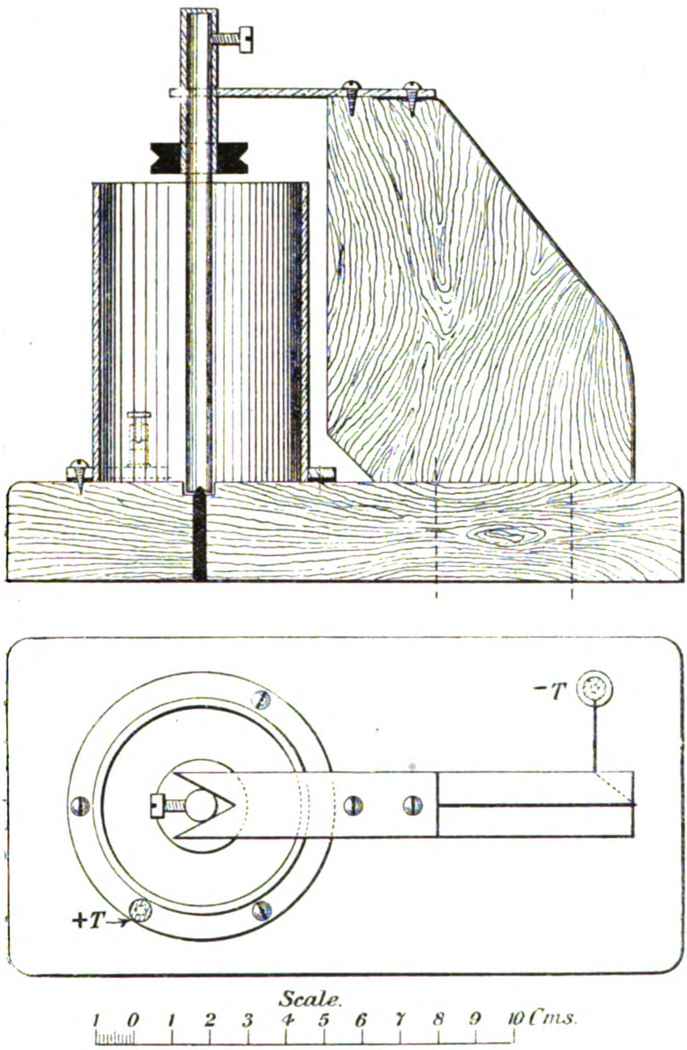

FIG. 1.—THE CYLINDRICAL VOLTAMETER.

part of the electrode downwards, quickly at first and then gradually more slowly. If the current is sufficiently high this

will be succeeded by the evolution of hydrogen. A voltmeter placed across the cell indicates an increase in voltage simultaneously with the deposition of the chocolate mass. This change in the character of the deposit is not due to the anode, since covering it with two or three layers of filter paper makes no difference. Neither is it due to the atmosphere, because in one experiment two identical cells were run in series, one having the electrolyte covered with a layer of oil. In both the effect appeared at the same time and to the same extent. Nor does the amorphous mass consist of copper oxide, since a portion of it, after being cleaned, dried and weighed, was reduced in a stream of hydrogen, and when re-weighed showed no loss in weight. The appearance of this deposit is similar to that of copper precipitated from a solution of copper sulphate by means of zinc. The same phenomenon is observed in other cells, the non-coherent deposit for zinc being bluish grey, for silver a dirty grey and for gold an orange red. In all cases, however, provided that the cathode current density is below a definite value, the action never takes place. On the other hand, if the current density (d) be raised above a certain value, the whole of the current cannot be carried by the copper ions, and some of it must be carried by the hydrogen ions. When the latter are deposited they interfere with the deposition of the copper, and hence give rise to the non-coherent deposit. When the rate of deposition becomes greater hydrogen will be evolved as gas. The copper thus deposited becomes so voluminous that it greatly increases the effective area of the cathode, consequently the cathode current density is reduced below the critical value for the deposition of hydrogen, and the copper is deposited in the metallic state again, the deposit then having the appearance of a gnarled oak stem. Hence, in order that the deposits may yield concordant results, it is necessary that the cathode density should be below a certain maximum or limiting value.

3. *The Methods of Obtaining the Limiting Current Density, "D."*

There are two ways of obtaining the value of D, the maximum or limiting cathode current density, namely, the weighing and voltmeter methods.

(*a*) *The Weighing Method.*—In a given cell systematically increased currents are run for the same length of time. The weights of the deposits are proportional to the currents. When, however, the copper is deposited by the secondary reaction

the deposit when weighed will no longer be proportional to the current. By a graphical construction this point of departure is found, and gives the maximum current which can be used. From this the limiting current density is deduced. This method is chiefly applicable to the case of rotating cathodes.

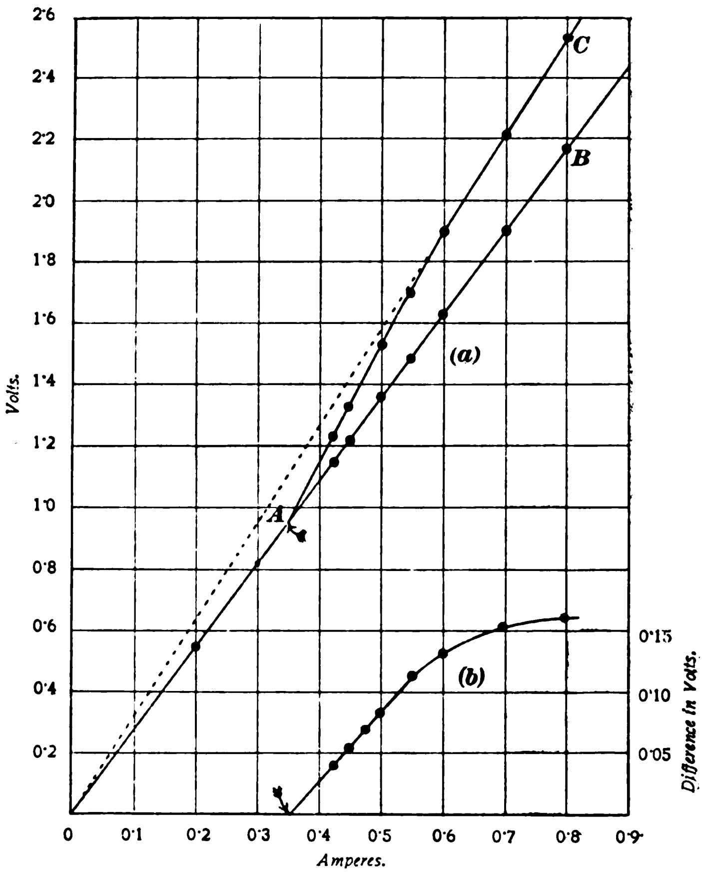

FIG. 2.—THE RISE IN VOLTAGE WHEN THE LIMITING CURRENT IS EXCEEDED.

(*b*) *The Voltmeter Method*.—A more expeditious method was found by placing a voltmeter across the cell, and noting its readings for steady increments of current. This gives the ordinary ohmic straight line passing through the origin, the line OAB of Fig. 2, which is the graph for one experiment.

But if the current is 0·8 ampere the needle, after standing at B for a time, begins to rise slowly at first, and then with increasing speed till the point C is reached, where it remains stationary or gradually falls. An examination of the cathode shows the presence of the non-coherent deposit. If the current is high enough the needle oscillates about the maximum reading, the amplitude of the oscillation giving a rough measure of the rate of the evolution of hydrogen. The curve passing through the maximum readings cuts the line OAB at A.

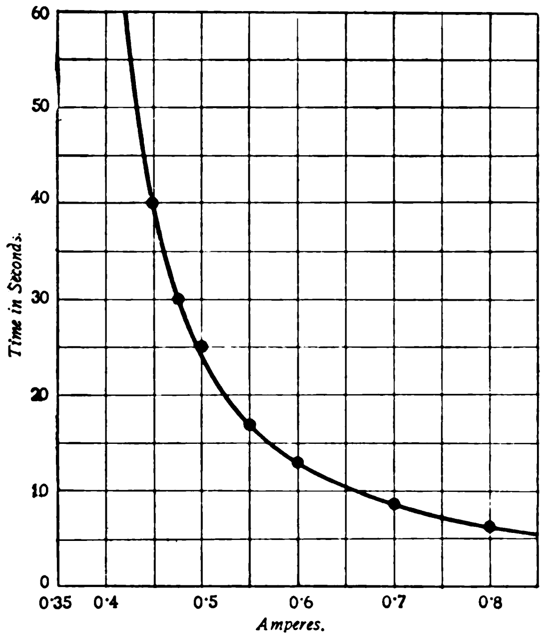

FIG. 3.—TIME FOR THE VOLTAGE TO RISE TO THE HIGHER VALUE WHEN THE LIMITING CURRENT IS EXCEEDED.

Below this point there is no trace of the non-coherent deposit. The point of intersection is taken as the limit to which the cell can be safely run. From this the value of D is deduced. In this experiment the cathode was cleaned before each reading. If this is omitted then the graph consists of the broken line OAC. Fig. 3 is for the same experiment, but in this case the ordinates represent the time in seconds for the voltage to rise to its higher value, after the switching on of the current. This curve can be represented by the equation $T = a\,(C - C_0)^{-b}$,

where C_0 is the distance of the vertical asymptote from axis of time, *i.e.*, with a current equal to, or below, C_0 the time for secondary deposition to take place is infinite. In this case the value for C_0, the limiting current, is equal to 0·35 ampere, the same as for the point A in Fig. 2.

The voltmeter method is capable of further modification to meet particular cases. By these methods the value of D was found for the voltameters of copper, silver, gold, and zinc under varying conditions of :—concentration of the electrolyte, height and radial thickness of the cell, change of temperature, the addition of acid and other impurities to the electrolyte, the inclusion of a porous pot, and the rotation of the cathodes.

4. *The Value of the Limiting Current Density*, "D."

The following empirical formula was found to give the limiting cathode current density for the electrolytes copper sulphate, zinc sulphate, silver nitrate, and gold chloride, the last containing acid,

$$D = ab^x c^y d^z, \qquad (1)$$

where D is measured in amperes per square centimetre. The values for the quantities are given in the table below.

Cell.	*a.*	*c.*	*d.*	*x.*	*b.*	*y.*	*z.*
Cu	0·0525	0·94	1·06	1·11	Concentration of the electrolyte in terms of normal strength.	Height of the cathode in cs. between the limits 3–7 cs.	Radial thickness of the electrolyte in cs. between the limits 2–4 cs.
Zn	0·0534	0·94	1·06	1·09			
Ag	0·094	0·94	1·06	1·33			
Au	0·050	0·94	1·06	1·14			

In this table *b*, *c*, *d*, *y* and *z* are independent of the particular metal, whilst *a* and *x* are peculiar to each metal. Further, it was found that in the case of Cu, Zn and Ag the limiting current density can be expressed by the formula

$$D = e(1-a)wc^y d^z, \qquad (2)$$

in which ab^x in equation (1) is replaced by $e(1-a)w$. Here $e(=2\cdot13)$ is a constant, the same for the three metals, $(1-a)$ is the un-ionised fraction of the salt in solution, and w is the mass of the metal per cubic centimetre. The equation (2), written in full, becomes

$$D = 2\cdot13\,(1-a)w(0\cdot94)^h(1\cdot06)^{r_1-r_2}. \qquad (2\text{A})$$

In 45 cases the calculated and experimental value of D agreed within the limits of experimental error. From equation (2A) it is evident that the value of D depends upon the geometry

of the cell, and also to a large extent upon the density convection currents.

Temperature.—All the above equations apply more particularly to the case in which the temperature of the laboratory and the electrolyte is about 18°C. If, however, the temperature rises, the new value of the constant (e) in equation (2) is given by

$$e_t = e_{18} + 0{\cdot}05(t - 18°). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Porous Pots.—If the electrodes are separated by porous pots the convection currents due to the removal of the metal ion are greatly restricted, and the allowable D falls in consequence. The new value for D is still given by the equations (1) and (2), but (z) is now reduced to zero.

Acids.—The allowable current-density is also cut down by the presence of acid in the cases of Cu, Zn and Ag, and if D be calculated from equations (1) and (2), which is for neutral solutions, the actual allowable current-density is found to be given by the equation

$$D_a = D(0{\cdot}93)^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where n is the concentration of the acid in terms of N/10 strength.

Rotation of the Cathodes.—The value of D rose when the cathode was rotated in the case of Cu and Zn, whether a porous pot was used or not, according to the equation

$$D_R = D(1 + 0{\cdot}0024R), \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where R is the number of revolutions per minute. The results for silver in this case were not conclusive.

It thus appears that, provided cylindrical cells are used, the maximum safe current can be calculated for all conditions imposed upon the experiment.

III. Crystalline Growths.

Having arranged that the value of the cathode current-density (d) shall be sufficiently small to satisfy the previous rules, it is still necessary to restrict the total amount of deposit in order to avoid difficulties from crystalline growths. This is given by the simple empirical equation

$$w\Delta = \text{constant}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where w is the total number of grammes deposited on a cathode

and Δ is the maximum current-density at which these crystalline growths are not produced. The value of the constant for different conditions of the cell is given in the accompanying table.

Metal.	Electrolyte.	Porous pot.	Rotation of cathode.	$w\Delta$.
Copper	Neutral	None	Stationary	0·025
	Neutral	Porous pot	Stationary	0·05
	$+\frac{1}{10}$% of H_2SO_4 by volume	Porous pot	Stationary	0·025
	Idem	Porous pot	1,000 r.p.m.	0·035
Silver	Neutral	None	Stationary	0·08
	Neutral	None	600 r.p.m.	0·08
	Neutral	Porous pot	700 r.p.m.	0·2

It is necessary, therefore, to use in any experiment a (d) less than either D or the Δ calculated from the above table.

IV. Impurities in the Cell.

1. *Anode Detritus.*

Very soon after the current is started a density convection current of the electrolyte is set up. This current, descending at the anode and ascending at the cathode, carries with it anode detritus. This gives rise to coarse coralline deposits at the bottom of the cathode. Covering the anode with a few layers of filter paper makes no improvement. Rotating the cathode diminishes this effect to an extent depending on the rate of revolution, but a trace of it is sometimes seen in the form of a spiral on the rod, which may in silver develop into fan-like plates. The trouble is eliminated by separating the electrodes with a porous pot. In this case copper oxide, sand and pumice powder may be held in suspension in the electrolyte without materially affecting the deposit. It seems, therefore, that only very small particles, which can pass through filter paper but not through a porous pot, can act as nuclei for the formation of the coralline bottom growths.

2. *Acid.*

Acid added to the cell clarifies the electrolyte and decreases the anode detritus. Besides lowering the value of D it modifies the character of the deposit considerably. The normal deposits of copper and silver consist of octahedral crystals. In the case of copper, if the current-density is above 0·01, the

crystalline deposit is destroyed. In its place we have a creamy non-crystalline surface, and on this hemispherical protuberances appear, increasing in number and size with time, till they become contiguous, and then develop into a coralline structure. Deposits from acidified solutions give concordant weighings, provided that the experiment is kept within the $w\Delta$ limits of equation (6). On the other hand, the crystalline deposit of silver is not destroyed, but in the upper region of the cathode needle-shaped crystals soon appear, which, branching, develop into beautiful lace-like patterns. For this reason, in the final determinations, only neutral solutions of silver nitrate were used.

3. *Other Added Impurities.*

The following impurities were introduced in the preliminary experiments to test the effect on the value deduced for the atomic weight. In the copper cell were introduced $ZnSO_4$, $FeSO_4$, $AgNO_3$ and $CaCl_2$; and in the silver cell $Cu(NO_3)_2$.

(*a*) The presence of $AgNO_3$ in the electrolyte was at once detected by the silver being precipitated from the solution on the electrodes. The $CaCl_2$ immediately blackens the cathode, and causes the needle of a voltmeter when placed across the cell to give very unsteady readings.

(*b*) The presence of $ZnSO_4$, $FeSO_4$ or $Cu(NO_3)_2$ in the electrolyte lowers the value of D the same as acids do according to equation (4).

(*c*) Neither Zn nor Fe is deposited in the copper cell. In a particular experiment the electrolyte contained equal parts of Cu and Zn. Using a current-density of 0·013, a deposit having a beautiful matt surface was obtained. This was peeled off, and 0·323 gramme of it dissolved. The copper precipitated as $Cu(CNS)_2$ yielded 0·322 gramme. Clearly, therefore, no zinc was deposited. Graham obtained a similar result with iron, the proportion of Cu to Fe in the electrolyte being 3 to 2.

Neither is Cu deposited from a silver cell. Repeating the experiment of Lord Rayleigh and Mrs. Sidgwick, but having a higher copper content, the proportion of Ag to $\frac{1}{2}$Cu being 5 to 1, a good deposit was obtained. 0·9437 gramme of the deposit was dissolved, and the silver, precipated as AgCl, gave 0·9436 gramme. These experiments cover practically all the impurities likely to be present in the two electrolytes.

4. *The Purity of the Metals and Salts used in the Final Determinations.*

In the final work no difference in the results was obtained when the electrodes consisted of the ordinary laboratory metal, or the extra pure of Kahlbaum, and the same is true for the salts copper sulphate, and silver nitrate.

V. An Account of Previous Work.

The solution of the Cu cathode by the electrolyte was first pointed out by Gore (1). Gray (2) also showed that this effect varied with the different conditions of the cell. Richards and Collins (4) found that the loss in weight of a cathode was proportional to its area. The loss was little affected by the presence of acid, but increased with the concentration of the electrolyte. This confirmed the work of Foerster and Seidel (19). They pointed out that the Cu dissolves in a strong $CuSO_4$ solution to form Cu_2SO_4, which may, especially in neutral solutions, yield too high a result, due to the presence of cuprous ions. Schuster (20) also showed that the solvent power of acid in the electrolyte was small. Experiment, however, showed that the solution effect cannot be corrected for by control experiments, especially as Adams and McNutt (3) proved that it is only the freshly deposited Cu which is so sensitive to the action of acid. The graphical method of correcting for solution under experimental conditions showed that the solvent power of the electrolyte does not increase with its concentration, but does increase with the acid content. The effect sometimes encountered with electrolytes previously boiled over Cu, where the weight of Cu deposited increases with the area of the cathode, may be looked upon as due to oxidation, which can be easily corrected for by the graphical method. If, however, Cu_2SO_4 is formed in the cathode space, its concentration and the amount of deposition by the cuprous ions will still be a function of the cathode area. Hence the graphical method of correction will cover this case, too. This method is preferable to that adopted by Beach (5) and Shephard (6), who added arbitrary amounts of ammonium chloride to their neutral electrolytes to prevent oxidation.

The amorphous deposit discussed in section II. was noted as far back as 1844 by Smee (7). Le Blanc (8) considered that both copper and hydrogen ions are present at the cathode, but the latter are not liberated until the concentration of the

copper ions is reduced to zero. Sand (9), using a vertical cell, came to the same conclusion. But Hittorf (10), as quoted by Sand, believed that the two ions are deposited at all current-densities. The hydrogen, however, reacts with copper sulphate with a definite reaction velocity, and is only evolved when its rate of deposition is greater than the reaction velocity. It seems from the experiments on the limiting current-density that more probably a definite cathode fall in potential is necessary for the deposition of hydrogen.

In order to increase the effective cathode current density Graham (11) circulated his electrolyte. Experiment shows, however, that better results are obtained when the cathode is rotated. The rotation of the cathode is employed by Cowper-Coles (12) for the deposition of copper on an industrial scale. He obtains a smooth deposit, which he maintains is due to the skin friction between the electrode and the electrolyte. Smooth deposits were not obtained in my experiments which may be due to the peripheral speed being insufficient. Also to eliminate the coral growths, due to anode detritus, he surrounds the bottom of the cathode with a baffle plate, placed a short distance away from it.

A résumé of the work on the silver voltameter is given by Guthe (13). This contains an account of the Guthe-Richards cell, which was used in the final determinations of the Atomic Weight in this Paper. The work of Lord Rayleigh and Mrs. Sidgwick (17), and that of Smith, Mather and Lowry (18), will be referred to in the next section.

The earlier determinations of the atomic weight of copper, chiefly performed by the reduction of CuO, were vitiated by systematic errors. This was pointed out by Richards (14). In his classic work he estimates the combining weights of Cu and Br in $CuBr_2$; the Cu, two determinations, by its deposition from 50 cc. of solution by electrolysis, and the bromine, six determinations, by precipitating it as AgBr. By combining these results 12 values for the atomic weight are obtained, whose mean is 63·57—the accepted value in the 1913 list of atomic weights, where O=16, Ag=107·88. Shaw (15), by the electrolytic deposition of Cu and Ag from cells in series, the Cu being deposited on 1 to 16 platinum wires, obtained the atomic weight 63·46. He endeavoured to correct for solution as a function of the current density. In the preliminary experiments of this Paper similar corrections gave abortive results. Using a silver cell of the Rayleigh and Sidgwick

type, and two copper cells of the parallel plate type, Richards and Collins (4) obtained the value 63·534, with a mean error of 0·02 for the last six consecutive determinations. This value is for the corrected weight of Cu found by extrapolating to zero area, the same as explained in this Paper. Unfortunately only two plates were used, and no note is given to say if they were simultaneously washed and dried. Subsequent work by Richards and Heinrod (21) showed that the weight of Ag deposited in the cell of the Rayleigh and Sidgwick type was greater than that obtained when a porous pot surrounds the anode. The correction for this raised the atomic weight of copper to 63·560. Also they found that by heating the Ag deposit to redness the deposit lost in weight. This further correction raised the atomic weight to 63·571. Lastly Pécheux from a similar electrolysis with neutral solutions, obtained the value 63·43. No details are given in the Paper referred to (16).

VI. The Atomic Weight of Copper.

In the following determinations two silver cells separated by four copper cells were run in series. This will correct for any short circuiting of the current. The ratio of the corrected weight of the deposits of copper to that of silver was found. From this the atomic weight of copper

$$=\frac{\text{Weight of copper}}{\text{Weight of silver}}\times 2\times 107\cdot 88.$$

The solution of the copper deposit was corrected for by the graphical method. This necessitated simultaneous washing and drying of the electrodes. The maximum cathode current density used must be less than that calculated from the equations (1) to (5). Also for a given weight of metal to be deposited the greatest current density used must be less than that given in equation (6).

1. *The Silver Cells.*

The cylindrical cell with a silver rod for cathode allowed only a small range of current densities to be used. As larger cathodes were not at my disposal the Guthe-Richard's cell was used. This is a modification of that of Lord Rayleigh and Mrs. Sidgwick (17). The cathode consists of a platinum dish, which is separated from the anode by a porous pot, the anode plate being tightly packed in with fine grains of silver, chiefly obtained from previous deposits. The silver nitrate

was freed from acid by fusing the salt before dissolving it. Although the equations already considered do not numerically apply to this cell yet similar phenomena are observed. The safe limits for the deposited weight and the current density were obtained from the preliminary experiments. The deposit, after being thoroughly washed and dried, was weighed. Afterwards it was heated to redness, and, when cool, reweighed. From the difference in the two weighings the correction for the occluded salt was obtained, after Lord Rayleigh and Mrs. Sidgwick.

2. *The Copper Cells.*

These were all of the cylindrical type. The determinations recorded (1) to (4) were the last of the series in which rotating cathodes were used, numbers 1 and 2 being carried out prior to the investigation of the effects of the porous pot. Determinations 5 to 10 are the last six experiments carried out. These were performed after all the preliminary work was completed and form an unbroken set of results, none of the weighings being omitted, except that in brackets in number 5. This is explained in the footnote at the end of the table. All the cells contain porous pots except those of the first two, 3 and 4 have red pots and the rest white ones. In the experiments 1 to 4, as soon as the current was broken rotation was stopped, and the rods were dropped into a similar copper sulphate solution as quickly as possible. The caps were removed, the rods simultaneously rinsed in tap water, and distilled water, allowed to stand in absolute aclohol, wiped with a cloth desiccated, and weighed. It was found that the areas of the cathodes did not vary sufficiently to plot the graph satisfactorily. In the experiments 5 to 10 with stationary cathodes the following modifications were adopted. On a single teak base were arranged four pillars of the same height, carrying the slotted brass plates, which could be swung out of position easily. The whole was heavily coated with shellac varnish. The cells consisted of two battery jars and two crystallising dishes, the latter being raised on teak blocks. White porous pots were added and paraffin wax run in until the appropriate depth was obtained. When the cells were in position, the tops of the porous pots, which reached half a centimetre above the cells, were on the same level. Three of the four cathodes consisted of thin brass tubes, closed at each end with brass discs. They were 12 to 16 cs. long and 2 cs. in diameter. The fourth cathode was a brass rod. The remaining portions of the cells

on r -5.	Notes.
)	1 rod stopped resulting in the current density being too high.
7	Cu reduced in hydrogen.
32 75	(*a*) 2 hollow tubes. (*b*) 3 rods.
1	......
6	During absence the circuit was broken after 240 mins., so electrodes stood in electrolyte for another 90 mins.
— 1·9	Deposit on copper.
— 4·4	......
	Anodes of Kahlbaum's Cu.
6	......
66	1 new porous pot for Ag, well boiled after chemical treatment

followed the type previously described under the heading "Cylindrical Cell." The copper sulphate solution was poured into the cells to the same distance from the top, so that the deposits were approximately 12, 24, 36 and 48 sq. cs. in area, the rod being used to take the smallest area. In order that the cathodes should be dealt with simultaneously they were treated as follows. After being weighed they were dropped into holes in a stand, the holes being arranged to correspond with the final position of the cathodes in the cells. A carrier was dropped over them and clamped them tightly by means of thumb screws. The electrodes were now transferred to the cells. They were lowered gently into the electrolyte, so that no portion was wetted which did not finally remain immersed, the displaced liquid meanwhile being siphoned into the anode compartment. The carrier was removed and the current started. To wash the deposits the brass plates were swung out of the way, the carrier fixed in position, and the whole quickly transferred to a tank of water placed under the bench. After being thoroughly swilled the tubes were lowered into absolute alcohol contained in four large mouthed bottles, and allowed to stand whilst the carrier was removed. The electrodes were now withdrawn, wiped with a cloth, and placed in desiccators.

In order to detect any sources of error not previously considered the experimental conditions for each determination were varied as much as possible. These are set out in the accompanying Table (I.) for the 10 determinations. It will be seen that the current density varies from 0·14 to 0·0033, and the time of the experiments from 33 to 430 minutes. It may be mentioned that in the earlier series of determinations the strength of the copper sulphate solution varied from 0·5 to 2·5 normal with acid added from 0 to 20 per cent. with concordant results. The solution factor—*i.e.*, the mean loss of copper per unit area of the cathode is shown in column 8. This was obtained by substituting in the equation

$$w = ma + w_0,$$

where w=the weight of the copper deposit,

a=the area of the cathode,

w_0=the intercept on the axis of zero area—*i.e.*, the value for the corrected weight of copper,

m=the slope of the line, or the solution factor.

This graph was compared to that drawn by eye; the latter was

sometimes difficult to draw where the experimental errors were large, due to the conditions imposed upon the experiment. In the majority of cases the two graphs agreed. In number (8) no satisfactory graph can be drawn, and therefore the solution factor was put equal to zero. The line for the three rods in number (3) as calculated has a small positive slope indicating oxidation. As the points were close together three colleagues were asked to draw the graph by eye. The mean value of the three intercepts was the same as that obtained from the equation. The factor for the tubes is also positive, but differs from that of the rods because they were not washed and dried at the same time. It is interesting to note that in these two cases the electrolyte has been boiled over copper for two hours, rendering it neutral. In determination (4), however, the electrolyte used in (3) has been treated in the same way three times. Its strength has dropped from 2 to 1·8 normal, whilst at the same time it has become acid, having a solution factor $-6{\cdot}1\times10^{-5}$. This clearly shows the necessity of correcting for solution under experimental conditions. The graphs as calculated and drawn by eye differ somewhat in determinations (6) and (7). The two values of the slopes are given, the calculated value being first. The mean value is used for the correction of the weights.

3. *The following is a typical example.*

Determination 10.

$AgNO_3$ solution=1·9N, filtered.

$CuSO_4$ solution=2·0N+0·1 per cent. acid, but used before in number 9.

Current=0·3 ampere.

Time=120 minutes.

Platinum Dishes.

—	(1) grs.	(2) grs.
1st weight	32·26799	32·68371
2nd weight	34·70764	35·12310
1st weight of Ag	*2·43965*	*2·43939*
Loss on heating to redness	0·00051	0·00037
∴ Weight of $AgNO_3$ occluded	0·00140	0·00104
∴ Weight of Ag deposited	*2·43825*	*2·43835*
Correction for weights	+1	+8
Correction for buoyancy	−6	−6
∴ Corrected weight of Ag	*2·43820*	*2·43837*
Mean weight of Ag	*2·43829*	

Mean error in atomic weight, due to	
Cu.	Ag.
0·002	0·009
0·002	0·000
0·004	0·010
0·003	0·002
0·004	...
0·016	0·000
0·011	0·022
0·018	0·005
0·002	0·026
0·008	0·002

high a weight for the
chemical treatment.
sit 3·24429 is in close
stimating the atomic

Copper.

Tubes	1.	2.	3.	Rod.
	Grs.	Grs.	Grs.	Grs.
1st weight	41·15292	37·81647	37·26446	32·95332
2nd weight	41·87035	38·53437	37·98221	33·67161
Weight of copper	*0·71743*	*0·71790*	*0·71775*	*0·71829*
Correction for buoyancy	+30	+28	+27	+7
Correction for weights	+ 9	− 3	+10	−9
Corrected weight of Cu (1)	*0·71782*	*0·71815*	*0·71812*	*0·71827*
Area of cathodes in s.cs.	*54·8*	*43·6*	*24·8*	*12·0*

These weights are plotted as ordinates against the corresponding areas as abscissæ. The line has a slope, which gives for the solution $0{\cdot}66 \times 10^{-5}$ grammes per square centimetre, and cuts the axis of zero area at the

corrected weight, 0·71831 gramme.

$$\therefore \text{ The atomic weight of Cu} = \frac{0{\cdot}71831}{2{\cdot}43829} \times 107{\cdot}88 \times 2.$$
$$= 63{\cdot}562.$$

4. *Experimental Errors.*

In order to determine the extent of the experimental error, each copper weight is corrected for solution by adding to it the product of the area and the constant $0{\cdot}66 \times 10^{-5}$. These corrected values are then compared with the mean silver as follows :—

Area of cathode.	Weight of Cu.	Correction for solution.	Corrected weight of Cu.	Atomic weight of Cu.
S.cs.	Grs.	Grs.	Grs.	
54·8	0·71782	+0·00036	0·71818	63·550
43·6	0·71815	0·00029	0·71844	63·573
24·8	0·71812	0·00016	0·71828	63·559
12·0	0·71827	0·00008	0·71835	63·567

Therefore the mean error in the atomic weight due to Cu $= \pm 0{\cdot}008$. Similarly, if the corrected weight of copper 0·71831 is compared with the two silver weights 2·43820, 2·43837, the atomic weights become 63·564 and 63·560. Therefore the mean error in the atomic weight due to Ag $= \pm 0{\cdot}002$. Table (II.) shows the weights, &c., for the 10 determinations, the atomic weight being calculated from the mean silver.

In determinations 7 and 9 the values for the atomic weight are vitiated by the large experimental errors of the silver. For this reason they are bracketed in the summary below and are omitted in determining the final value. These errors cannot

be accounted for, the deposits in both cases appearing remarkably good. Smith and Lowry (reference 18) pointed out that a strong solution of $AgNO_3$ has the power of dissolving $AgNO_2$ formed during the fusion of the salt. The nitrite may become insoluble in the weaker cathode solution and be deposited with the silver. Corrections made for occluded nitrite instead of the nitrate will raise the value for the atomic weight, but does not decrease the experimental errors. The source of error may be due to the detritus from the porous pots. If such is the case the error would increase with the duration of the experiment, and this seems to be the case.

The errors in the atomic weight due to copper depend largely on the state of the electrolyte.

For neutral solutions the error=0·01, and greater still when CuO is added.

If 0·1 per cent. acid is added the error=0·003 when the solution is used once.

If 0·3 per cent. is added the error=0·002 when the solution is used several times.

Probably the errors are due to the presence of CuO formed by hydrolysis. This can be temporarily removed by filtering through fine Swedish filter paper, and permanently by adding enough acid or by electrolysing the solution. Any of these will impart to the solution the well-known clear sapphire colour.

5. *Results.*

Summarising the mean values for the 10 determinations we have :—

(1)	63·567
(2)	63·560
(3)	63·563
(4)	63·559
(5)	63·560
(6)	63·572
(7)	[63·558]
(8)	63·562
(9)	[63·556]
(10)	63·562

∴ **The atomic weight of copper = 63·563**

when Ag =107·88

Mean error in the atomic weight = ±0·003

Mean error due to copper = ±0·007

Mean error due to silver (omitting 7 and 9) = ±0·003

Probable error due to weighing = ±0·003

In these 10 determinations no less than 100 results can be obtained for the atomic weight whose mean value is 63·562.

In conclusion, the author takes this opportunity of putting on record his indebtedness to Dr. J. H. Vincent, in whose laboratory this work was done.

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ABSTRACT.

Four copper cells separating two silver cells were run in series. The areas of the four copper cathodes increased from 10 to 50 s.cs. By plotting the weights of the copper deposits against the corresponding areas of the cathodes, and extrapolating to zero area, the weight of the deposit is corrected for under experimental conditions.

$$\text{The atomic weight of copper} = \frac{\text{corrected weight of Cu}}{\text{mean weight of Ag}} \times 107{\cdot}88 \times 2.$$

The mean atomic weight for 10 determinations = 63·563, with a mean error of ±003. To obtain a uniform coherent deposit of pure metal the following points were considered :—

1. Cylindrical cells with stationary and rotating cathodes were used.

2. The cathode current density must be kept below a certain *limiting* value to prevent the formation of non-coherent deposits, due to secondary deposition. This was found to depend upon the geometry of the cell, the concentration of the electrolyte, the presence of acid and other impurities, the addition of a porous pot and

the rate of revolution of the cathode. Formulæ are given by which the *limiting cathode current density* can be found for all conditions of the cell for Cu, Ag, Au and Zn.

3. To prevent the formation of loose crystalline clusters the current density must also be kept below a certain value depending upon the weight to be deposited. (Formulæ are given.)

DISCUSSION.

Mr. F. E. SMITH congratulated the author on the results of his work. He appeared to have triumphed over many difficulties, and for his particular purpose had converted the copper voltameter from an instrument of error into an instrument of precision. In future work he hoped that Mr. Shrimpton would avoid the use of common porous pots. They might produce trouble, and were not necessary, as all voltameter work could now be carried out without the introduction of any medium between anode and cathode. He should be pleased to give Mr. Shrimpton full particulars. The statement respecting deposition of hydrogen was, he thought, incorrect. It was not logical to assume that hydrogen ions were first deposited because of the lack of copper in the neighbourhood of the cathode, and then to state that as soon as they were deposited they went into solution again, and in doing so displaced copper from the solution. With regard to crystalline growths at the bottom of the cathode, he believed these were due to the high current density at the base due to the passage of the current through the liquid descending from the anode. There was also a possibility that the electrolyte was not quite pure. In the silver voltameter he had obtained striated deposits with impure electrolytes, but not with pure solutions. Mr. Shrimpton also stated that silver deposits from acidified solutions were normal; this was not his experience, nor that of other investigators. He believed the deposit to be less in mass when the electrolyte was acid. A rather important question arose with regard to any Cu_2SO_4 which might be formed in the cathode space. Richards (who also extrapolated to zero area, as Mr. Shrimpton had done) remarked in his Paper: " A value obtained in this way must correspond to a deposit of copper slightly too great; for the mode of correction (*i.e.*, extrapolation to zero area) does not take account of the growing, although slight, presence of cuprous salts." Mr. Shrimpton said the extrapolation to zero area covered this case. The speaker thought the point was one that should be settled. Of course, when one reduced the area the current density at the cathode was increased, and so also was the fall of potential near the cathode; this might have some influence.

Dr. J. H. VINCENT mentioned that after working out the conditions necessary for satisfactory results, the author had performed the 10 determinations quoted in the Paper straight away, no results being rejected. As they all came within 1 part in 6,000 of the mean there appeared to be no doubt that the correct conditions for satisfactory working had been obtained.

Dr. S. W. J. SMITH thought the author's views on secondary deposition could not be correct as they stood, although it was conceivable that sudden variations in the surface concentration of the copper sulphate, due to irregularities of convection, might make it possible for hydrogen to be deposited at one moment and to go into solution again at the next. Non-coherent deposits could be explained without invoking the aid of hydrogen. He thought the author's empirical conclusion, that the limiting current density depends upon the concentration of non-ionised molecules, had some theoretical support.

Mr. SHRIMPTON (communicated remarks): In section II. 2, line 34, as originally written, the author, following Hittorf, described the non-coherent deposit as being due to secondary reaction. As the deposit when the limiting current density was exceeded had the appearance of precipitated copper, it was assumed as a working hypothesis that the hydrogen after being deposited went again into solution, thereby displacing Cu from the unionised $CuSO_4$. In the discussion it was pointed out that if the similar deposit of the Ag voltameter be carefully collected and weighed, there is a loss in weight, thus indicating the deposition of H. It seems, therefore, that the above hypothesis is unnecessary, the mere presence of deposited H materially affecting the deposition of the metal. For this reason the text has been altered. As regards the presence of acid in the silver cell, the author meant that the crystalline character of the deposit was not destroyed as in Cu, Zn and Au (the last in aqua regia). In the Ag cylindrical cell acid must not be present because non-coherent needles soon develop at the top of the deposit. Lastly, the author considers that Cu_2SO_4, if formed, was completely corrected for, since its formation must be a function of the area of the cathode and the duration of the experiment. In the determinations the current density varied from 0·14 to 0·003, and the duration of the experiments from 33 to 430 minutes, yet no difference appears in the values for the atomic weight.

XXXII. *Note on an Improvement in the Einthoven String Galvanometer.* By W. H. Apthorpe.

Communicated by Mr. R. S. Whipple.

Received May 7, 1914.

Since the String galvanometer was introduced by Prof. Einthoven in 1903* it has been found useful for many purposes where an extremely sensitive galvanometer is required; but perhaps it has been put to the greatest use in physiological work in recording small changes in electric potential, due to the movements of the heart. This is done by connecting the patient to the galvanometer terminals and projecting an image of the string by means of an arc lamp and a suitable optical system on to a photographic plate.

As it is necessary to record the cardiac potential from three or more points of the body, and in conjunction with these the phonocardiogram† for a complete examination of the heart, it will at once be appreciated how much more readily the records can be studied, and the exact relation between the phenomena determined if two or more records are obtained simultaneously on the same photographic plate. Up to the present this has only been possible by using two separate string galvanometers,‡ each having its own optical system and lamp for illuminating the string, the optical systems being so adjusted that the projected images fall side by side on the camera.

It has been possible in this way to obtain good results after expending a considerable amount of time and patience in making the necessary adjustments.

In December, 1912, Dr. Avery Newton suggested that the same result could be obtained more easily by placing two strings in the same magnetic and optical field, so that the two images were projected side by side on to the photographic

* "Annalen der Physik," 1903, XII., 1,059.

† A record of the sound of the heart beating which is produced by converting it into an electric current by means of a stethoscope, a microphone and transformer.

‡ It should be mentioned that an oscillograph, with two or more moving systems in the field of an electromagnet, has been made by Messrs. Bock and Thoma of Munich, and that it has been used for taking simultaneous electrocardiograms.

plate. This, however, was found to be practically impossible, owing to the closeness with which the two strings had to be mounted (0·04 mm.) in order that the two images should be sufficiently close together under a magnification of 600. This large magnification is found necessary in cardiographic work.

In order to overcome this difficulty the following device was

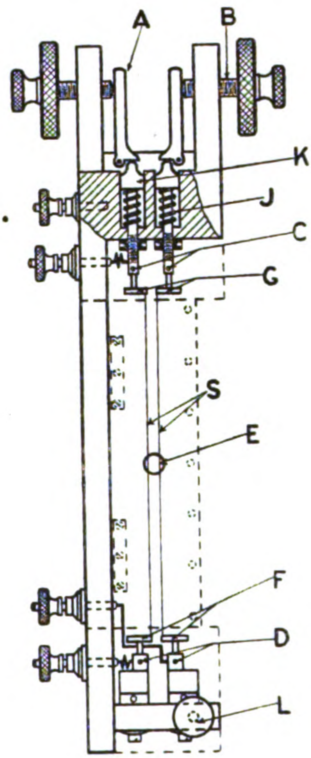


Fig. 1.

recently constructed, which has been found to give very satisfactory results.

A special holder, as shown diagrammatically in Fig. 1, has been made for holding two strings parallel and in a plane at right angles to the magnetic field. The strings S, which are made of silvered glass fibres, are soldered to T-shaped brass ends F and G, which are held in position by set screws at C and D.

Provision is made for adjusting the tension by means of the screws and bell crank levers A and B. By screwing in the screws the levers depress the plungers J and K to which the strings are attached to one end, and so decrease the tension and thus alter the sensitivity.

A further adjustment was provided at L for moving one string in relation to the other, so as to bring the parts of the strings under observation into the same focal plane. The strings are placed at a distance of about 0·5 mm. apart, which was found to be convenient for manipulation and which allows both of them to come well within the field of a 16 mm. Zeiss objective.

The moving systems are entirely enclosed in a thin metal case (shown dotted) and a small mica window E allows the movements to be observed at the centre.*

As it is desirable to record the deflections of the strings on a photographic plate 3¼ in. (82 mm.) wide, it is necessary to reduce the distance between the projected images. This is done most conveniently by deflecting the projected beam by means of an achromatic prism of about 15 deg. as shown at M and N Fig. 2.

The final adjustment of the distance between the images is obtained by rotating the prisms through a small angle about their common vertical axis O.

By means of this device not only is the manipulation simplified, but as the same time marker is used, any slight inaccuracy in timing will affect both records to the same extent, and can usually be neglected. Both strings are influenced equally by any change in the magnetic field.

The actual method of mounting the fibre case, the micrometer heads for altering the tension of the fibres, and the arrangement for supporting the prism are shown clearly in Fig. 3.

Fig. 4 is part of a record of an electrocardiogram and a phonocardiogram obtained simultaneously from a patient by means of this apparatus.

The upper record is the electrocardiogram obtained from

* The closed metal case for holding the moving system of a string galvanometer was originally designed by Mr. W. Duddell in 1905, for the Cambridge Scientific Instrument Co., Ltd. It is by the courtesy of the company that I am able to describe this improvement which has been developed in their workshops. I may also mention that patents have been applied for to protect the invention.

Fig. 4.

To face page 316.]

Provision is made for adjusting the tension by means of the screws and bell crank levers A and B. By screwing in the [illegible]

[illegible] Scientific Instrument Co., Ltd. It is by the courtesy of the company that I am able to describe this improvement which has been developed in their workshops. I may also mention that patents have been applied for to protect the invention.

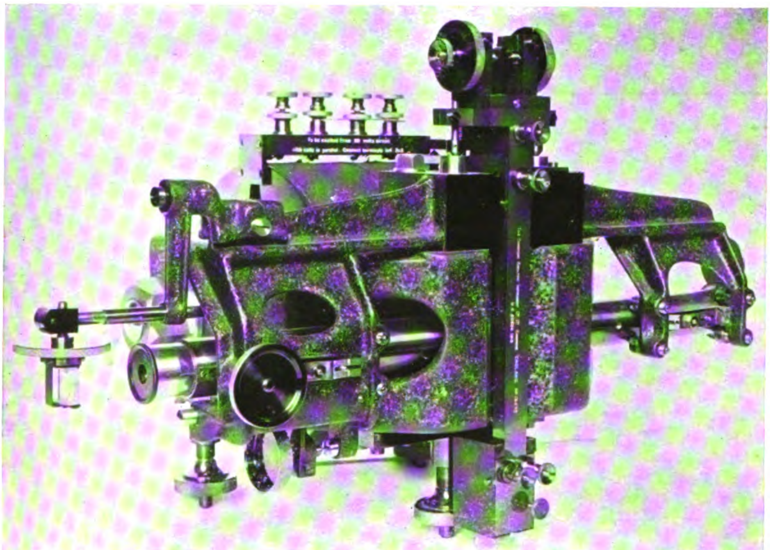


FIG. 3.

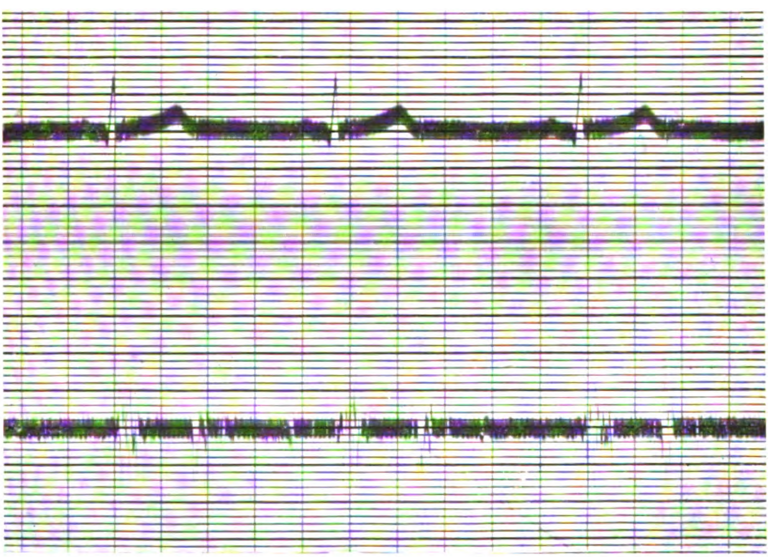


FIG. 4.

To face page 316.]

lead I.* of a normal patient, the lower record being the phonocardiogram obtained simultaneously with the electrocardiogram.

The horizontal lines are produced by lines engraved on the cylindrical lens of the camera, each division representing 0·0001 volt. The vertical lines are time intervals of one-fifth of a second.

Although this apparatus was designed for electrocardiographic work it appears to the author that it might be employed with advantage for other purposes, such as recording two radio-telegraphic signals of different wave length simultaneously by means of a camera in which a band of photographic paper is made to move continuously.

* Following the nomenclature of Einthoven, the different circuits through the patient and the galvanometer, which are generally used, are called "Leads," and are numbered as follows :—

Lead I., that obtained with the horizontal connection (right arm, left arm).
Lead II., that obtained with the oblique connection (right arm, left leg).
Lead III., that obtained with the vertical connection (left arm, left leg).

XXXIII. *On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves Round the Earth's Surface.* *By* J. A. Fleming, *M.A., D.Sc., F.R.S.*

Received May 28, 1914.

Apart altogether from true diffraction or from any reflective or retractive effects due to atmospheric ionisation the normal refraction of the atmosphere due to the decreasing density of the constituent gases as we rise above the earth's surface exercises a bending effect upon rays of light, and therefore also upon electric radiation sent out from any point in various directions. If a ray of light is sent out from a source on the earth's surface, starting in a direction tangential to the terrestrial spheroid, it follows a curved path of varying curvature with concavity directed towards the earth, as it passes outwards through the atmosphere. This curvature is such that, very roughly speaking, a tangent to the ray is bent through an angle of about half a degree in its course.

Conversely, the atmospheric refraction bends incoming rays from stars or celestial bodies so that their apparent zenith distances are less than the true ones.

It becomes, therefore, an interesting matter to inquire under what conditions this ordinary atmospheric refraction taken alone could operate to carry a ray, starting from any point on the earth's surface in a tangential direction, round the earth parallel to its surface. The question is not by any means a new one, and has already been considered in connection with some astronomical problems.* It has, however, become increasingly clear of late years that the full explanation of the phenomena of long-distance radio-telegraphy, such, for instance, as the transmission of wireless signals a quarter of the way round the earth from Ireland to Argentina, is only to be found in atmospheric action on the transmitted waves. The most complete mathematical investigations, such as those of Prof. H. M. Macdonald,† March and Rybczynski,‡ show that it is not possible for true diffraction to contribute more than a certain

* *See* Dr. August Schmidt, "Die Strahlenbrechung auf die Sonne," Stuttgart, 1891. Also Miss Agnes Clerke, "Problems in Astrophysics," p. 165.

† "The Transmission of Electric Waves around the Earth's Surface," "Proc." Roy. Soc., A., Vol. XC., p. 50, 1914.

‡ "Ann der Phys," Vols. XXXVII. and XLI.

fraction of the total effect at the distant receiving station. On the other hand, whilst there are indications that some of the effect is propagated along or through the earth's crust and follows round any irregularities in it, it is also clear that the major part cannot arrive in this manner or it would not be so dependent upon changes in the state of the atmosphere as regards solar illumination, as we find it to be.

We are, therefore, thrown back upon the consideration of the question of the relation of the atmosphere to electric wave propagation through it. The extraordinary vagaries of long electric wave transmission and the manner in which our power of transmitting such waves varies from hour to hour and day to day through the year points to some continually fluctuating atmospheric condition as a principal factor in it. Such a condition is found in the ever-varying ionisation of the atmosphere under solar light.

Dr. W. H. Eccles has provided the foundation for a valid theory of ionic refraction and an explanation of the manner in which ionisation of the upper air can cause an electric ray to be bent round so as to follow more or less the earth's curvature in its path.*

Also he has considered the influence of what may be called the normal refraction of the atmosphere, and suggested some modes of variation of refractive index with height above the earth's surface which would tend to fulfil the requirements of circular propagation of the ray round the earth.†

The whole question of long-distance radiotelegraphy is, therefore, intimately bound up with fundamental questions of meteorology and atmospheric composition, and it may be well to refer briefly to our knowledge on the subject. A very valuable summary of information on it from the point of view of chemistry and spectroscopy was given by Sir James Dewar in his Presidential Address to the British Association at Belfast in 1902, and also in a Friday Evening Discourse at the Royal Institution on April 11, 1902, on "Problems of the Atmosphere." (*See* "Proc." Roy. Inst., Vol. XVII., p. 223.)

He showed that, broadly speaking, the atmosphere may be divided into two regions. First, a lower region in which winds

* *See* Dr. W. H. Eccles, "Proc." Roy. Soc., A., Vol. LXXXVII., p. 79, June, 1912.

† *See* "British Association Report," Birmingham, 1913. "Proc." Sec. G., and "The Electrician," Sept. 19th, 1913. "Atmospheric Refraction in Wireless Telegraphy."

and convective motions keep the constituents so well mixed that although there is a gradual decrease of density as we ascend the chemical composition is not very much changed, except in the percentage of water vapour and carbon dioxide.

This convective region extends to a height of 10 or 20 miles, more or less.

Above that height we pass into a second region in which convection ceases to operate and differences of density in the gaseous constituents and other more obscure actions cause a separation to take place. The denser gases oxygen and nitrogen gradually disappear at some height dependent on the temperature gradient, and above this the atmosphere consists practically of hydrogen and helium, with possibly the other more volatile atmospheric gases—neon, krypton, &c.—present in some degree.

Sir James Dewar gave at his Royal Institution lecture above mentioned diagrams representing the atmospheric composition at various heights on certain assumed temperature gradients. He stated that on any permissible temperature gradient hypothesis the atmosphere above 60 miles or so would be substantially composed of hydrogen and helium.

If we assume a homogeneous atmosphere and neglect the variation of gravity with height, and assume temperature to be constant or to decrease upwards according to some straight line law, it is a comparatively easy matter to calculate the density at any height in a column of gas in equilibrium under the forces of gravity and its own elasticity. Thus, if p_0 and q_0 are the pressure and density of the gas at the earth's surface, and p and q the same at any height h above it, and if T is the absolute temperature, g the acceleration of gravity and G the gas constant, then we have

$$p = \mathrm{GT}q. \quad \ldots \ldots \quad (1)$$

If we consider a horizontal slice of thickness, δh, of a vertical column of the gas of unit cross-section, then the equation of equilibrium of the element is

$$-\frac{dp}{dh}\delta h = gq\delta h, \quad \ldots \ldots \quad (2)$$

or

$$\frac{dq}{q} = -\frac{g}{\mathrm{GT}}dh. \quad \ldots \ldots \quad (3)$$

If R is the mean radius of the earth, and if the mean value of gravity at the surface is 980, then $g = 980\mathrm{R}^2/(\mathrm{R}+h)^2$ nearly.

Since the variation of gravity is only about 5 per cent. in a height of 100 miles we make no great error in considering it as a constant. There is more difficulty in finding a function of h to express the temperature. One of the results brought to light by modern observations with sounding balloons is that the atmosphere may be divided into two regions in which the distribution of temperature is very different. In the lower region there is a continual fall in temperature up to a height of about 10 km., or 6 miles or so, at the rate of about 6°C. per kilometre. Above this level the temperature becomes constant for a certain unascertainable range. Beyond that it is reasonable to suppose there is a second gradient of temperature falling towards the temperature of space. The lower region in which the first temperature gradient takes place, which is more or less identical with the region in which convection and winds keep the atmospheric gases well mixed, is now called the *troposphere*. The region of more constant temperature is called the *stratosphere*. The height at which the stratosphere is reached de pends on the latitude, being lower in polar than equatorial regions.

If we consider the temperature and gravity constant at various heights the integral of equation (3) is

$$q=q_0\varepsilon^{-\frac{g}{GT}h}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

showing that the density decreases in accordance with an exponential law.

If we assume a linear temperature gradient of β°C. per kilometre, so that T is a function of the height h of the form,

$$T=T_0\left(1-\frac{\beta}{T_0}h\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\beta=6$ nearly, and T_0 is the absolute temperature at the earth's surface, then equation (3) takes the form

$$\frac{dq}{q}=-\frac{g}{GT_0}\frac{dh}{(1-\beta h/T_0)}$$
$$=\frac{g}{G\beta}\frac{d(1-\beta h/T_0)}{(1-\beta h/T_0)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The integral of this is

$$q=q_0\left(1-\frac{\beta}{T_0}h\right)^{g/G\beta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

subject to the condition that h must not be greater than T_0/β.

If β is taken as the temperature fall in absolute degrees per kilometre, then since the dimensions of g/G are the inverse of a length, and since 1 km.$=10^5$ cm., we have to multiply $g/G\beta$ by 10^5 if we take the unit of length as the kilometre. Again, if we take gravity as 980 and the absolute density of air as $1/800$ at a pressure of 10^6 dynes per square centimetre, then we have

$$\frac{g}{G\beta}=\frac{980\times T_0\times 10^5}{800\times\beta\times 10^6}=\frac{49}{400}\frac{T_0}{\beta},$$

the kilometre being the unit of length.

If we take the average temperature of the air at the earth's surface to be $300°$abs.$=T_0$, and if $\beta=6°$ per kilometre then $g/G\beta=49/8=6{\cdot}125$. This is rather a high value for T_0, but we take 300 to get a round number.

We have then

$$q=q_0\left(1-\frac{h}{50}\right)^{6\cdot125} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

This last expression, which takes account of the temperature gradient, is really more simple than the exponential expression (4), but (7) is limited in range to T_0/β km., at which $q=0$, whereas (4) gives $q=0$ at an infinite distance. It is clear, however, that β is not constant throughout the whole height of the atmosphere.

Throughout the region of the troposphere the formula (7) gives a ratio for q/q_0 numerically less than that given by (4). Thus, if $h=5$ km., and if T_0 and β have the values given above, we have $g/GT=0{\cdot}1225$, and (4) gives us

$$\frac{q_0}{q}=\varepsilon^{0\cdot6125}=1{\cdot}845,$$

whereas $(7a)$ gives

$$\frac{q_0}{q}=\left(\frac{10}{9}\right)^{6\cdot125}=1{\cdot}898.$$

If $h=10$ km., then (4) gives the ratio $q_0/q=3{\cdot}404$, whilst $(7a)$ gives it $3{\cdot}920$.

This is as it should be, because lowering the temperature causes the density to degrade more rapidly.

We can next consider the atmospheric condition above the so-called convective region. In the latter the ingredients of the atmosphere are kept so well mixed by winds that the

relative proportions will not be altered with height, although the absolute density will be reduced.

If, then, we insert in (7a) the value $q_0=0{\cdot}001293$, this being the density of air at the earth's surface, it will give us the density q at any height below 10 km. fairly accurately.

Above the convective region Dalton's law will begin to assert itself, and the constituent gases will attenuate independently, each gas attenuating in density according to the law as expressed by equations (4) or (7). Since the gas constant varies inversely as the density of the gas it follows that the denser gases fade away soonest.

Thus, if we take the average temperature gradient throughout a range of 150 km. to be 2° per kilometre then in formula (7) we have to put $\beta=2$. Also $g/G\beta=q_0 14{,}700$; if we take the surface temperature as 300° abs. Therefore, we have

$$\frac{q}{q_0}=\left(1-\frac{h}{150}\right)^{q_0\,14{,}700},$$

or

$$\log_{10}\frac{q}{q_0}=q_0\,14{,}700\,\log_{10}\left(1-\frac{h}{150}\right)$$

Consider the atmospheric composition at a height of 100 km.

Then, $1-\frac{100}{150}=\frac{1}{3}$. But $\log_{10} 1=0$, and $\log_{10} 3=0{\cdot}47712$.

Hence $\log_{10}\frac{q_0}{q}=7013{\cdot}7\,q_0$.

Table I. below gives the absolute densities of various atmospheric gases at 10^6 dynes per square centimetre pressure and 0°C.

TABLE I.

Gas.	Relative density.	Absolute density.
Hydrogen	1	0·00008837
Helium	2	0·00017674
Neon	10	0·0008837
Nitrogen	14	0·0012393
Oxygen	16	0·0014107
Argon	20	0·0017674
Krypton	40·88	0·0036125
Xenon	64	0·0056428
Air	—	0·001293

If we multiply these absolute densities by 7013·7 and find the anti-logarithms we have the values of q_0/q, that is, the ratio

of the density at the surface to the density at a height h km., as follows :—

TABLE II.

Gas	q_0/q at 100 km.
Hydrogen	4·167
Helium	17·36
Neon	1,570,000
Nitrogen	475,500,000
Oxygen	8,257,000,000

The above figures show without further explanation that at a height of 100 km. or 60 miles the atmosphere consists, as stated by Sir James Dewar, substantially of hydrogen and helium. At 50 km. height there is, however, a very large proportion of nitrogen and some oxygen.

Sir James Dewar recalls an observation of Pickering on the spectrum of a meteorite just entering the earth's atmosphere, which showed strongly the lines of hydrogen and helium, as confirmatory of the above conclusions as to the composition of the atmosphere at great heights.

These two formulæ (4) and (7) are therefore practically in agreement with the formulæ of Heinrich and Ferrel, from which the charts were prepared, given in Sir James Dewar's Royal Institution Discourse (*loc. cit.*). By the summation of expressions such as (7), with various values for β, we could represent the change in density due to any assigned mode of temperature variation.

We are now in a position to consider the refractive effects of the atmosphere at various heights. In gases the refractivity $(\mu-1)$ is connected with the density q with fair accuracy by Gladstone and Dale's law $(\mu-1)/q=\text{A}=$a constant. The constant A may be called Gladstone and Dale's constant. Its value for various gases are given in Table III. We can then calculate the curvature of a ray passing through the atmosphere as follows :—

The atmosphere may approximately be considered as formed of concentric layers of gas having refractive indices which decrease as the altitude increases. Let us take the centre of the earth as origin and consider the curved path of any ray travelling to or from the earth (*see* Fig. 1). Then if a tangent is drawn at any point P to this curve, and if l is the length of the perpendicular let fall from the origin on the tangent, and μ the refractive index at the point of contact of the tangent with the curve, it can easily be shown that all along the path of the ray the product $l\mu$ is constant (*see* Parkinson's "Optics," p. 109).

Again, if r is the radius vector to that point, and if ϱ is the radius of curvature at the same point, it is easily shown that $\varrho = rdr/dl$.

Also by Gladstone and Dale's law, the quotient $(\mu-1)/q = \mathrm{A}$, where q is the density at that point, will be a constant different for each gas. Taking the expression (4) to represent the

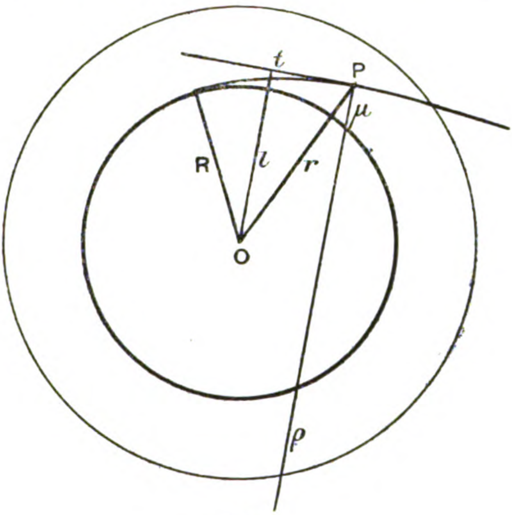


FIG. 1.

variation of density with height, we have the following three equations* :—

$$l\mu = \mathrm{C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\frac{\mu-1}{\mathrm{A}} = q = q_0 \varepsilon^{-a(r-\mathrm{R})}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\varrho = r\frac{dr}{dl}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where C and A are constants and r is the radius vector from the centre of the earth to any point on the ray at which the refractive index of the air is μ, whilst l is the length of the perpendicular from the centre of the earth on the tangent to the ray at that point. Also ϱ is the radius of curvature of the ray at that point, and R is the mean radius of the earth and

* Dr. Eccles starts with three similar equations his investigation on "Atmospheric Refraction in Wireless Telegraphy" (*loc. cit.*, Note, p. 319).

$(r-\mathrm{R})$ is the height above the earth's surface. The symbol a stands for g/GT, where g is the acceleration of gravity, G is the gas constant, and T the absolute temperature.

From (10) we have

$$\varrho = r\frac{dr}{dl} = r\frac{dr}{d\mu}\frac{d\mu}{dl}, \quad \ldots \ldots \quad (11)$$

and from (8)

$$\frac{d\mu}{dl} = -\frac{\mathrm{C}}{l^2} = -\frac{\mu^2}{\mathrm{C}}. \quad \ldots \ldots \quad (12)$$

Hence

$$\varrho = -\frac{r}{d\mu/dr} \cdot \frac{\mu^2}{\mathrm{C}}. \quad \ldots \ldots \quad (13)$$

But by (9) we have

$$\mu = 1 + \mathrm{A}q_0\varepsilon^{-a(r-\mathrm{R})}. \quad \ldots \ldots \quad (14)$$

Therefore

$$\frac{d\mu}{dr} = -a\mathrm{A}q_0\varepsilon^{-a(r-\mathrm{R})}. \quad \ldots \ldots \quad (15)$$

Hence substituting (15) in (13) we have

$$\varrho = \frac{r\varepsilon^{a(r-\mathrm{R})}}{\mathrm{C}a\mathrm{A}q_0}(1+\mathrm{A}q_0\varepsilon^{-a(r-\mathrm{R})})^2, \quad \ldots \ldots \quad (16)$$

or by (9)

$$\varrho = \frac{r}{\mathrm{C}a}\frac{\mu^2}{\mu-1} = \frac{r}{la}\frac{\mu}{\mu-1}. \quad \ldots \ldots \quad (17)$$

Now

$$a = \frac{g}{\mathrm{GT}} = \frac{gq_0}{p_0} = \frac{980q_0}{10^6}.$$

Also q_0 the density of the air at the earth's surface under a pressure of 10^6 dynes per square centimetre is nearly equal to $1/800$. Hence, $a = 98/(8\times 10^7)$.

The dimensions of a are inversely as a length. Hence, if we take the centimetre as unit of length $1/a = 8\times 10^5$ nearly. If, however, we reckon lengths or heights in kilometres then the numerical value of $a = 1/8$ nearly.

Accordingly the following simple expression gives the radius of curvature of the ray

$$\varrho = \frac{8r}{l}\frac{\mu}{\mu-1}. \quad \ldots \ldots \quad (18)$$

Consider, then, a ray starting horizontally from a point on the earth's surface. We have at that point $r = l = \mathrm{R} =$ the earth's

radius, and $\mu=1\cdot000294$. Hence, $\mu/(\mu-1)=10{,}000/3$ nearly, and the radius of curvature of the ray at starting $=80{,}000/3$ $=26{,}666$ km., or 16,000 miles, or about four times the radius of curvature of the earth.

As the point considered is taken farther from the source the radius of curvature of the ray at that point increases. For we have

$$\varrho=\frac{r}{la}\,\frac{\mu}{\mu-1}=\frac{r\mu}{la\mathrm{A}q}. \qquad (19)$$

But μ is very nearly unity, and A and a are particular constants for each gas, whilst l is never very much greater than r, and the density q continually diminishes with increasing altitude.

Hence, ϱ rapidly increases with increase of distance of the point on the ray considered, as we take it farther from the source on the earth's surface.

Since a is equal to gq_0/p_0, it is clear, if lengths are measured in kilometres, that $a=98q_0$. Also $\mu_0-1=\mathrm{A}q_0$. Accordingly, taking the source at the earth's surface, where l and r are equal, and considering the ray emitted tangentially by the earth's surface, we have

$$\varrho=\frac{\mu_0}{98\mathrm{A}}\,\frac{1}{q_0^2}. \qquad (19a)$$

But μ_0 for all gases is very nearly unity, and A has values for various gases as in Table III. below.

The constant A may be called the Gladstone and Dale constant, and q_0 is the density of the gas at 0°C. and 10^6 dynes per square centimetre pressure.

TABLE III.

Gas.	Refractive index μ_0 at 0°C. and 10^6 dynes/cm.²	$\mathrm{A}=\frac{\mu_0-1}{q_0}$.
Hydrogen	1·000138	1·56
Helium	1·000035	0·197
Neon	1·0000687	0·0777
Nitrogen	1·000300	0·242
Oxygen	1·000272	0·193
Argon	1·000285	0·161
Krypton	1·000424	0·117
Xenon	1·000693	0·123
Air	1·000293	0·227

In the formula (19a), if we substitute the values for air, viz., $\mu_0=1\cdot000293$, $\mathrm{A}=0\cdot227$ and $q_0=0\cdot001293$ we find $\varrho=27{,}000$ km. as before, or 4·1 times the earth's radius.

Again, if we suppose the atmosphere consisted wholly of

hydrogen, then inserting in formula (19a) the values $\mu_0 = 1{\cdot}000138$ A$=1{\cdot}56$ and $q_0 = 0{\cdot}00008837$, we find $\varrho = 880{,}000$ km. or 136 times the earth's radius.

If, however, the atmosphere consisted wholly of krypton, for which $\mu_0 = 1{\cdot}000424$, A$=0{\cdot}117$ and $q_0 = 0{\cdot}0036125$, we find $\varrho = 6{,}682$ km., or about the same as the earth's mean radius.

Finally, if the atmosphere consisted wholly of xenon, for which $\mu_0 = 1{\cdot}000693$, A$=0{\cdot}123$, $q_0 = 0{\cdot}005643$, then we find $\varrho = 2{,}768$ km. which is much less than the earth's radius.

On the assumption, therefore, above made as to uniform temperature we find the following remarkable result.

If the earth's atmosphere consisted wholly of krypton a ray of light sent out tangentially to the earth's surface would be refracted round the earth parallel to the surface, and never escape at all from the atmosphere. If, therefore, the atmosphere consisted entirely of krypton, wireless telegraphy right round the earth might be easily possible in consequence of this circular refraction of a tangentially emitted ray.

This curious result can be confirmed in another way. If we consider a plane wave travelling round the earth with wave front always vertical or in a radial direction then the condition for such a wave progress is that the velocity of the wave v at a height h above the earth's surface must be to its velocity v_0 at the surface as $h+\mathrm{R}$ is to R, where R is the earth's mean radius. If μ and μ_0 are the refractive indices at these places then for ray transmission round the earth parallel to the surface we must have

$$\frac{h+\mathrm{R}}{\mathrm{R}} = \frac{v}{v_0} = \frac{\mu_0}{\mu}. \qquad (20)$$

But $\mu - 1 = \mathrm{A}q$, hence

$$\frac{h}{\mathrm{R}} + 1 = \frac{\mathrm{A}q_0 + 1}{\mathrm{A}q + 1} \qquad (21)$$

or

$$\frac{h}{\mathrm{R}} = \frac{\mathrm{A}(q_0 - q)}{\mathrm{A}q + 1}. \qquad (22)$$

Differentiate this last equation with regard to h, and we have

$$\mathrm{A}q + 1 + \mathrm{A}h\frac{dq}{dh} = -\mathrm{AR}\frac{dq}{dh} \qquad (23)$$

or

$$-\frac{dq}{dh} = \frac{q + \mathrm{A}^{-1}}{h + \mathrm{R}}. \qquad (24)$$

Hence at the surface of the earth where $h=0$ and $q=q_0$ we have

$$-\frac{dq}{dh}=\frac{q+A^{-1}}{R}. \quad \ldots \ldots \quad (25)$$

But now by equation (4) we have

$$q=q_0\varepsilon^{-\frac{g}{GT}h}. \quad \ldots \ldots \quad (26)$$

Hence, differentiating (26) with regard to h, we have

$$-\frac{dq}{dh}=\frac{gq}{GT}. \quad \ldots \ldots \quad (27)$$

Therefore equating (25) and (27)

$$\frac{q+A^{-1}}{R}=\frac{gq}{GT}. \quad \ldots \ldots \quad (28)$$

Hence

$$GTq+\frac{GT}{A}=gRq. \quad \ldots \ldots \quad (29)$$

But

$$\mu-1=Aq.$$

Therefore,

$$\mu-1=\frac{1}{\frac{Rg}{GT}-1}. \quad \ldots \ldots \quad (30)$$

But $g/GT=\alpha=98A$, as already shown, and since Rg/GT is always very much greater than unity, the formula (30) is equivalent to

$$R=\frac{1}{98Aq^2}. \quad \ldots \ldots \quad (31)$$

This is practically identical with formula (19a), when $\rho=R$. Hence (31) gives us the value of q required for circular refraction. Accordingly if the terrestrial atmosphere consisted wholly of a gas having a density about twice that of air, but the same Gladstone and Dale constant A, then the radius of curvature at starting of a ray of light sent out tangentially to the earth's surface would be identical with the curvature of the earth, and the ray would follow round the earth and never leave it.

It remains to consider the modification which will be introduced into the expression (19a) for the radius of curvature of the ray if we take into account the temperature gradient.

If in place of the expression (4) for the density at various

heights, which neglects temperature gradient, we use the expression (7) which assumes a uniform temperature gradient, and employ (7) in place of (4) in obtaining the formula for the radius of the curvature of the ray it is quite easy to show that we obtain the result

$$\rho=\frac{r}{la}\frac{\mu}{\mu-1}\left(1-\frac{\beta}{T_0}\overline{r-R}\right), \quad . \quad . \quad . \quad . \quad (32)$$

where the letters have the same signification as in equations (8) to (19). Hence when $h=r-R=0$ the last expression becomes identical with (19*a*) and also with (31).

The same expression is therefore obtained in three different ways for the radius of curvature of a horizontally emitted ray at the point of emission.

There appears to be a considerable amount of evidence that many of the spectral lines in the aurora spectrum are due to the more volatile constituents in the air. The question is discussed at length in the Presidential Address and lecture of Sir James Dewar, already mentioned. If any auroral lines due to electric discharges at a height above 40 km. or 50 km. are lines of krypton or xenon then the question presents itself: How do these dense rare gases rise to such heights? A very similar anomaly is seen in the solar atmosphere. The outer portions of the sun's atmosphere, like that of the earth, are composed principally of hydrogen and helium. But calcium, which has an atomic weight 40 is also present in the chromosphere, and no answer has yet been given to the question, by what actions an element of this molecular weight can rise so high. If one may be so bold as to hazard a rash suggestion, perhaps the neon and krypton are manufactured *in situ* by atmospheric electric discharges passing through the rarefied hydrogen, in the manner suggested by recent researches of Profs. Collie and Patterson.

Another question which suggests itself for consideration is, whether the ionisation of the upper regions of the atmosphere, upon which the actual bending of radio-telegraphic waves partly depends, is more easily effected because of the presence there of these non-valent gases neon, helium, &c. The high conductivity and small dielectric strength of these gases must be connected with ease of ionisation. It might be worth while to test the facility for ionisation of different gases under ultra-violet light.

In conclusion, the effect of a variation in the earth's diameter

may be pointed out. Referring to equation (30) since Rg/GT is always large compared with unity, the equation can be written

$$Rg = \frac{GT}{\mu_0 - 1} = \frac{GT}{Aq_0}. \qquad (33)$$

But at 0°C., $GT = 10^6/q_0$. Hence for the same temperature and gas the product Rgq_0^2 is constant.

Now gravity at the surface of a planet varies as the radius, if the mean density of the planet remains constant. Hence for the same chemical constitution and the same pressure and temperature of the atmosphere the conditions for the circular refraction of a tangential ray or its following closely round the surface will be complied with if the radius of the planet varies inversely as the surface density of the atmosphere.

If the Gladstone and Dale constant A had a constant value of 0·25 then on our earth, it would be necessary for the atmosphere to have double its present density to fulfil the condition for circular refraction. Therefore, for the same density as at present it would be sufficient if the earth had double its present radius also to fulfil it.

This shows that on a large planet such as Jupiter, having also a dense atmosphere, no ray of light emitted at or near the horizontal direction at any point could escape from the atmosphere. Also it follows that, except in so far as distant vision was hindered by absorption of light, it would be possible to see objects on the surface at a much greater angular distance than is possible on our earth.

These conclusions have merely a theoretical interest, but they show how closely the possible range of long-distance radiotelegraphy is connected both with the size of the planet and the nature of the atmosphere in which it is conducted.

Having regard to the great variations which exist in planetary atmospheres it is quite possible our earth is unique in this respect, as in many others, in being perhaps the only planet on which long-distance wireless telegraphy is possible.

ABSTRACT.

In this Paper the author considers the conditions under which true atmospheric refraction would be sufficient to carry a ray of light or electromagnetic radiation sent out horizontally from any point on the earth's surface round the earth parallel to its surface. It is now generally agreed that pure diffraction is insufficient to account for all the phenomena of long-distance wireless telegraphy, but that we

have to postulate some action of the atmosphere which tends to curve the radiation round the earth. The theory of ionic refraction has been put forward, which is based on the theoretical conclusion that in ionised air the velocity of long electric waves is increased. We know as a matter of fact that the atmosphere decreases in density as we rise upwards, and this alone produces a decrease of refractive index and an increase in velocity.

The first part of the Paper is concerned with the deduction of formulæ expressing this variation of density with heights taking into account as far as possible the known temperature variation with increase of height. It is shown that at a height of 100 km. the terrestrial atmosphere must consist substantially of hydrogen and helium. An expression is then obtained for the radius of curvature at any point of a ray of light sent out horizontally from the earth's surface, and it is shown that this radius at the starting point is given by the formula $\rho=\mu_0/(98Aq_0^2)$, where μ_0 and q_0 are the refractive index and density at the surface of the earth, and A is the Gladstone and Dale constant for the gas which forms the atmosphere. From known values for various gases it is shown that for air ρ is four times the earth's radius, for hydrogen 136 times, and for krypton equal to the earth's radius. Accordingly if the terrestrial atmosphere consisted wholly of krypton a ray sent out horizontally would be refracted round the earth, and in such an atmosphere wireless telegraphy to the Antipodes would be possible. The above formulæ are deduced in three ways. It is also shown that for the same atmospheric density and constant A this circular refraction would result if the earth were twice its present diameter.

The question of atmospheric composition is then considered in the light of what is known about the auroral spectrum. The suggestion is made that perhaps the non-valent gases neon and krypton are manufactured at great atmospheric heights by electric discharges occurring in the rarefied hydrogen atmosphere. Also that by their ease of ionisation they contribute to produce the ionised layer demanded by the theories of Heaviside and Eccles to account for the actual achievements of long-distance wireless telegraphy.

Finally, it is suggested that our earth is perhaps unique in being the only planet on which such long-distance radiotelegraphy is possible.

DISCUSSION.

Mr. DUDDELL considered it was very difficult to follow what was going on in long-distance transmission. One difficulty was that it seemed probable that the heavier gases were absent from the upper atmosphere, yet we had to assume their presence either to get the refraction effect or the Eccles effect.

Dr. C. CHREE thought Prof. Fleming's Paper emphasised the importance of the field common to meteorology and wireless telegraphy. With regard to the relations between temperature, pressure and altitude in the atmosphere, aqueous vapour was an element which meteorologists had to take serious account of; its variability exercised a great influence on the results within 2 km. or 3 km. of the ground. With respect to the constitution of the upper atmosphere, more or less successful attempts had been made of late years to obtain samples by means of apparatus sent up by pilot balloons. On the theoretical side, there were Papers more

recent than the pioneer one by Sir James Dewar quoted by Dr. Fleming. In particular he would call attention to two Papers by Wegener in the "Physikalische Zeitschrift" for 1911. A fundamental point was whether the auroral spectrum did or did not connote a gas different from all hitherto isolated at the earth's surface. Wegener thought it did, and believed the unknown gas thus indicated to be a very light one, which at great heights was even more important than hydrogen. The fact that Prof. Störmer's recent photographic determination of auroral heights had in some cases supplied heights well over 300 km. was evidence that an atmosphere of some kind extended to a very great height The fact emphasised by Dr. Fleming that wireless results by day and night differed markedly certainly seemed to support strongly his contention that the upper atmosphere played a most important part in the phenomenon. A similar conclusion had been drawn in the case of the ordinary diurnal variation of the elements of terrestrial magnetism, where normally changes were much larger by day than by night, and much larger in summer than winter. It had been found, however, that while the difference between midsummer and midwinter existed in high latitudes, the difference between night and day was there much reduced, even at the equinoxes. Systematic wireless observations in the Arctic or Antarctic might throw a great deal of light upon the whole question.

Prof. G. W. O. Howe congratulated Prof. Fleming on his interesting and suggestive Paper, and pointed out that although the upper atmosphere undoubtedly had a profound effect on the transmission of electromagnetic waves over long distances, they were still uncertain to what extent long-distance radiotelegraphy would be possible without any assistance from the upper atmosphere. As successive mathematicians attacked the problem errors were discovered in the previous work and up to the present the effect of the correction seemed to have been in every case to increase the amount of energy diffracted around the globe.

Prof. C. H. Lees mentioned that in the formula $(\mu-1)/q=A$, the value assumed for μ was for sodium light, whereas the value required was for $\lambda=500$ metres.

Prof. Marchant mentioned that in some work recently published it was shown that the difference between the carrying power of signals by night and by day was very much greater in summer than in winter. He asked within what limits of accuracy Gladstone and Dale's law held.

Prof. Fleming, in reply, said that Dr. Chree's remarks were of importance, and the questions raised required to be cleared up. In reply to Dr. Lees, he said that for all gases the square root of the dielectric constant for steady voltage practically agreed with the refractive index for sodium light. It would be useful to have the known facts as to the nature of the atmosphere collected for the use of wireless investigators.

XXXIV. *Atmospheric Electricity Observations made at Kew Observatory, by* GORDON DOBSON, *B.A.*

COMMUNICATED BY DR. CHREE. RECEIVED JUNE 5, 1914.

DURING the past few years the observations of atmospheric electricity at Kew Observatory have included measurements with Prof. Ebert's apparatus for the number of ions per cubic centimetre and for the electric conductivity of the air, and also with Mr. C. T. R. Wilson's apparatus for the electric conductivity of the air and the vertical air-earth current. As some doubt had been raised as to the exact nature of the results given by this latter instrument, and also about the method of using it which had been employed at Kew, some observations were made to try to settle these points.

In the instrument designed by Prof. Ebert air is drawn through a vertical earthed tube, in the axis of which is an insulated rod connected to an electrometer and charged to about 200 volts. As the air passes through the tube ions of the opposite sign to the charge on the rod will be caught by it, and the loss of charge on this rod measures the charge received from the ions in the volume of air drawn through. Allowance is made for leakage over the insulators supporting the charged rod.

The mobility of the ions is measured by placing a short rod, charged to a low potential, in the air current before it reaches the main rod. Some ions will be caught by this rod, so that the number caught by the main rod will be smaller than before. Assuming the total number of ions to remain constant, the difference in the number caught by the main rod in the two cases will be equal to the number caught by the auxiliary rod. Since the proportion of ions which are caught on the auxiliary rod depends on its potential and the mobility of the ions the mobility of the ions can thus be obtained.

With this apparatus we can, therefore, measure both the charge per unit volume of air and also the mobility of the ions. The conductivity of the air depends on these two values alone. If λ_+ be the conductivity due to the positive ions, E_+ the charge on them per unit volume, and u_+ their mobility, we have

$$\lambda_+ = E_+ u_+,$$

and, similarly, for the negative ions

$$\lambda_- = E_- u_-.$$

The total conductivity of the air is given by

$$\lambda=\lambda_{+}+\lambda_{-}.$$

Unfortunately, the average conductivity of the air at Kew is very small, so that, with the electrometer which is usually fitted to the Ebert apparatus, it frequently happens that the conductivity is too small to be measured by this method. Also, since the value for the mobility depends on the difference between the results of two observations, and, since the number of ions is always changing, it not infrequently happens that the conductivity comes out negative.

The conductivities obtained by the Ebert apparatus at Kew are, therefore, only very approximate values, but the mean of a large number of observations may be fairly reliable. Table I. shows how often zero or negative values are given by this instrument at Kew :—

TABLE I.

——	Year.	Winter.	Summer.
Total number of observations.........	107	41	66
Number of cases in which zero or negative values were obtained for both $u+$ and $u-$	14	12	2
Ditto for $u+$	25	17	8
Ditto for $u-$	43	25	17
Ditto for either $u+$ or $u-$	54	31	23

In Mr. Wilson's apparatus * the electric current is measured which enters a small test plate, freely exposed to the earth's field and kept at zero potential. This test plate is surrounded by a guard-ring and may be kept at any desired potential by a charged sliding condenser or "compensator." The potential gradient immediately above the test plate can be obtained by measuring the charge induced on it when freely exposed. If the instrument could be used so that the test plate were level with, and practically continuous with the surface of flat open ground, then both the potential gradient above it and the current entering it would be the same as for any other part of the surrounding ground.

In practice it is not convenient to use the instrument under these conditions, and it is, therefore, placed on a stand. In the observations taken at Kew the test plate has always been approximately 135 cm. above the ground. Under these con-

* Cambridge Phil. Soc. "Proc.," Vol. XIII., pp. 184 and 364.

ditions the potential gradient above the test plate is very greatly increased and, if the conductivity of the air is everywhere the same the current entering the test plate will be increased in the same proportion. If the test plate were always used at the same height above the ground it would be possible to obtain a factor showing how much the potential gradient above the test plate was greater than that above the surface of the ground. If, therefore, we assume that the conductivity is everywhere the same we can apply this factor to the measured current entering unit area of the test plate and obtain the current entering the ground.

For various reasons this method has not been used at Kew; and advantage has been taken of the fact that the potential gradient immediately above the test plate can easily be measured, and the conductivity has been calculated from the current entering the test plate and the potential gradient immediately above it. This method gives a result which can be directly compared with the conductivity as given by the Ebert apparatus. If the air-earth current be required it can be obtained from this conductivity and the potential gradient given by the recording electrograph.

It has been assumed by Dr. Lutz * that, since in this apparatus the current measured is due to the positive ions, it will measure only the part of the total conductivity denoted above by λ_+. Assuming λ_+ and λ_- to be equal—a result which seems to be approximately true at many stations, though, as will be seen from tables 10 and 11, it is not true at Kew—it is supposed that Wilson's apparatus measures about half the conductivity and half the current. It has further been suggested that when this instrument is used on a stand, since the potential gradient above the test plate is thereby greatly increased, conditions approaching saturation may be obtained, and the value given for the conductivity be, therefore, too small.

It seems probable that if the Wilson apparatus could be used with its test plate practically continuous with the general surface of the ground all these doubtful points would disappear. The total current passing through the air must be equal to that entering the ground, and the potential gradient could not be changed from its normal value, so that no saturation effects could occur. It was not found practicable to use

* Luftelektrische Messungen, Munich, 1905-10.

the instrument in a pit with its test plate level with the ground; so observations were made with an entirely different test plate. This was made of wood covered with tinfoil, and insulated on sulphur. The test plate was 29 cm. square, and was surrounded by a guard-ring 15 cm. broad. The whole was in the form of a very shallow box, 4 cm. deep. Connection could be made with the test plate through one side of the box. The box was sunk into the ground, so that the test plate and guard-ring were level with the surrounding ground. The test plate was connected to the electrometer of the ordinary Wilson apparatus, which was placed on a low stand some little distance away. The connecting wire was carried on sulphur insulators inside an earthed metal tube, as it was necessary to shield it from the earth's field. A metal cover could be placed over the test plate, and the apparatus was used in the same way as that

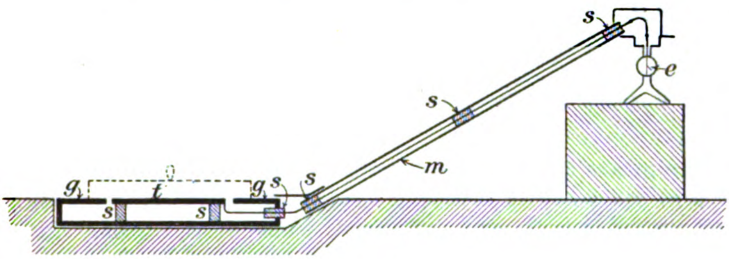


FIG. 1.

having the usual small test plate. The general arrangement of the apparatus is shown in Fig. 1.

During the spring of 1912 some observations were made with this apparatus, alternate measurements being made using this large test plate, and using the instrument with its usual small test plate, and placed on the stand which is ordinarily used. Owing to the constant variation of the electrical conditions, it was found that a very long series of observations would be necessary to get satisfactory results by this method. In the summer a second Wilson apparatus was obtained, and the experiments were begun again in September, simultaneous observations being made with the two instruments, one connected to the large test plate at ground level and the other used in the normal way on the stand. When the large test plate is used, the deflections of the electrometer are smaller than when the instrument is used in the ordinary way, so that

the readings are less accurate. Every set of observations consists of six or more simultaneous observations with the two instruments. Both instruments in turn were used in connection with the large test plate.

At first, observations were made with the two instruments on two stands a short distance apart, both being used in the ordinary way with their usual test plates. When the insulation was satisfactory on each instrument the results were in good agreement; the means of four sets, each of six simultaneous observations taken on four different days being as follows :—

TABLE II.

Conductivity.	
Instrument "A."	Instrument "B."
$0{\cdot}51 \times 10^{-25}$ E.M.U.	$0{\cdot}50 \times 10^{-25}$ E.M.U.
0·44 ,,	0·52 ,,
0·75 ,,	0·76 ,,
0·47 ,,	0·46 ,,
0·54 ,, (Mean)	0·56 ,, (Mean)

The means of each series of observations probably agree as well as could be expected, but the individual observations are not always in such good agreement. This may be partly due to the fact that the observations are not strictly simultaneous, one instrument being read first and the other as soon afterwards as possible. As a typical set of observations, the first of the above table is given below :—

TABLE III.

Conductivity.		
——	Instrument "A."	Instrument "B."
Obs. No. 1.........	$0{\cdot}55 \times 10^{-25}$ E.M.U.	$0{\cdot}71 \times 10^{-25}$ E.M.U.
,, 2.........	0·66 ,,	0·55 ,,
,, 3.........	0·46 ,,	0·45 ,,
,, 4.........	0·52 ,,	0·46 ,,
,, 5.........	0·45 ,,	0·46 ,,
,, 6.........	0·45 ,,	0·40 ,,
Means	0·51 ,,	0·50 ,,

The results of the simultaneous observations with two Wilson instruments, one being connected with the large test plate at

ground level, and the other used on the stand in the ordinary way, are given in Table IV. :—

TABLE IV.

Number of observations.	Instrument which was used with large test plate.	Conductivity.		Ratio of conductivity from Large test plate / Small test plate.
		Large test plate at ground level.	Usual test plate instrument on stand.	
7	A	0·28 × 10⁻²⁵ E.M.U	0·28 × 10⁻²⁵ E.M.U	1·00
5	B	0·25 ,,	0·23 ,,	1·09
6	B	0·37 ,,	0·31 ,,	1·19
6	A	0·39 ,,	0·25 ,,	1·56
6	B	0·55 ,,	0·47 ,,	1·17
5	B	0·48 ,,	0·38 ,,	1·26
Means		0·39 ,,	0·32 ,,	1·22

It appears that the values for the conductivity given by the apparatus with the large test plate at ground level are some 20 per cent. larger than those given by the Wilson apparatus when used in the normal way on the stand. Tables II. and III. show the agreement between two similar instruments, and it could not be expected that the accuracy of the readings when using the large test plate would be quite so great as when using the instrument in the ordinary way, since the observations are more difficult to make. There is, however, no reason to suppose that there is any systematic error in these observations ; so that the means should be trustworthy. The following table shows how a typical set of observations (the fifth in the above table) agree among themselves :—

TABLE V.

	Conductivity.	
Observation No.	Large test plate.	Usual test plate.
1	$0{\cdot}62 \times 10^{-25}$ E.M.U	$0{\cdot}41 \times 10^{-25}$ E.M.U.
2	0·60 ,,	0·43 ,,
3	0·50 ,,	0·54 ,,
4	0·62 ,	0·56 ,,
5	0·51 ,,	0·42 ,,
6	0·43 ,,	0·50 ,,
Means	0 55 ,,	0·47 ,,

Table VI. gives the values obtained, in the spring of 1912, for the conductivity, using the large, and usual, test plates.

At this time, only one Wilson instrument was available, and observations were, therefore, taken alternately, first with the instrument on its usual stand with its ordinary test plate, and then with the electrometer connected to the large test plate at ground level. These figures are, naturally, not so accurate as the results obtained by taking simultaneous observations with two instruments. They show, however, decidedly higher values when the large test plate is used :—

TABLE VI.

Number of pairs of observations.	Conductivity.		Ratio of conductivity $\frac{\text{Large test plate}}{\text{Small test plate}}$.
	Large test plate at ground level.	Usual test plate instrument on stand.	
3	$0 \cdot 20 \times 10^{-25}$ E.M.U.	$0 \cdot 11 \times 10^{-25}$ E.M.U.	1·82
3	0·15 ,,	0·11 ,,	1·36
4	0·23 ,,	0·17 ,,	1·35
3	0·16 ,,	0·22 ,,	0·73
4	0·31 ,,	0·20 ,,	1·55
4	0·40 ,,	0·18 ,,	2·22
4	0·15 ,,	0·18 ,,	0·84
4	0·36 ,,	0·25 ,,	1·44
5	0·32 ,,	0·31 ,,	1·03
Means	0·25 ,,	0·19 ,,	1·32

It would have been more satisfactory if more observations could have been made, but, unfortunately, the weather during the summer of 1912 was not good for these experiments. Each observation lasts 10 minutes, and the time occupied in getting a set of six observations, including setting up and taking down the apparatus, is over two hours. Although the accuracy of the observations is not so great as could be wished for, yet it seems fairly certain that, when the conductivity is measured by using the large test plate at ground level, higher values are obtained than when the instrument is used in the ordinary way on the stand. The difference seems to be about 20 per cent.

A few sets of observations were made, in which both instruments were used in the normal way with their ordinary test plates, but one instrument stood on the ground while the other stood on the usual stand. In each set of observations half the observations were made with one instrument on the stand

and half with the other on the stand. The following values of conductivity were obtained :—

TABLE VII.

Number of observations.	Conductivity.		Ratio
	Instrument on ground.	Instrument on stand.	$\frac{\text{Ground}}{\text{Stand.}}$
12	$0{\cdot}365 \times 10^{-25}$ E.M.U.	$0{\cdot}330 \times 10^{-25}$ E.M.U.	1·11
12	0·255 ,,	0·225 ,,	1·13
9	0·275 ,,	0·240 ,,	1·15
Means ...	0·295 ,,	0·265 ,,	1·12

When the instrument stands on the ground the test plate is about 35 cm. above the ground. The observations show very good agreement among themselves, and also show the increased value of the conductivity when measured near the ground.

In the cases when the large test plate was used at ground level, or when the ordinary instrument stands on the ground, the potential gradient is much less, and the equipotential surfaces above the test plate are much less curved than when the instrument is used on the stand. It, therefore, seemed of interest to put a very large guard-ring round the ordinary test plate, using the instrument on the ordinary stand. In this way the potential gradient immediately above the test plate is reduced, and the equipotential surfaces must be nearly flat immediately above it. For this purpose a wooden guard-ring, 30 in. square and covered with tinfoil, was placed on the ordinary guard ring. Simultaneous observations were made with two instruments, both on stands of the usual height, one instrument having the large guard-ring and one having only its ordinary small guard-ring. The large guard-ring was half the time on one instrument and half on the other. The values for the conductivity obtained in the two ways are given in Table VIII. :—

TABLE VIII.

Number of observations.	Conductivity.		Ratio
	Large guard-ring.	Small-guard ring.	$\frac{\text{Large}}{\text{Small}}$
10	$0{\cdot}390 \times 10^{-25}$ E.M.U.	$0{\cdot}295 \times 10^{-25}$ E.M.U.	1·31
7	0·280 ,,	0·245 ,,	1·13
12	0·275 ,,	0·250 ,,	1·10
Means	0·315 ,,	0·265 ,,	1·19

The observations are not in very good agreement, but they show that the value found for the conductivity is certainly greater when a large guard-ring is used than when the ordinary small guard-ring is used.

If there were any real difference between the conductivity of the air near the ground and that a few metres above it we should expect that there would be a change in the number of ions, since it is hardly likely that the mobilities would be different in the two places. A difference in the number of ions in the air near the ground and in that a few metres higher might be caused either by the migration of ions under the electric potential gradient or by highly ionised air coming out of the ground. It seems probable that, with even a moderate wind, the air would be stirred up sufficiently to prevent the formation of any layer of different ionisation near the ground. At Dr. Chree's suggestion, several observations of the number of ions were made with the Ebert apparatus on calm days, taking air from the surface of the ground and from about 2 metres above it. The results are given in Table IX., the last column of which table gives the direction and estimated Beaufort force of the wind at the time when each set of observations were taken :—

TABLE IX.

Positive charge per c.c. E.M.U. $\times 10^{20}$.		Negative charge per c.c. E.M.U. $\times 10^{20}$.		Wind
Ground.	2 metres.	Ground.	2 metres.	
150	180	180	140	S. 0 to 1
380	360	...	...	W. 1 to 2
...	...	340	270	W. 1
...	...	390	420	Calm
...	...	320	330	S. 1
400	400	300	330	S. 1
440	330	330	360	S.W. 1
140	180	180	180	N.W. 1
302	290	291	290	Means.

Although the observations were made on days with very little wind no appreciable difference in the ionisation of the air from near the ground and that from 2 metres above it is shown.

It is interesting to compare the values of the conductivity given by the Wilson apparatus with those from observations made at the same time with the Ebert apparatus. Unfortunately, as stated above, owing to the low average conduc-

tivity at Kew, the individual values given by the Ebert apparatus are not satisfactory. But, by taking a large number of observations for the conductivity by the Ebert and by the Wilson instruments we should be able to get a fairly satisfactory comparison from the means. Table X. gives values obtained from the Ebert apparatus for λ_+, λ_- and $(\lambda_+ + \lambda_-)$, and also the conductivity given by the Wilson apparatus at the same time. All days were excluded from this table on which the value of either λ_+ or λ_- was zero or negative. The Wilson instrument was used on its stand, with its ordinary guard-ring, in these observations. The values obtained from it, therefore, require a correction of 20 per cent.; this correction has been applied only to the final mean :—

TABLE X.

Conductivity in E.M.U. $\times 10^{25}$

Date.	Ebert.			Wilson.	Date.	Ebert.			Wilson.
	λ_+	λ_-	$\lambda_+ + \lambda_-$			λ_+	λ_-	$\lambda_+ + \lambda_-$	
1912									
Feb. 27	0·26	0·21	0·47	0·36	June 19	0·67	0·28	0·95	0·75
Apr. 19	0·52	0·28	0·80	0·72	21	0·70	0·50	1·20	0·66
22	0·22	0·27	0·49	0·37	27	0·44	0·20	0·64	0·58
25	0·27	0·14	0·41	0·39	July 8	0·75	0·47	1·22	0·51
May 2	0·22	0·35	0·57	0·47	10	0·96	0·43	1·39	0·81
6	0·41	0·48	0·89	0·70	11	0·27	0·67	0·94	0·77
15	0·51	0·52	1·03	0·73	12	0·37	0·30	0·67	0·98
22	0·52	0·32	0·84	0·55	18	0·68	0·34	1·02	0·45
24	0·42	0·14	0·56	0·44	23	0·23	0·54	0·77	0·65
June 11	0·62	0·20	0·82	0·34	26	0·78	0·22	1·00	1·02
17	0·45	0·55	1·00	0·51	30	0·80	0·41	1·21	0·66
18	0·54	0·44	0·98	0·44	Aug. 1	0·20	0·68	0·88	0·56
					Sept. 30	0·28	0·41	0·69	0·49
					Means	0·484	0·374	0·858	0·593
					Wilson value + 20% =				0·712

The mean from this table gives a value for λ_+ from the Ebert apparatus, which is about 35 per cent. below the conductivity (corrected by 20 per cent.) given by the Wilson apparatus; while the value for $(\lambda_+ + \lambda_-)$ is somewhat above the corrected Wilson value. Owing to the inaccuracy of the reading with the Ebert apparatus we may suppose that the values given by it are as often too high as too low. Now, many of the readings which are too low will be zero or negative and are not included in this table, while all the readings which are too high are included. This will make the means from the Ebert apparatus

somewhat too high. It is difficult to know how much the values from the Ebert apparatus in Table X. are increased by selecting only those days with positive values for λ_+ and λ_-. If we divide the sum of the measurable conductivities by the total number of days on which observations were made we shall get a value which was too low, since it is highly improbable that the conductivity was ever really zero. If, however, the days with zero or negative values are few it would seem that the mean thus obtained should be only slightly below the true value. In Table XI. the observations on 56 days are given. These are all the observations made during the seven months of the year when the conductivity is greatest. On

TABLE XI.

Conductivity in E.M.U. $\times 10^{-25}$

Date.	Ebert.			Wilson.	Date.	Ebert.			Wilson.
	λ_+	λ_-	$\lambda_+ + \lambda_-$			λ_+	λ_-	$\lambda_+ + \lambda_-$	
1912.									
Mar. 14	0·06	—	0·06	0·31	May 28	—	0·28	0·28	0·52
20	0·44	—	0·44	0·24	29	0·49	—	0·49	0·63
22	0·30	—	0·30	0·30	31	0·13	0·07	0·20	0·51
27	0·26	0·21	0·47	0·24	June 5	—	0·07	0·07	0·50
28	—	0·35	0·35	0·45	10	0·62	0·20	0·82	0·34
29	0·21	—	0·21	0·28	13	0·35	—	0·35	0·21
Apr. 2	—	—	—	0·21	14	0·20	—	0·20	0·38
3	—	—	—	0·29	17	0·45	0·55	1·00	0·51
11	0·03	—	0·03	0·15	18	0·54	0·44	0·98	0·44
12	0·24	0·04	0·28	0·26	19	0·67	0·28	0·95	0·75
15	—	—	—	0·24	20	0·07	0·48	0·55	0·47
16	0·30	—	0·30	0·26	21	0·70	0·50	1·20	0·66
18	0·38	—	0·38	0·47	27	0·44	0·20	0·64	0·58
19	0·52	0·28	0·80	0·72	July 6	—	0·46	0·46	0·24
22	0·22	0·27	0·49	0·37	8	0·75	0·48	1·23	0·51
23	0·23	—	0·23	0·44	10	0·96	0·43	1·39	0·81
24	0·06	—	0·06	0·34	11	0·27	0·64	0·91	0·77
25	0·34	0·14	0·48	0·39	12	0·37	0·30	0·67	0·98
30	—	0·14	0·14	0·56	18	0·68	0·34	1·02	0·45
May 1	0·41	—	0·41	0·48	23	0·23	0·53	0·76	0·65
2	0·21	0·35	0·56	0·47	25	0·56	—	0·56	0·79
3	0·43	—	0·43	0·42	26	0·78	0·25	1·03	1·03
6	0·41	0·48	0·89	0·70	30	0·80	0·41	1·21	0·66
15	0·51	0·52	1·03	0·73	Aug. 1	0·20	0·07	0·27	0·56
16	0·07	0·14	0·21	0·55	Sept. 18	0·86	—	0·86	0·36
22	0·52	0·32	0·84	0·55	19	0·28	—	0·28	0·13
23	0·14	—	0·14	0·58	20	0·39	—	0·39	0·29
24	0·42	0·14	0·56	0·44	30	0·28	0·41	0·69	0·40
					Means	0·336	0·192	0·538	0·475
					Wilson value + 20% =				0·570

most of the days both λ_+ and λ_- had positive values. The mean gives a value for λ_+ which is very much below the corrected value from the Wilson apparatus; but the sum of λ_+ and λ_- is only very slightly below the Wilson value. If we take only the observations made in the summer months there are very few observations when either λ_+ ot λ_- had either zero or negative values. The mean for $(\lambda_+ + \lambda_-)$ for the three summer months is $0{\cdot}873 \times 10^{-25}$ E.M.U.; while that for λ_+ is $0{\cdot}498 \times 10^{-25}$ E.M.U.; and the corresponding value from the Wilson apparatus (corrected by 20 per cent.) is $0{\cdot}752 \times 10^{-25}$ E.M.U. A set of days has also been selected on which the conductivity as measured by the Wilson apparatus was high, those days with values above $0{\cdot}40 \times 10^{-25}$ E.M.U. being used. These days gave a mean for $(\lambda_+ + \lambda_-)$ equal to $0{\cdot}659 \times 10^{-25}$ E.M.U., that for λ_+, $0{\cdot}389 \times 10^{-25}$ E.M.U., and the corrected value from the Wilson apparatus $0{\cdot}712 \times 10^{-25}$ E.M.U.

There seems to be little doubt, therefore, that the corrected conductivity obtained from the Wilson apparatus corresponds with $(\lambda_+ + \lambda_-)$ as measured by the Ebert apparatus. This is, indeed, what we should have expected from the results with the large test plate at ground level.

In using the Ebert apparatus, all the small fast-moving ions will be caught on the main rod, but some of the large, slow-moving ions will also be caught. If we assume the figures given by Langevin—namely, that the number of the large ions is about 50 times that of the small ions, while their mobility is about 1,000 times less—then, as the Ebert apparatus is used at Kew, about 20 per cent. of the value found for the number of ions per unit volume of air, is due to large ions. The value of the mobility, however, is decreased nearly 20 per cent. below the true mobility of the small ions by the presence of the large ions; the effect on the measured conductivity being that it is about 1·7 per cent. greater than if no large ions were present. Under the same conditions about 1·7 per cent. of the current entering the test plate of the Wilson apparatus will be due to the large ions, so that the conductivity, as measured by it, is also 1·7 per cent. greater than if no large ions were present.

Mr. Wilson has described * experiments to test whether placing grass on the test plate of his apparatus had any effect on the results. In these experiments a small piece of turf was placed on the ordinary small test plate of the apparatus,

* "Proc." Roy. Soc., Vol LXXX., p. 537.

which stood on a low stand. The observations showed that the grass had no appreciable effect. No attempt, however, was made to measure the current which entered a large test plate at ground level. The experiments described above do not, therefore, contradict the results of these observations, but seem to point to the fact that the assumption is not quite correct that "if the ratio of the current to the charge on the exposed surface were found to be the same in the two cases" (with and without grass on the test plate) "it might with some confidence be assumed that the same ratio would hold for the current to the charge per unit area of the ground."

Some observations were begun to measure the conductivity of the air, using the large test plate, both it and its guard-ring being covered with turf, and sunk into the ground, so that the turf was practically continuous with that surrounding it. The observations in this way were considerably more difficult to make, and it was thought better to make observations at first with the large test plate not covered with turf. These observations took all the available time during the fine weather of the summer of 1912, so that it has not been possible to make further observations with the large test plate covered with turf. The results of Mr. Wilson's experiments with turf on the ordinary test plate of the instrument would lead us to expect, however, that there would be no difference if the large test plate were covered with turf.

ABSTRACT.

The Paper gives an account of experiments made to determine the accuracy of the results obtained with the apparatus designed by Mr. C. T. R. Wilson for measuring the electric conductivity of the air, and the electric current passing from the air to the earth. Observations were made (1) using the standard instrument on a stand according to the usual practice and (2) using an experimental apparatus level with the ground, which was assumed to give correct results. It was found that it was necessary to apply a small correction to the results obtained with the standard apparatus when used in the ordinary way.

A comparison was also made of the electric conductivity of the air as measured by Mr. Wilson's apparatus and that designed by Prof. Ebert. The results given by this latter apparatus appear to be too inaccurate to allow any satisfactory conclusions to be drawn.

DISCUSSION.

Prof. C. H. Lees said it was important to settle which was the better of the two methods. He thought the large Langevin ions were at the root of the trouble.

Dr. Chree said the large ions considerably affected the charge per unit volume, but he did not think they would affect the value of the earth-air current.

XXXV. *Thermal and Electrical Conductivities of Some of the Rarer Metals and Alloys.* By THOMAS BARRATT, *A.R.C.S., B.Sc.*

RECEIVED JUNE 3, 1914.

I. *Introduction.*

THE following investigation was undertaken, not with the idea necessarily of giving more accurate determinations of the quantities involved than those given in the most recent researches,* but in the first place because the method can be employed for many of the rarer metals and alloys, and, secondly, because the mathematics, and especially the formulæ used, are exceedingly simple.

In most of the investigations on thermal conductivity one end of a rod or wire enclosed in a water jacket,† or open to the air,‡ is heated, and measurements taken of the amount of heat supplied to one end of the specimen, and of two or more temperatures at various points along its length. For example, Lees measured the heat given (electrically) and two temperatures; while Jäger and Diesselhorst measured three temperatures at fixed points along the rod. In each of the two researches just mentioned one end of the rod was securely fastened to a copper block forming part of the enclosure. The presence of the thermometers necessary for measuring the temperatures at various points along the rod involved elaborate precautions and corrections, which in the present research are avoided, the specimen being completely bare from end to end. Another type of arrangement of apparatus is that introduced by F. A. Schulze,§ and employed by Grüneisen || and others. In this method, which may be termed the "variable temperature" method, the rod is allowed to assume a constant temperature, usually that of the enclosure, and then one end is suddenly cooled—*e.g.*, by a stream of cold

* C. H. Lees, Bakerian Lecture, Roy. Soc., 1908; W. Jäger and H. Diesselhorst, "Abh. d. Phys., Tech. Reich.," 3, 269, 1900; L. Lorenz, "Ann. d. Physik," XIII., p 422, 1881.

† Lees, Jäger and Diesselhorst, *loc. cit.*; Wiedemann and Franz, "Ann. d. Phys.," LXXXIX., p. 497, 1853.

‡ Forbes, Edin. "Trans.," XXIV., 73, 1867. R. W. Stewart, "Proc.," Roy. Soc. Lond., LIII., 151, 1893; Biot, "Traite de Phys.," 1816.

§ F. A. Schulze, "Wied. Ann.," LXVI., 2, p. 207, 1898.

|| E. Grüneisen, "Ann. d. Physik.," III., 1, pp. 43-74, Sept., 1900.

water. The temperature is then observed at regular intervals, a few centimetres from this end, by means of a thermo-junction. Unfortunately, the results obtained by this method do not, as a rule, agree at all well with those given by the "stationary temperature" method. For example, Schulze obtained as a mean result for copper the value 0·6108, which is 30 per cent. lower than the present accepted value. It has been pointed out by Schaufelberger * that the discordant results given by Schulze's method are, partly at any rate, due to the unjustifiable assumption that the end of the heated bar acquires the temperature of the water by which it is cooled. This assumption will easily lead to an error of 10 per cent.

Still another method of procedure has recently been employed by Angell † at the suggestion of C. E. Mendenhall. A rod of the metal was electrically heated, and the temperature measured at the centre and at the circumference of the rod by means of platinum and platinum-rhodium couples. The method was used principally for the determination of thermal conductivities at high temperatures. Gray ‡ used specimens (as in the present research) in the form of thin wires, but the heat was supplied by steam, and the amount of heat flowing along the wire was measured by the rise of temperature of a copper sphere of known thermal capacity suspended from the lower end of the wire. His method was a combination of the "stationary" and "variable" types. His results were given as at a temperature 10°C. to 97°C.

Reference is made to some of the results obtained by the above-mentioned experimenters in the tables given towards the end of the present Paper.

An exhaustive account of researches on thermal conductivity up to 1906 is given in Winkelmann's "Handbuch der Physik," volume "Wärme"; also in a French edition of Chwolson's "Physics," which has been recently published. At present there appears to be no English publication giving anything like an adequate account of the subject.

II. *Apparatus and Measurements.*

The method employed in the present research is a new one, of the "stationary temperature" type. Measurements are

* W. Schaufelberger, "Ann. d. Physik.," VII., 3, pp. 589-630, 1902.
† M. F. Angell, "Phys. Rev.," XXXIII., pp. 421-432, Nov., 1911.
‡ Gray, "Proc." Roy. Soc. London, LVI., 205, 1894.

made of (*a*) the heat supplied electrically to one end of the metal; (*b*) the temperatures of this end and of the enclosure. The method is particularly applicable to metals, in the form of wires, which cannot easily be obtained in bulk, owing to scarcity or cost. Such metals as tungsten, molybdenum, iridium, rhodium and tantalum have, therefore, been tested. Platinum-iridium and platinum-rhodium have also been included, as a knowledge of their thermal conductivity is likely to be useful to experimenters employing them as thermo-electrical thermometers. The elasticity of the method is illustrated by its extension to a non-metal—viz., graphite—and experiments are already in progress with a modified form of apparatus, on the measurement of the thermal conductivity of other more or less badly-conducting substances, including, if possible, liquids. Experiments have been carried out at laboratory temperatures and at 100°C., and the method could easily be extended to temperatures above or below those mentioned.

It will be shown in the next section that the thermal conductivity k is given by the equation,

$$k=\frac{H^2}{pqhV^2}\coth^2 al,$$

where H is the amount of heat flowing into the wire at the hot end; p is the perimeter, q the cross-sectional area of the wire; V the excess of temperature of the hot end over that of the enclosure; h the amount of heat lost per second from 1 sq. cm. of the wire when its temperature is 1°C. above that of its surroundings; and $a=\sqrt{\frac{hp}{qk}}$. When l is great the equation reduces to the very simple form:—

$$k=\frac{H^2}{pqhV^2}$$

The wires employed were nearly all of diameter about 1 mm., and of lengths ranging from 30 cm. upwards, so that either the simple form of equation could be employed or the factor $\coth^2 al$ was an exceedingly small correction.

IIa. *Measurement of H and V.*

Fig. 1 illustrates the method employed in the determination of H, the amount of heat flowing into the specimen at its

hot end, and of V, the excess of temperature of this end above that of the enclosure. The specimen AB was "coppered" at the end A by electrolytic deposition, and this end carefully filed until it fitted tightly into a slightly conical hole, 1·3 mm. diameter at its widest part, bored in a small copper cylinder of length 9 mm. and diameter 6 mm. at the end A. This cylinder was enclosed by a hollow cylinder, AC, made of thin copper, and of length 4 cm. Round the latter was wound just 3 metres of single silk-covered pure platinum wire, of gauge 36, whose resistance at 17°C. was 24·25 ohms. Through this platinum coil a measured current, C, was passed. The platinum wire coil was covered with fine silk paper, thinly coated with shellac varnish.

The E.M.F. (E) at the ends of this wire was carefully measured by means of a Rayleigh potentiometer, every coil of which has been calibrated in terms of a standard resistance. The current C was also measured at the same time and in the

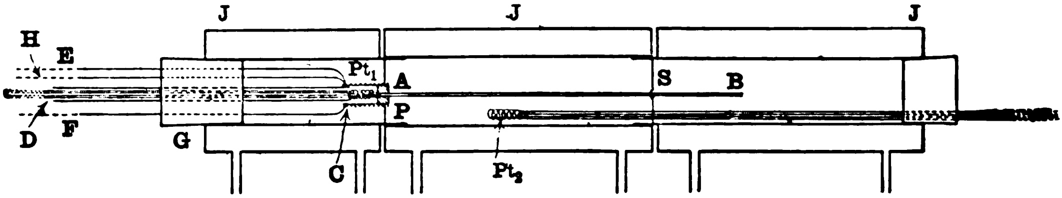


Fig. 1.

same way by obtaining the E.M.F. at the ends of a standard resistance coil O (Fig. 3), through which the current also passed. Thin silk-covered copper wires E and F, of gauge 30, joined to the ends of the platinum coil round AC were brought through the cork G (Fig. 1), as was also a third wire, H, of platinoid No. 30, to serve as potential lead. Inside the cylinder AC was a platinum thermometer, Pt_1, which, in connection with a Callendar-Griffiths bridge, G (Fig. 3), gave the temterature of the hot end A of the wire. Another similar thermometer, Pt_2, indicated the temperature of the enclosure. This consisted of a brass water-jacket, JJJ, in three parts, of inner diameter 9 cm., and total length 60 cm., enclosed also in cotton wool. The corresponding readings of the two platinum thermometers were frequently verified. Each was provided with compensating leads exactly similar to those in connection with the corresponding thermometer, and lying

side by side with them. The four leads from Pt_1 of thin double silk-covered copper wire, gauge 30, were carried through a glass tube CD of length 25 cm., which also served to keep the end A of the wire in the middle of the water-jacket. The specimen could be taken out of the copper cylinder, or replaced,

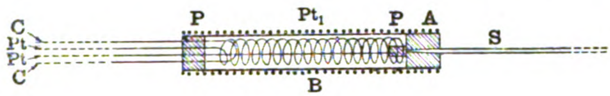


FIG. 2.

in a few seconds. It was held in position in the centre of the water-jacket by a very thin silk thread at S.

Fig. 2 illustrates on a larger scale the relative positions of the specimen wire S, copper cylinders A and B, platinum

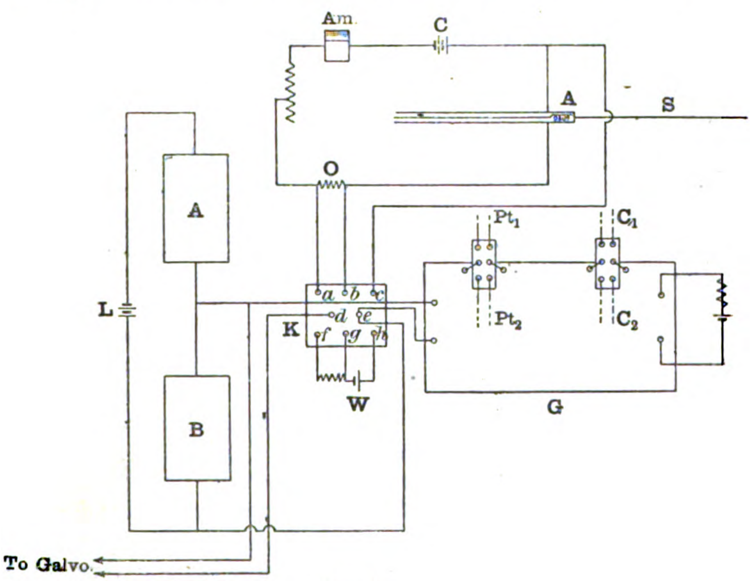


FIG. 3.

coil PP, platinum thermometer Pt_1 and compensating leads CC.

The electrical connections are shown in Fig. 3, G being the Callendar-Griffiths bridge, AB the Rayleigh potentiometer, O the standard 1 ohm coil, and K a key to enable one to com-

pare the current C and E.M.F. E with a Weston cell, W. The arrangement allows one to use the same galvanometer for all purposes. E is obtained in terms of W by connecting *bd*, *ce*, and then *fd* (or *gd*), *eh*. C is obtained in terms of W by joining *ad*, *be*, and then *fd* (or *gd*), *eh*.

The excess of temperature V of the hot end of the wire over that of the enclosure was given by the readings of Pt_1 and Pt_2. The latter thermometer also gave the actual temperature of the enclosure. For the measurement of H, the heat flowing in at the hot end of the wire, the latter was adjusted with this end in the copper block, and the amount of heat CE/J (where CE is in "Joules" and $J \doteq 4{\cdot}185$) that was required to raise the temperature of the copper cylinder V° above that of the jacket was measured, ample time being given after the current was switched on to allow temperature conditions to become steady. The specimen was then removed. It was found that a considerably smaller amount of heat, C′E′/J, was now required to maintain the same temperature difference, V°, between the end A of the wire and the enclosure. The value of the expression (CE—C′E′)/J gives the amount of heat H which is given to the wire, and is lost by convection and radiation from the sides and the other end of the wire. It is worthy of notice that the heat lost from the copper cup and the coil surrounding it is the same whether the specimen is in place or not (apart, that is, from the heat that is given to the wire), as the temperature difference between the coil and its surroundings is maintained the same in both cases. Thus, all the heat given by the expression (CE—C′E′)/J flows into the specimen, and is transferred thence to the water jacket. The method, therefore, has the advantage of being a null one. In addition to this, it may be mentioned that no corrections are necessary (as in other methods) for loss of heat from thermometer leads, which would also involve a distortion of the lines of flow of heat. The wire is entirely uncovered, all the necessary temperatures being measured at points quite apart from the metal itself. Finally, the experiment takes a very short time once the conditions have become steady, and, owing to the very simple formula, the calculations involve very little time or labour.

III. *Theory of the Experiment.*

When a thin rod of cross-section q and perimeter p is heated at one end, and loses heat from its surface and the other end, the

excess of temperature v at a distance, x, from the hot end over that of its surroundings is given in the steady state by

$$\frac{d^2v}{dx^2}=\frac{hp}{qk}\cdot v=(\text{say})\ a^2v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where k is the thermal conductivity, h the heat lost per second per unit surface area per 1°C. difference of temperature between the rod at that point and its surroundings, and $a=\sqrt{\frac{hp}{qk}}$.

The solution of this equation is

$$v=\mathrm{A}\cosh ax+\mathrm{B}\sinh ax. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

At the point $x=0$ let $v=\mathrm{V}$; then $\mathrm{A}=\mathrm{V}$.

Hence
$$v=\mathrm{V}\cosh ax+\mathrm{B}\sinh ax., \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

and
$$\frac{dv}{dx}=a\mathrm{V}\sinh ax+a\mathrm{B}\cosh ax. \quad . \quad . \quad . \quad . \quad . \qquad (3\text{A})$$

If the rod is of length l, we have at the point $x=l$,

$$kq\frac{dv}{dx}+hqv=0.. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3\text{B})$$

$$\therefore\quad ka(\mathrm{V}\sinh al+\mathrm{B}\cosh al)+h(\mathrm{V}\cosh al+\mathrm{B}\sinh al)=0$$

or
$$\mathrm{B}=-\mathrm{V}\frac{ka\sinh al+h\cosh al}{ka\cosh al+h\sinh al}.$$

Hence, from (3),

$$v=\mathrm{V}\left\{\cosh ax-\frac{ka\sinh al+h\cosh al}{ka\cosh al+h\sinh al}\cdot\sinh ax\right\}. \quad . \qquad (4)$$

This equation holds whether l is great or small. If l is great the equation reduces to

$$v=\mathrm{V}e^{-ax}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

If not, we have from equation (4)

$$v=\mathrm{V}\frac{ka\cosh a(l-x)+h\sinh a(l-x)}{ka\cosh al+h\sinh al}. \quad . \quad . \quad . \qquad (4\text{A})$$

At the point $x=0$, $\mathrm{H}=kq\left(-\frac{dv}{dx}\right)$. Hence, from (4A),

$$\mathrm{H}=kq\mathrm{V}a\frac{ka\sinh al+h\cosh al}{ka\cosh al+h\sinh al},$$

$$=kq\mathrm{V}a\tanh al, \text{ if } h/ka \text{ is small.*}$$

* In the most unfavourable case employed in the present research an error of about one part in 10,000 is involved in this assumption.

Hence $$H^2 = kqV^2hp \,.\, \tanh^2 al.$$

$$\therefore\ k = \frac{H^2}{pqhV^2} \,.\, \coth^2 al, \qquad (6)$$

which becomes, when l is large,

$$k = \frac{H^2}{pqhV^2}. \qquad (7)$$

IV. *Experiments to Test the Applicability of the Formulæ.*

(*a*) In the equation $k = \frac{H^2}{pqhV^2} \coth^2 al$,

if k, p, q, h and l are constant at a given temperature of the enclosure, H/V should remain constant; *i.e.*,

$$\frac{CE - C'E'}{Pt_1 - Pt_2} \text{ should be constant.}$$

Many experiments with various wires were carried out to verify this, and provided the excess of temperature V of the hot end of the wire was made not greater than 10°C. or 12°C., the results were exceedingly good. It would appear, however, that for greater temperature differences the value of h is not quite constant, but tends to increase slightly with the temperature. Some results for a eureka wire are given below:

November 13*th*, 1913.—*Eureka Wire.*

Diameter, 0·0995 cm.; length, 40 cm.

Temperature of enclosure, 15·85°C.

In arbitrary units,

$$(1)\ \frac{CE - C'E'}{Pt_1 - Pt_2} = \frac{6{,}817 \times 234{\cdot}73 - 6{,}643 \times 228{\cdot}78}{13{\cdot}0260 - 12{\cdot}8372},$$

$= 425$ for temperature difference 5·73°C.

$$(2)\ \frac{CE - C'E'}{Pt_1 - Pt_2} = \frac{8{,}692 \times 297{\cdot}1 - 8{,}465 \times 289{\cdot}5}{13{\cdot}1425 - 12{\cdot}8320},$$

$= 424$ for temperature difference 8·96°C.

Similarly, for differences of temperature of 9·952°C. and 15·680°C., the quotients came out as 425 and 428 respectively.

(*b*) Again, from equation (6), it appears that for a given temperature difference, V, the amount of heat H received at the hot end of a given wire varies in such a way as to keep H coth al constant. This, again, was tested, and the results were entirely satisfactory.

Below are given details of an experiment made on a copper wire (commercially pure), the length being varied from 45 cm. down to 5 cm.

December 9th and 10th, 1913.—Copper Wire.

Diameter, 0·1024 cm.; k, 0·75; $a=\sqrt{\frac{hp}{qk}}=0{\cdot}1665$.

Temperature, 17°C. (Arbitrary units are given.)

1.	$l=$	5 cm.	10 cm.	20 cm.	30 cm.	45 cm.
2	coth $al \propto$	1·468	1·074	1·0025	1·000009	1
3	$\therefore$ H $\propto$	$\frac{1}{1{\cdot}468}$	$\frac{1}{1{\cdot}074}$	$\frac{1}{1{\cdot}0025}$	$\frac{1}{1{\cdot}000009}$	1
4	Experimental H=	$\frac{1}{1{\cdot}458}$	$\frac{1}{1{\cdot}075}$	1	1	1

The third line in the table gives the relative values of H as calculated from the formula. The agreement of the fourth line (experimental determinations of H) with the third is exceedingly good. In the case of lengths of wire from 20 cm. to 45 cm. no difference whatever could be detected in the amount of heat H flowing into the wire for a given temperature difference V. For a length 10 cm. experiment gave the result correct to one part in 1,000; and for a length only 5 cm. the error was one part in 100, a result which must be considered very satisfactory. These results indicate that the thermal conductivity can be measured accurately with quite small lengths of wire, especially if the substance has a low conductivity. In the case of platinum, for example, if a length of only 5 cm. were employed, the diameter being only 1 mm., the factor $\coth^2 al$ is 1·1025. For a length 10 cm. it is only 1·0056.

The temperatures at various points along a long uniform rod heated at one end in an enclosure at constant temperature are readily calculated from equation (5)—

$$v=Ve^{-ax}.$$

In the case of a silver wire of 1 mm. diameter, at tempera-

ture 17°C., $k=1$, $r=0{\cdot}05$, $h=0{\cdot}000533$, $a=0{\cdot}1456$, we obtain :—

Distance from hot end.	Value of v.
x cm.	$Ve^{-0{\cdot}1456x}$
5 ,,	$V\times0{\cdot}491$
10 ,,	$V\times0{\cdot}236$
15 ,,	$V\times0{\cdot}114$
20 ,,	$V\times0{\cdot}055$
30 ,,	$V\times0{\cdot}013$
40 ,,	$V\times0{\cdot}003$
50 ,,	$V\times0{\cdot}0007$

Again, the heat H lost from the first l cm. of wire is given by

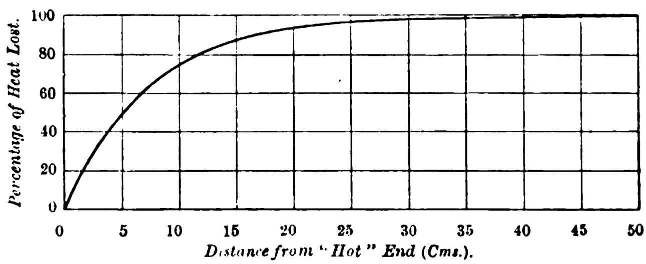
$$H=\int_0^l 2\pi rhv dx=C(1-e^{-al}),$$

where C is put for the constant quantity $\frac{2\pi rhV}{a}$.

In the case of the silver wire above we have :—

Distance from hot end.	Heat lost.
l cm.	$C(1-e^{-al})$
5 ,,	$C\times0{\cdot}5094$
10 ,,	$C\times0{\cdot}764$
15 ,,	$C\times0{\cdot}886$
20 ,,	$C\times0{\cdot}945$
30 ,,	$C\times0{\cdot}987$
40 ,,	$C\times0{\cdot}997$
50 ,,	$C\times0{\cdot}999$

The curve given below illustrates these results graphically. It is evident from the figure that in the case of a thin wire most of the heat given to the hot end of the specimen is lost from the first few centimetres.



(c) The resistance at the joint between the wire and the copper block into which it fits was investigated, and the con-

sequent fall of temperature there measured in the following way* :—

A "double joint," of precisely the same kind as employed in the main experiments, was constructed as shown in Fig. 4. A brass wire, RR, of rather larger diameter than the other specimens used, because of the difficulty of accurately boring the thinner wires, was fitted into the conical space ABCD. A hollow cone of thin brass (shaded in the figure) could also be inserted into this space. The wire then fitted into the hollow cone in exactly the same way as into the copper block. The two joints were, therefore, of precisely the same pattern and size.

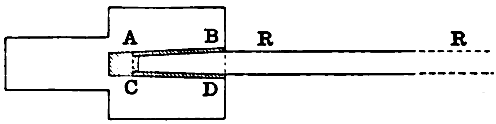


FIG. 4.

Determinations were made of the heat given to the wire :—

1. With the hollow cone in place.
2. The hollow cone being removed.

With the same notation as before, if the heat given to the rod

in case (1) is $$H = \frac{CE—C'E'}{W^2J},$$

and in case (2) is $$H_1 = \frac{C_1E_1—C'E'}{W^2J},$$

then $$\frac{H}{H_1} = \frac{CE—C'E'}{C_1E_1—C'E'} \quad . \quad . \quad . \quad . \quad . \quad (A)$$

Also $$k = \frac{H^2}{pqhV^2}\coth^2 al.$$

Hence, for a constant temperature difference, V, (k, p, q, h, a and l being also invariable), H is proportional to V.

Hence, from equation (A),

$$\frac{V}{V_1} = \frac{H}{H_1} = \frac{CE—C'E'}{C_1E_1—C'E'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

Measurements were made at laboratory temperatures and at 100°C., and various temperature differences, V (given, as

* The method here described was suggested by Prof. C. H. Lees, and the apparatus was carefully made for me by G. F. Goodchild, Esq., M.A.

usual, by the simultaneous readings of the two platinum thermometers), were tried. It was found that the percentage fall of temperature was the same, within the errors of experiment, for all values of V, both at 100°C. and at the temperature of the laboratory. The constancy of the results of the experiments recorded in section IV. (*a*) also verifies this conclusion. Assuming that the "double joint" has the effect of doubling this fall of temperature, the value of the latter is given by $V - V_1$.

In the tables below are recorded two of the experiments, the first at about 17°C., the second at 100°C.

I. Radius of wire, 0·15 cm. Length, 35 cm. Temperature, 17°C. Temperature difference, (V), 12·47°C.		
Wire in place without cone.	Wire in, with cone.	Wire out.
$C=9{,}283.$	$C_1=9{,}254$	$C'=7{,}437.$
$E=399{\cdot}0$	$E_1=396{\cdot}9$	$E'=318{\cdot}8$

This gives $$\frac{V}{V_1}=\frac{H}{H_1}=\frac{CE-C'E'}{C_1E_1-C'E'}=1{\cdot}023.$$

II. Temperature, 100°C. Temperature difference (V), 11·32°C.		
$C=10{,}271$	$C_1=10{,}233$	$C'=8{,}117$
$E=361{\cdot}2$	$E_1=359{\cdot}7$	$E'=285{\cdot}6$

Here $$\frac{V}{V_1}=\frac{CE-C'E'}{C_1E_1-C'E'}=1{\cdot}022.$$

The mean value of V/V_1, as the result of several concordant experiments, was 1·025. This indicates a fall of temperature of $2\frac{1}{2}$ per cent. of the measured temperature difference V. The values of V given by the thermometers are therefore higher by $2\frac{1}{2}$ per cent. than the actual differences of temperature between the hot end of the wire and the enclosure. In the results given in Table I., column X., the necessary corrections in the values of V have been made.

V. *Determination of h.*

As the formula for the thermal conductivity k involves a knowledge of "h," the amount of heat lost by convection and radiation per second from 1 sq. cm. of surface of the wire when its temperature is 1°C. above that of the enclosure, careful experiments were made for the purpose of determining this quantity at the temperatures employed—viz., 17°C. and 100°C. It has been frequently assumed that the quantity h is not

dependent on the shape or size of the body from which heat is lost. In the present research most of the wires employed were of circular section, and about 1 mm. diameter. Some, however, had a diameter of only 0·5 mm. and others were of square section. It was of importance, therefore, to determine the quantity h for wires of sections corresponding to all those employed. Nickel and platinum wires were experimented upon, and gave practically identical results. The diameter of the wires (between the limits employed—0·5 to 1 mm.), seemed to have very little influence on the value of h. This is in agreement with the experiments of Ayrton and Kilgour,* whose curve connecting "emissivity" and

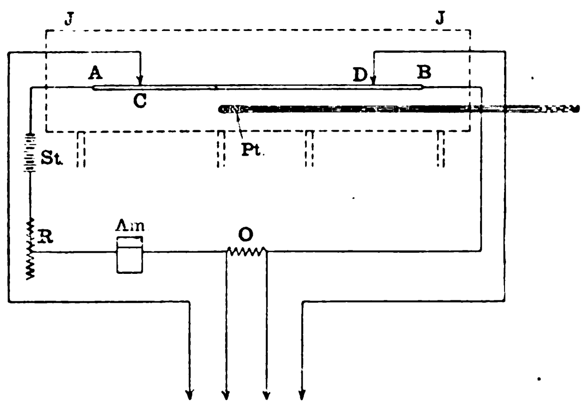


FIG. 5.

"radius of wire" becomes practically parallel to the latter axis when the diameter of the wire is 14 mils (0·356 mm.) or more.

Thin copper wire, gauge 30, was soldered to the extreme ends of the platinum wire (of length 40 cm.), and two wires of very thin platinum, or in some cases, platinoid, were welded or soldered with very great care—and little solder—to two points about 30 cm. apart. The latter wires served as potential leads. The wire was then placed in the middle of the water jacket, just as in the main experiment; a platinum thermometer also resting within the enclosure just opposite the middle of the wire (*see* Fig. 5).

* Ayrton and Kilgour, Phil. Trans., clxxxiii., Part 1, p. 371, 1892.

Va. *Measurement of c, the Temperature Coefficient of the Wires.*

A small current, C, measured roughly by an ammeter, A*m*, and accurately by means of a standard 1 ohm coil, and Rayleigh potentiometer, was sent through the platinum wire AB. The P.D. E between the points C and D was measured in the same way. R(=E/C) then gives the resistance of the wire between C and D at the temperature of the enclosure (or slightly higher, unless the current is very small), as given by the platinum thermometer. The temperature was then raised to about 100°C. by passing steam through the jacket, and the resistance was again measured as before. From the two values of the resistance, and those of the temperature, the temperature coefficient *c* was calculated. The constancy of the value of *c* obtained when different strengths of current were used afforded evidence that the heat lost down the potential leads was too small to be taken into account.

Special experiments were done to find the actual temperature coefficients between the limits (1) 17°C. to about 27°C.; (2) 100°C. to about 110°C. For this purpose the platinum wire was placed within a copper cylinder 60 cm. long (replacing the water-jacket mentioned above), round which was wound as evenly as possible a coil of manganin wire. The temperature, as shown by the platinum thermometer inside the cylinder, was then raised to various values up to 120°C. by sending an electric current through the manganin wire till the required temperature was attained. When this was quite steady the resistance of the platinum was measured as before. No difference in the temperature coefficients could be detected at temperatures within the range of the present series of experiments.

The mean value obtained for *c* for a platinum wire of diameter 1 mm., as the result of several concordant experiments, was 0·00386, with an agreement to within about 1 part in 300. Below is recorded one of the experiments :—

September 20, 1913.—*Platinum Wire.*

Circular section, diameter 0·1006 cm. Distance between potential leads, 28·95 cm.

1. Water in jacket.	2 Steam through jacket.
Pt = 16·446°C. *E = 208·88 *C = 5352 R = 0·03903 ohm.	99·603°C. = Pt. 271·37 = E.* 5341 = C.* 0·05081 = R.

* E and C are readings on the Rayleigh potentiometer, in ohms, corresponding to the P.D. between the points C and D on the wire, and the current through the wire respectively. The resistance R is obtained by the division of E by C.

Hence $$\frac{5081}{3903}=\frac{1+99{\cdot}603C.}{1+16{\cdot}446C.},$$

and $$C=0{\cdot}00386.$$

Vb. *Measurement of "h" for Platinum Wire of Diameter* 1 *mm.*

A comparatively large current, C (about 2 to 2·5 amperes), was now sent through the platinum wire, placed in precisely the same position as in the main experiment, described in Section IIa. The P.D. E and the amount of heat Q given to the part CD were also determined.

$$Q=CE/J. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

The temperature (t') of the enclosure was taken at the same time by the platinum thermometer. A small current (C') was now sent through the wire, and the P.D. (E') measured between C and D, the wire being first allowed to cool to a temperature that would be equal to (or very slightly higher than) that of the enclosure. The change of resistance of the wire is, of course, given by the expression $E/C—E'/C'$.

With a knowledge of the temperature coefficient c of the wire the excess of temperature of the latter over that of the enclosure can be accurately determined. The amount of heat given to that part of the wire between C and D, and thence to the enclosure, is given by

$$(CE-C'E')/J. \quad . \quad . \quad . \quad . \quad . \quad (II.)$$

Hence if s is the surface area of the wire between C and D, and $(t—t')$ the excess of temperature of the wire over that of the enclosure, we have

$$h=\frac{CE-C'E'}{J(t-t')s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (III.)$$

Experiments were conducted at temperatures 17°C. and 100°C. The mean results obtained for this particular wire, from a large number of experiments, were

$$h=0{\cdot}000533 \text{ at } 17°C.; \text{ and } 0{\cdot}000634 \text{ at } 100°C.$$

It is assumed that h has the same value for all the metals (of identical cross-section) employed in the present research. In this connection it may be noted that the loss of heat is almost entirely due to convection, so that the difference in emissivity of the various metals (which is not likely to be great, however), would scarcely affect the value of h. It has been already stated that nickel and platinum, at any rate, give

identical values for this quantity. Below are given examples of the determination of h, together with the method of reducing the results.

September 23rd, 1913.—*Platinum Wire.*

Circular section, diameter 1 mm. Surface area of platinum between potential leads, 9·149 sq. cms.

I.—Large current.	II.—Small current.
Reading of ammeter 2·05 amps.	0·125 amp
Pt. .... 17·694°C.	17·700°C.
E .... 202·74	37·343
C 5178	981·41
W......................2576	7821
R .... 0·03915 ohm.	0·03805 ohm.

If t=temperature of platinum when current 2·05 amperes flows through it we have, $\frac{R_t}{R_{17\cdot7}}=\frac{3,915}{3,805}=\frac{1+0\cdot00386\times t}{1+0\cdot00386\times17\cdot7}$,

from which $t=25\cdot701$°C., and excess of temperature of wire over that of enclosure=8·007°C.

$$\text{Also}\quad Q=\left\{\frac{5,178\times202\cdot74}{(2,576)^2}-\frac{981\cdot41\times37\cdot343}{(7,821)^2}\right\}\times0\cdot2479^*$$

$$\therefore\ Q=0\cdot03904 \text{ calories.}$$

$$\therefore\ h=\frac{Q}{s(t-t')}=0\cdot000533.$$

In precisely the same way the value of h at 100°C. was found to be 0·000634 for the same wire.

The corresponding results for a platinum wire of square cross-section were as follows:—

April 24th, 1914.—*Platinum Wire.*

Square section; diameter, 0·795 mm. Temperature coefficient of resistance, 0·00358. Surface area of wire between potential leads, 8·43 sq. cms.

I.—Large current.	II.—Small current
Reading of ammeter, 1·72 amp.	0·22 amp.
Pt .... 16·851°C.	16·851°C.
E .... 320·73	39·72
C 6587	839·0
W......................3814	3814
R .... 0·04869 ohm.	0·04735 ohm.

$$^*0\cdot2479=\frac{(\text{E.M.F. Weston Cell})^2}{J}=\frac{(1\cdot0186)^2}{4\cdot185}$$

Excess of temperature of wire over that of enclosure $=8{\cdot}412°C$. $Q=0{\cdot}03541$ calories.

$$\therefore\ h=\frac{0{\cdot}03541}{8{\cdot}412\times 8{\cdot}43}=0{\cdot}000499 \text{ at } 17°C.$$

Similarly the value of h at 100°C. was 0·000594.

In connection with these results, an interesting Paper by Dr. Alexander Russell,* which throws valuable light on the variation of h under different conditions, is worthy of mention. It appears that h is not under all conditions proportional to the difference of temperature, θ, between the hot body and its surroundings. For example, L. Lorenz† obtained, mathematically, the result, that in the case o a heated strip, $h \propto \theta^{\frac{1}{4}}$. On the other hand, Boussinesq‡ deduced that the convection of heat by a stream of liquid from a cylinder maintained at a constant temperature is given by $h \propto \theta$. This result is confirmed by results obtained by P. Compan,§ and also by Kennelly,‖ in the case of the cooling of cylindrical wires.

Again, as Russell remarks, Newton's law ($h \propto \theta$) leads to results in several practical applications that are found to be in close accordance with experiment.

The results of the present experiments, *e.g.*, those given in section IV. (a), indicate that with this particular arrangement of apparatus, Newton's law is strictly applicable, at any rate for values of θ up to 10°C. or 12°C.

VI. *Determination of Thermal Conductivity k.*

The following is a full account of the determination of the remaining quantities necessary for obtaining "k," together with the method of reducing the results, the particular case taken being that of pure platinum at the temperature of the laboratory. For temperatures 100°C. the only difference in procedure was to send steam through the outer jacket.

* A. Russell, "Proc." Phys. Soc., XXII., p. 432, 1909.

† L. Lorenz, "Ann. der Physik," XIII., p. 582, 1881.

‡ Boussinesq, "Comp. Rend.," CXXXIII., p. 257.

§ P. Compan, "Ann. de Chim. et Phys.," XXVI., p. 488, 1902.

‖ Kennelly, "Amer. Inst. Elec. Engin. Proc.," July, 1909.

November 4th, 1913.—*Platinum.*

Radius of wire, 0·0503 cm. $\therefore$ $p.q.h.=2\pi^2r^3h=1339\times10^{-9}$. Length of specimen, 35·1 cms.

Platinum out.	Platinum in.
$Pt_1=Pt_2 \therefore V=12\cdot177$°C. $E'=7618$. $C'=310\cdot8$. $W=3839$.	Pt_1-Pt maintained 12·177°C. $E=8140$. $C=331\cdot7$. $W=3839$.
E.M.F. Weston = 1·0186 volt.	

Hence, true current $=C/W\times1\cdot0186$.

True E.M.F. $=E/W\times1\cdot0186$.

$$\text{and heat given } =\frac{EC}{JW^2}\times(1\cdot0186)^2, \text{ with wire in.}$$

$$\text{and}=\frac{E'C'}{JW^2}\times(1\cdot0186)^2, \text{ with wire out.}$$

$$H \text{ (measured in calories)}=\frac{CE-C'E'}{JW^2}\times(1\cdot0186)^2.$$

$$\therefore H^2=508\cdot5\times10^{-6}\times6\cdot146\times10^{-2}.$$

$$\left(\text{N.B.}—6\cdot146\times10^{-2}=\left[\frac{1\cdot0186^2}{J}\right]^2\right).$$

$$H^2=31\cdot26\times10^{-6}.$$

$$V^2=(11\cdot880)^2=141\cdot13.$$

$$\therefore k=\frac{H^2}{pqhV^2}=0\cdot165.$$

In Table I. the thermal conductivities of all the metals examined are tabulated, together with the principal measurements involved. The figures given are the corrected ones, the principal corrections being those due to (1) the calibration of the coils of the resistance boxes used in the Rayleigh potentiometer, (2) the standardisation of the 1-ohm resistance coil and of the Weston cells, (3) the fall of temperature at the junction of the wire and the copper block. In the calculation of V in column X., where V is the difference of temperature as given by the readings of the two platinum thermometers Pt_1 and Pt_2, a curve was drawn on a large scale connecting the simultaneous readings of the two thermometers at various temperatures from 0°C. to 100°C. It was therefore a simple matter to convert the readings of Pt_2 to terms of Pt_1, obtain the value of V in terms of platinum, and thence reduce to gas

temperatures. This comparison of the readings of the two thermometers was frequently made during the course of the research, and it was satisfactory to find no appreciable alteration in their relative resistances within the range of temperature employed. The wire used for the thermometer Pt_1 was from the same reel as one whose δ coefficient had previously been obtained, and had the value of 0·000152 in the formula,

$$t - Pt = \delta t(t - 100),$$

where t is the gas temperature, Pt the platinum. The fixed points taken for the determination of δ were, as usual, those of ice, steam and boiling sulphur.

In cases where results by other investigators are available they are given in column XIV. for the sake of comparison. As a whole, the results of the present Paper are in good agreement. In the case of the majority of the metals experimented upon, however, no previous determinations of the thermal conductivity appear to have been made.

VII. *Electrical Resistances of Wires.*

The electrical resistances of the same wires (including also graphite) were determined at laboratory temperatures and at about 100°C. in the following way:—

The wire was placed on two copper knife-edges, EE (Fig. 6), and fastened down firmly with silk thread, TT. Two thin silk-covered copper wires (gauge 30) were soldered to the knife-edges, which were kept at a fixed measured distance apart by

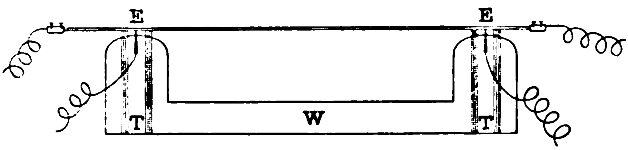


FIG. 6.

being inserted in parallel vertical grooves made in a piece of hard wood, W, shaped as in the figure.

A current, C, from two storage cells was sent through the specimen, and the E.M.F. (E) between the knife-edges measured, as in the heat experiments, by means of a potentiometer and a Weston cell, the current being determined also in a similar way. The metals were placed in the same enclosure as in the heat experiments, and measurements made at about

17°C. and 100°C., the temperatures being given by a platinum thermometer placed midway between EE. The same current, approximately, was used at both temperatures and was too small to cause an appreciable rise of temperature of the wire. Details are given below of an experiment on platinum-iridium (10 per cent. iridium), and Table II. gives the measurements similarly obtained with the remaining wires at both temperatures.

November 25th, 1913.—*Platinum-Iridium* (10 *per cent.*).

Diameter, 0·1025 cm.; distance l between potential leads, 28·14 cm.

Temperature, 16·53°C.

Current C (in terms of potentiometer and Leclanché cells), 568·8 ohms.

E.M.F. E (in terms of potentiometer), 50·24 ohms.

Resistance R=E/C=0·08845 ohm.

Specific resistance

$$(\rho)=\frac{R\,.\,\pi r^2}{l}=\frac{0\cdot08845\pi(0\cdot05125)^2}{28\cdot14}=25\cdot88\times10^{-6}\text{ohm per cm.c'be.}$$

The same result for the value of R to four significant figures could nearly always be obtained, using different values of the current and of the E.M.F. This afforded satisfactory evidence of the correctness of the calibration of the coils of the resistance boxes used in the Rayleigh potentiometer. A 1/100th ohm resistance box was included in the latter.

In Table II. the "length" given in column IV. is, of course, the distance apart of the knife-edges between which the P.D. E was measured. The resistances tabulated in column VII. are then obtained from the quotient E/C. Columns IX. and X. are obtained by a slight extrapolation of the results from column VIII. The comparatively high values obtained for $_0\alpha_{100}$ in the case of platinum, tungsten, palladium, tantalum and molybdenum are evidences of the purity of those metals. All the metals employed (except the tungsten) were supplied by Messrs. Johnson, Matthey & Co., and were of the highest purity obtainable.

VIII. *Discussion of the Results.*

In order to enable the results of the present Paper to be compared with the electronic theories the values of k/KT are given in Table III. k is the thermal conductivity, K

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TABLE III.—*Values of* k/KT* *at* 273° *and at* 373° (*Abs. Temp.*)

I.	II.	III.	IV.	V.	VI.
Metal.	Thermal conductivity k.	Electrical conductivity $\mathrm{K} \times 10^{-4}$	Abs. temp. T.	$\mathrm{KT} \times 10^{-6}$	$k/\mathrm{KT} \times 10^{8}$
Platinum	0·691 0·711	10·24 7·35	273 373	27·95 27·40	2·47 2·59
Platinum—10% iridium	0·310 0·314	3·98 3·53	273 373	10·86 13·17	2·85 2·38
Platinum—15% iridium	0·234 0·247	3·49 3·19	273 373	9·53 11·90	2·45 2·08
Platinum—20% iridium	0·176 0·176	3·02 2·81	273 373	8·23 10·48	2·14 1·68
Platinum—10% rhodium	0·301 0·306	4·46 3·92	273 373	12·16 14·63	2·48 2·10
Iridium.........	0·590 0·565	12·21 9·44	273 373	33·33 35·22	1·77 1·60
Rhodium	0·879 0·804	20·78 16·10	273 373	56·7 60·1	1·55 1·33
Gold	2·95 2·93	44·09 32·52	273 373	120·4 121·3	2·45 2·41
Eureka (Constantan)	0·222 0·234	2·179 2·180	273 373	59·5 81·3	3·73 2·87
Tungsten ("Pladuram")	1·99 1·97	19·21 13·22	273 373	52·44 49·33	3·79 3·99
Palladium (commercial)	0·423 0·418	5·62 5·39	273 373	15·34 20·10	2·70 2·08
Palladium (pure)	0·603 0·598	10·16 7·39	273 373	27·74 27·56	2·18 2·16
Tantalum......	0·544 0·540	6·92 5·21	273 373	18·89 19·43	2·88 2·78
Molybdenum	1·45 1·39	17·22 11·74	273 373	47·01 43·80	3·08 3·17
Graphite	0·155 0·159	0·00513 0·00552	273 373	0·0140 0·0206	1,110 770

* k = Thermal conductivity in watts per centimetre-degree Cent.
K = Electrical conductivity in reciprocal ohms per centimetre-cube.
T = Absolute gas temperature.

the electrical conductivity, and T the absolute temperature on the hydrogen scale. As a general rule, the values given in column VI. tend towards a common value somewhere about 2·5, though two or three of the metals, notably tungsten, rhodium and iridium, differ rather widely from this generalisation. Graphite is, of course, a very pronounced exception, for its thermal conductivity is very nearly as great as that of platinum 20 per cent. iridium, while its electrical conductivity is about 600 times less. Lees has observed a parallel case in the comparison of the conductivities of bismuth and quartz, the latter being one of the very best of the electrical insulators, while having at the same time a thermal conductivity greater than that of bismuth. The platinum-iridium series given in Table III. is interesting, as affording a good example of the influence of "impurities" on the thermal and electrical properties of a metal. Grüneisen * showed that addition of foreign substances reduces the electrical conductivity to a greater extent than the thermal—at any rate in cases where the amount of impurity is comparatively small. The following table is compiled from some of his measurements. Cu indicates pure copper, $CuAs_1$ copper with a small percentage of arsenic, $CuAs_2$ copper with a larger percentage of arsenic:—

Metal.	Electrical conductivity, $K \times 10^5$.	Thermal conductivity, λ.	$\lambda/K \times 10^{-1}$.
Cu	57·4	0·934	163
$CuAs_1$	19·1	0·340	178
$CuAs_2$	5·03	0·0995	198

Below is given a similar table derived from Table III.:—

Metal.	Electrical conductivity, K.	Thermal conductivity, λ.	λ/K
Platinum	10·24	0·691	675
Pt-10% Ir.	3·98	0·310	779
Pt-15% Ir.	3·49	0·234	670
Pt-20% Ir.	3·02	0·176	583
Iridium	12·21	0·594	487

As in Grüneisen's results for copper, the 10 per cent. alloy shows an increase in λ/k over that given by the pure platinum. As the "impurity" increases, however, the thermal conductivity decreases more rapidly than the electrical. The values

* E. Grüneisen, Ann. de Physik., 3, 1, pp. 43-74, Sept., 1900.

obtained for the thermal conductivity of two specimens of palladium are rather striking. A commercially "pure" specimen gave values of 0·101 and 0·100 at 0°C. and 100°C. respectively, while the pure metal gave 0·144 and 0·143 at corresponding temperatures. The presence of what was presumably quite a small amount of impurity reduced the conductivity by over 30 per cent.

Lees' observation that for pure metals the thermal conductivity decreases with rise of temperature, while for alloys it increases as the temperature rises, is, on the whole, well borne out in the present experiments. Platinum (as in Jäger and Diesselhorst's experiments) forms an exception to this rule. The changes of conductivity, however, are in many cases so slight as to be almost within the errors of experiment.

Perhaps the least satisfactory measurement of those involved is that of the radius of the specimens. With the wires employed, however, which had been used for no other purpose previously, there was surprisingly little variation in diameter, which could be measured correctly at any rate to one-third of 1 per cent. As the formulæ employed involve the cube of the radius this means a possible error of about 1 per cent.

IX. *Particulars of Instruments Employed.*

The *Micrometer Screw Gauge* used in the measurement of the diameters was one by the Brown & Sharpe Manufacturing Co. It was tested in the National Physical Laboratory, and certified to be correct (at 20°C.) to within 0·005 mm. at eight points from 0·5 mm. to 4·0 mm.

The *Standard Resistance Coil* was supplied by Messrs. J. J. Griffin, and was found to have a resistance of 0·99859 true ohm at 20°C., as compared with the N.P.L. standards. It was kept in paraffin oil, whose temperature during the research varied very little from 17°C.

Three *Standard Cadmium* (*Weston*) *Cells* were employed, which agreed with each other to within one or two parts in 10,000. One was made by myself according to specifications given by Mr. F. E. Smith,* of the National Physical Laboratory, and the other two were supplied by Messrs. J. J. Griffin. One of these cells was certified by the N.P.L. as having an E.M.F. of 1·0186 volt at 20°C.

The *Galvanometer*, which was used in connection with the

* F. E. Smith, "Phil. Trans.," A, 207, pp. 393-420, 1908.

Callendar-Griffiths' bridge for platinum temperature measurements, and also with the potentiometer, was of the Ayrton-Mather moving-coil pattern, by the Cambridge Scientific Instrument Co. Its resistance at 15°C. was 19·4 ohms, its period 8·6 seconds, and its deflection in millimetres at 1 metre for a current of 1 micro-ampere was 350.

The Callendar-Griffiths' Bridge, used in connection with the platinum thermometers, was carefully calibrated in the usual way. With its aid temperatures could be accurately measured within the ranges employed to 0·001°C.

The figures given in the various tables of the present Paper have all been corrected according to the specifications given above.

The research has been carried out at the Wandsworth Technical Institute, and my thanks are due to the Principal (Mr. G. F. Goodchild, M.A., B.Sc.) for the kind interest he has taken in the work; also to Mr. J. J. Risdon, of the firm of Messrs. Johnson, Matthey & Co., for his never failing courtesy in preparing and supplying the specimens used.

I am also very deeply indebted to Prof. C. H. Lees for valuable hints and criticisms during the course of the research.

ABSTRACT.

A new method of the "stationary temperature" type is employed for measuring the thermal conductivities of some of the rarer metals, including tantalum, molybdenum, rhodium, iridium and tungsten, at air temperatures and at 100°C.

It is shown that if a rod of metal is of length l, perimeter p, and cross-section q, then its thermal conductivity k is given by

$$k=\frac{H^2}{pqhV^2}\coth^2 al,$$

where H is the heat given per second to one end of the rod, V the excess of temperature of this end over that of the enclosure, h the heat lost per second from 1 sq. cm. of surface when its excess of temperature over that of its surroundings is 1°C., and $a=\sqrt{\frac{hp}{qk}}$.

If l is large the equation reduces to the very simple form $k=\frac{{}^2}{pqhV^2}$, this latter equation being employed in nearly every case.

Experimental details and the method of working out the results are given, and it is shown that the thermal conductivity of non-metals can also be determined in the same way.

Electrical conductivities of the same specimens have also been measured, and, for purposes of comparison with electronic theories, the values of k/KT have been worked out, where k=thermal conductivity, K=electrical conductivity and T=absolute temperature.

DISCUSSION.

Prof. C. H. Lees thought the method extremely useful and simple. The accuracy obtained was quite sufficient to allow of verification of the ionic theories on which the author touched. His figures differed sufficiently from the theoretical value to show that the theory was only approximate in its present state.

Dr. Harker asked about the uniformity of temperature in the air space. The apparatus was horizontal, and if the heat supplied to the wire was very great there were bound to be considerable irregularities of temperature in the chamber.

Mr. Barratt, in reply to Dr. Harker, said there was sometimes a very slow change of temperature of the air in the enclosure, but as the actual experiments, once conditions had become steady, took only a few minutes, the small change in temperature would not matter. Moreover, this change would affect both the hot end of the specimen and the cold one, so that the value of V, which was the object of the measurement, would not be altered. In addition, the heat supplied to the wire was never great enough to raise the temperature of its hottest part more than 10°C. above that of the enclosure. In fact, the greater part of the wire was at practically the same temperature as that of the water jacket.

XXXVI.—*Some Investigations on the Arc as a Generator of High Frequency Oscillations.* *By* F. Mercer, *B.Eng.*

Communicated by Prof. Wilberforce.
Received June 3, 1914.

Introduction.

These investigations were undertaken in the first instance with the view of determining what effect an increase on the gas pressure in the containing vessel would have on the power and efficiency of a copper-carbon arc when used as a generator of high-frequency oscillations.

A series of experiments on the high-frequency arc were published in the "Bureau of Standards" for May, 1907, by Mr. L. W. Austin. One of the most interesting points in the Paper was that dealing with the effect on the high-frequency current of increasing the pressure of gas surrounding the arc. Consideration of the underlying principles leads one to the conclusion that the effect of this should be to increase the amount of useful power that could be taken from the high-frequency circuit, and Mr. Austin's work shows that this actually does occur. A curve connecting the pressure in atmospheres and the secondary current in amperes shows that up to two atmospheres and beyond five the curve is practically flat, but in the intermediate portion the shunt current increases enormously, rising from 2·5 amperes at two atmospheres to 12 at five atmospheres. For his experiments Mr. Austin used silver-tipped electrodes, and an atmosphere of compressed air. The electrodes were hollowed out and cooled by means of a running stream of water. The nature of these results leads one to the conclusion that further investigation is necessary, and for this reason the author made a series of similar experiments on the ordinary copper-carbon arc. In the course of these some other properties peculiar to the high-frequency arc were brought to light, and these will be referred to in due course.

Apparatus.

The arc was encased in a cast-iron cylinder, capable of withstanding mechanical pressure of 200 lbs. per square inch. The copper electrode and the brass holder for the carbon electrode pass through the ends of the cylinder, being insulated from them by ebonite, and are held in position by glands lined with

ebonite fitting into annular spaces in the ebonite insulation of the ends. Asbestos packing is placed between the gland and the ebonite of the cover, and can be compressed by tightening up the gland, thus preventing leakage and at the same time allowing either electrode to be moved.

In regulating, the carbon only was moved, the copper electrode remaining stationary.

The main leads (Fig. 1) are taken to a double-pole switch, and the primary circuit completed through the inductance L, the ammeter A*a*, the resistance R and the arc. The shunt circuit contains an ammeter A*s*, inductance L*s* and capacity C. The condensers used were of the Moscicki type, the respective capacities of the four available being 0·00201, 0·00212, 0·00299, 0·00314 microfarad. The inductance L*s* consisted of 2·65 mm. wire wound on a square frame of 80 cm. side. The copper

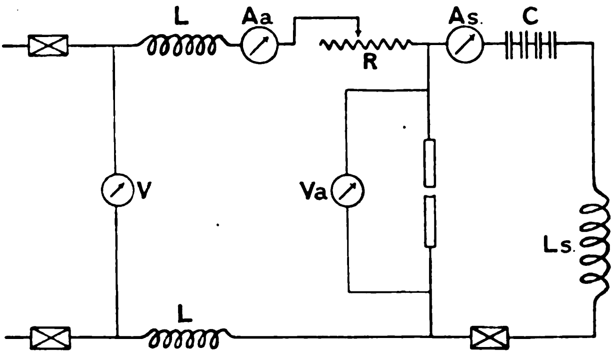


FIG. 1.—DIAGRAM OF CONNECTIONS.

electrode was 19 mm. and the carbon 20 mm. in diameter. A Lorentz wave-meter was used to measure the oscillation frequency. All connections in the shunt circuit were made with thick copper wire in order to reduce its resistance to a minimum.

Experimental Results.

A few preliminary tests were carried out to find the best conditions for producing oscillations. It was found that for steady burning—

(1) The copper electrode should be negative; this conclusion is in agreement with observations made by Barreca.*

* "The Electrician," Vol. LX., pp. 522-523, Jan. 17, 1908.

II 2

(2) The magnetic field should be dispensed with, as steadier readings can be obtained without it.

(3) The arc length should be carefully adjusted. If this length is too short the oscillations will not pass, while if too long they are so irregular that no readings can be taken. There is, however, a definite length, varying from about 0·5 mm. to 2 mm., which not only gives steady oscillations but which is least liable to be affected by change in the conditions in the course of any particular experiment.

In the first place experiments were made with a view to determining the effect of arc length on the frequency and magnitude of the current in the shunt circuit. As it is a matter of considerable difficulty to measure accurately the distance between the two electrodes, and knowing that the arc current

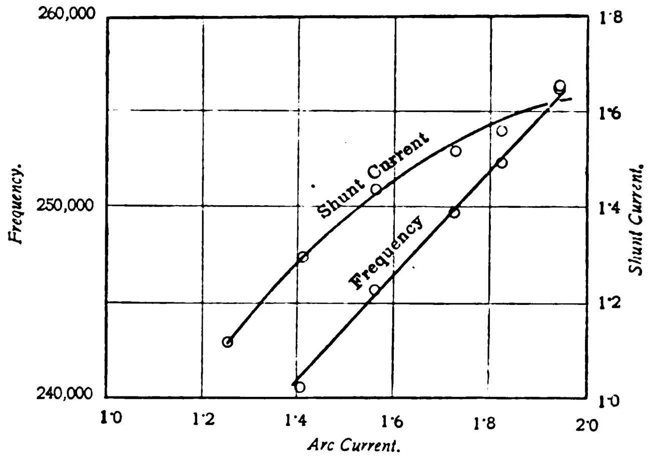


Fig. 2.—Curves obtained by altering the Arc Length and taking Readings of Arc Current, Shunt Current, and Frequency.

increases as the arc length is diminished and vice versa, we can deduce from the curves in Fig. 2, which are obtained by altering the arc length and taking simultaneous readings of arc current, shunt current and frequency, that both the magnitude and frequency of the shunt current decrease as the arc length is increased. Now, with a copper-carbon arc it is a practical impossibility to ensure that the arc length should remain constant, even for a short time, owing to slight irregularities in the structure of the carbon, and to its burning away, though the latter defect can be remedied to a certain extent by slowly

rotating the carbon. Therefore, even with the most carefully prepared arc of this description, the values of frequency and shunt current are continually altering. By taking certain precautions before any experiment, however, these variations can be cut down to such an extent that their effect is hardly appreciable.

Similar curves can be obtained by altering the resistance in series with the arc. In this case as the current through the arc increases the frequency increases and vice versa.

These facts are borne out by the formula

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{L\,.\,C}-\frac{R^2}{4L^2}},$$

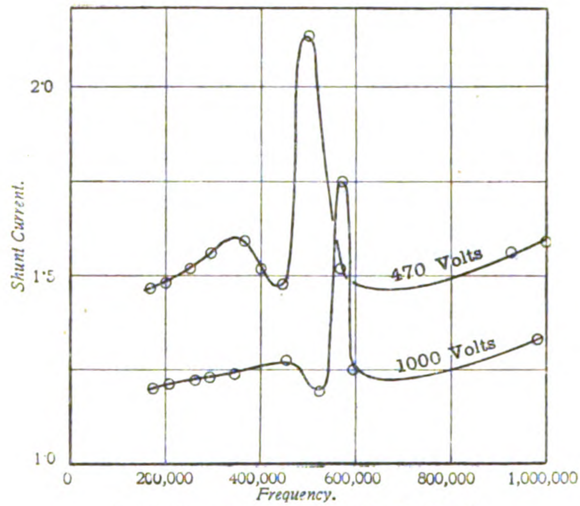
where f=frequency of oscillations,
L=inductance of circuit in henries,
C=capacity in farads,
R=resistance in ohms,

since it is fairly safe to assume that increasing the arc length or decreasing the current through the arc would increase its resistance and vice versa.

Effect of Ratio of Self-induction to Capacity.

It has been shown that the ratio of self-induction to capacity has an important influence on the production of oscillations. In order to examine this effect the following experiment was carried out. Keeping the capacity constant the inductance was slowly decreased one turn at a time. The curve connecting frequency with the current in the shunt circuit was found to be of the nature of a resonance curve, examples of which are shown in Figs. 3 and 4.

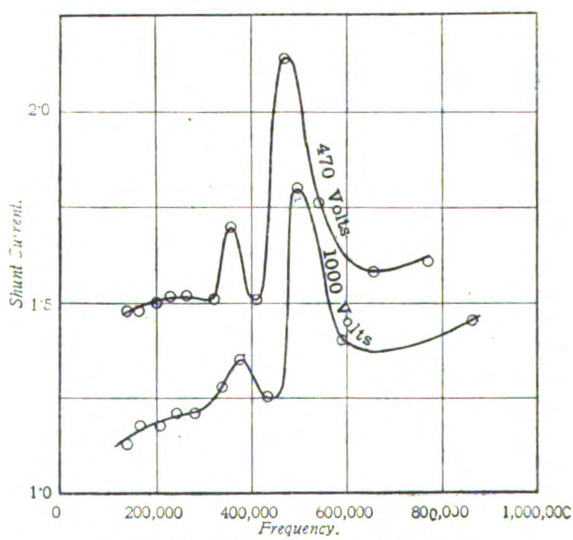
With fairly high capacities in the shunt circuit a gradual increase in current was observable as the inductance was decreased, no apparent maxima occurring. This state of affairs might have been expected, for, in decreasing the inductance, the resistance of the shunt circuit is also decreased by a certain amount. This resistance naturally has considerable influence on the strength of oscillations, as has been shown experimentally by Austin, an increase in resistance giving a decreased current and vice versa. However, in Figs. 3 and 4, in which condensers of smaller capacity were employed, the shunt current had decided maxima, two or even three of these occurring in certain cases. With a given capacity in the shunt circuit



Constant capacity in shunt circuit=0·0012 mfd.

FIG 3.—CURVES SHOWING THE EFFECT OF ALTERING INDUCTANCE, KEEPING CAPACITY CONSTANT.

Arc current at	470	volts	=	1·4	amperes.
,, ,, ,,	1,000	,,	=	1·46	,,
,, volts ,,	470	,,	=	62	
,, ,, ,,	1,000	,,	=	74	



Constant capacity in shunt circuit=0·00201 mfd.

FIG 4.—CURVES SHOWING THE EFFECT OF ALTERING INDUCTANCE, KEEPING CAPACITY CONSTANT.

Arc current at	470	volts	=	1·4	amperes.
,, ,, ,,	1,000	,,	=	1·48	,,
,, volts ,,	470	,,	=	62	
,, ,, ,,	1,000	,,	=	65	

there is then a definite value of inductance which gives high-frequency oscillations of maximum power. The inductance necessary to give this effect, and the corresponding shunt current, and the frequency, are plotted against capacity in Fig. 5. The steadiness of the oscillations depends on the values of the inductance and capacity, being very poor with either small capacity or small inductance. In order to obtain the conditions for maximum output from the arc, one or other of these two quantities has to be small and consequently steadiness of

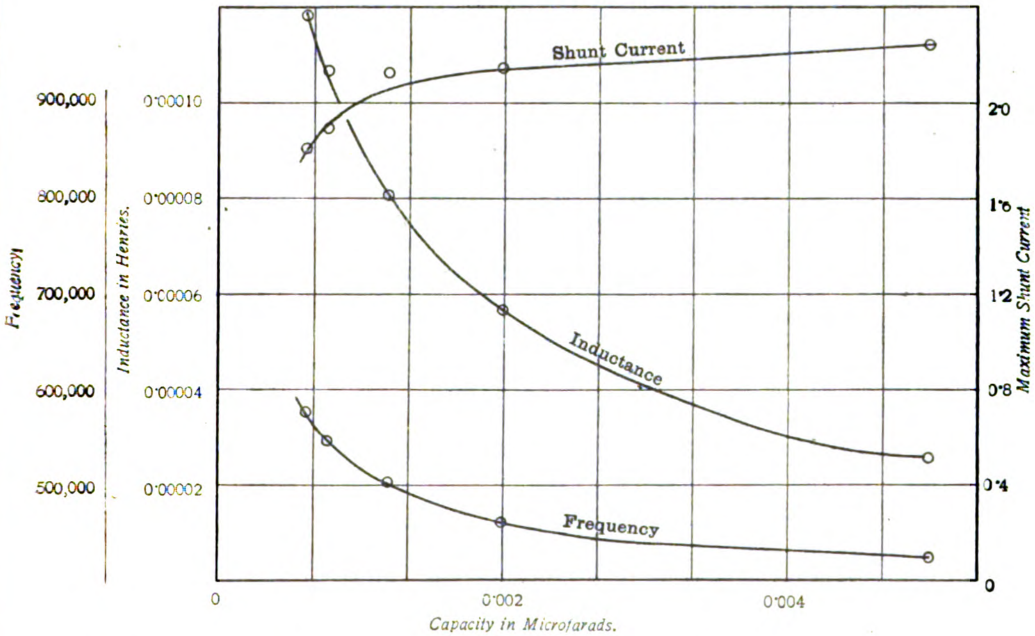


FIG. 5.
CURVES GIVING THE CONDITIONS FOR MAXIMUM SHUNT CURRENT FOR DIFFERENT CAPACITIES.

operation is sacrificed to output. The two curves shown in Figs. 3 and 4 are for 470 and 1,000 volts respectively. It will be seen that the two are similar, except that they reach their maxima at different frequencies. This, however, might be accounted for by differences in the arc length and current.

Effect of Gas Pressure.

The experiments carried out to determine the effect of gas pressure were not conclusive. The maximum electrical

pressure available was about 3,000 volts (direct current), which was obtained by connecting a 2,000 and a 1,000-volt generator in series. The maximum current given by the former was 1 ampere. When using 1,000 volts and over the

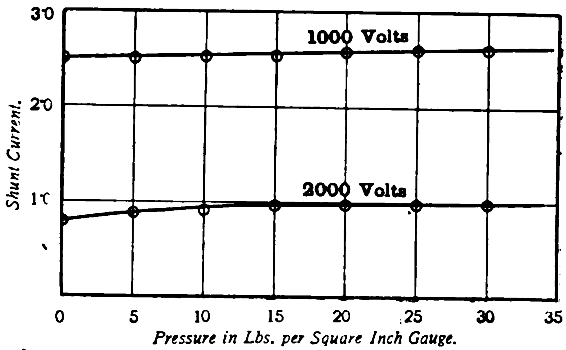


FIG. 6.—CURVE SHOWING THE EFFECT OF ALTERING THE GAS PRESSURE.

copper electrode was earthed. At the lower voltages (230 and 460 volts) it was found that altering the gas pressure had little or no effect. At 1,000 volts, however, a slight rise in current

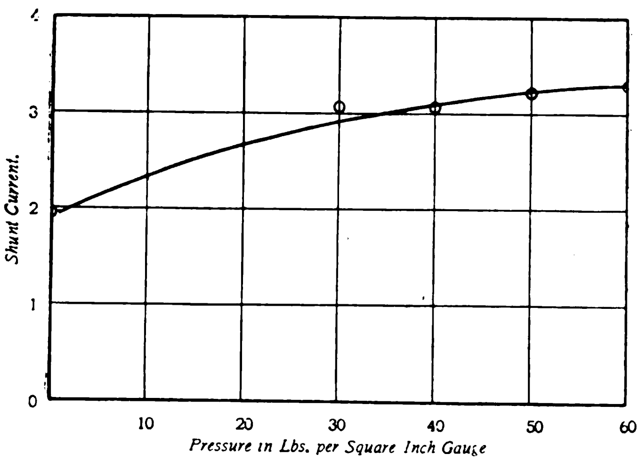


FIG. 7—CURVE SHOWING THE EFFECT OF ALTERING THE GAS PRESSURE.

could be observed, while at 2,000 volts the curve between shunt current and gas pressure was comparatively steep. When a pressure of 3,000 volts was tried, no steady readings could be obtained. However, a comparison of the results obtained with

230, 460, 1,000 and 2,000 volts leads to the conclusion that the shunt current obtainable increases as the electrical pressure is increased, and it is quite possible that at 5,000 volts, if mean readings could be taken, similar results could be obtained with the copper-carbon arc as were obtained by Austin using silver-tipped electrodes. The magnitude of the effect is dependent on the values of inductance and capacity in the shunt circuit. The plotted results in Fig. 6 were taken with a capacity of 0·0103 mfd. and inductance 0·0002 henry. In Fig. 7 it will be seen that the curve rises more steeply than in either of the previous cases. The inductance in this case was about 0·000025 henry, the capacity being the same, viz., 0·0103 mfd. Otherwise the conditions were similar to those obtaining in the lower curve of Fig. 6.

Conclusions.

(1) There is a definite value of inductance for any given capacity, which gives a maximum current in the shunt circuit.

(2) The effect of increasing the gas pressure becomes more marked as the electrical pressure is increased, but as the gas pressure rises the steadiness of the arc diminishes. The effect is somewhat similar to that obtained by increasing the arc length.

(3) Any effort made to increase the output by the use of a magnetic field, or by altering the arc length or the resistance in series with the arc, is detrimental to the steadiness of operation.

The foregoing experiments were carried out at Liverpool University, and before concluding the author wishes to express his thanks to Prof. Marchant for assistance and advice throughout the work, and also to Mr. N. P. Devonald, B.Eng., who very kindly assisted in carrying out some of the experiments.

ABSTRACT.

This Paper contains the results of a series of experiments on the copper-carbon arc when used as a generator of high-frequency oscillations. The copper electrode was water-cooled, and the whole enclosed in a cast-iron cylinder filled with hydrogen under pressure.

Preliminary experiments were carried out to determine the conditions which would give maximum steadiness of burning, and in the ensuing experiments these conditions were adhered to as rigidly as was possible.

The first experiments deal with the effect of varying the arc length, and also the arc current (by changing the resistance in series with the arc), on the magnitude and frequency of the shunt current. The

deduction is drawn that the effect on frequency arises from a change in the resistance of the arc.

The second experiment refers to the effect on the shunt current of altering the ratio of inductance to capacity. Keeping the latter constant and varying the former a series of curves were obtained, showing that for a given capacity there is a certain value of inductance giving maximum current in the shunt circuit.

Experiments were also made to determine the effect which the pressure of the hydrogen would have on the magnitude of the shunt current. This is dependent on the P.D. used. With small P.D.s the effect is negligible, at 1,000 volts it is just appreciable, and at 2,000 volts a marked increase is apparent. Beyond this it was impossible to obtain readings with any degree of accuracy, as the oscillations were very unsteady.

DISCUSSION.

Prof. FLEMING mentioned that Mr. Bairsto had used an arc under pressure as a source of high-frequency oscillations over a year ago. He then described a more satisfactory method recently used by himself and Mr. Coursey. A number of carbons were fixed vertically to a metal plate which was suspended near the bottom of an iron vessel containing heavy oil. The carbons just project above the oil. Over each carbon tip is suspended a copper cylinder, the walls of which go down into the oil, and the thick top of which is perforated. Arcs are struck by lowering the cylinders until they make contact with the carbons, and then raising them with a screw. In this way very steady oscillations were obtained.

Mr. DUDDELL said that the Society was indebted to Dr. Marchant, who had come from Liverpool purposely to read Mr. Mercer's Paper. Many of the observations mentioned in the Paper were common knowledge to those who had worked with arcs. He would like, however, to call attention to the peculiar shape of the curves connecting the current in the shunt circuit and the frequency which the author had obtained, and in this connection he would like to ask the author whether the oscillations were of what are known as the first or the second type. If the latter were the case, it might be possible to explain the peaks on the curve, and that their position would probably depend on the relation between the self-induction in the main circuit and the capacity in the shunt circuit.

Mr. E. H. RAYNER asked what was the source of supply, and whether large amounts of inductance and resistance were used between the arc and source of power, as it might be that the character of the curves obtained might be modified if the "ballasting" in this part of the circuit were small. He also asked whether any attempt was made to load up the oscillating circuit with resistance to represent power radiated or otherwise employed. One would expect some modification in the results as compared with a "frictionless" oscillating system.

Prof. MARCHANT, in reply, expressed his interest in Prof. Fleming's generator. The author had found that when the power was too great the arc became unstable. He thought Mr. Duddell's explanation was probably correct, as the oscillations were of the second type. It was noticed that with a big capacity, the peak was considerably less pronounced. There was a constant inductance in the circuit, and the current was reduced by adding resistance.

Mr. F. MERCER (communicated) stated in reply to Mr. Rayner that the source of supply was, for the low-voltage experiments, the supply mains, and, for the higher voltage experiments, two high-tension direct-current generators giving pressures up to 3,000 volts when connected in series. The inductance in the primary circuit was about 1 henry and the resistance 290 ohms when working at 470 volts, and 630 ohms when working at 1,000 volts. No additional resistance was included in the secondary circuit.

*Report of the Committee on Nomenclature and Symbols.**

The Committee consists of Prof. Sir J. J. Thomson, O.M., F.R.S. (President), Prof. H. L. Callendar, F.R.S., A. Campbell, Esq., B.A., Dr. C. Chree, F.R.S., Prof. G. Carey Foster, F.R.S., Dr. W. Eccles (Secretary and Convener), Sir George Greenhill, F.R.S., Dr. Alexander Russell, M.A., Prof. the Hon. R. J. Strutt, F.R.S., Prof. S. P. Thompson, F.R.S., Dr. W. Watson, F.R.S.

There have been three meetings—namely, on March 27th, April 24th and June 11th—and the following recommendations have been arrived at respecting electric, magnetic and certain associated quantities.

TYPOGRAPHICAL.

Capitals and Small Letters.—For electrical quantities varying harmonically, capitals should stand for the amplitude and small letters for the value at any instant.

Greek Letters.—Where possible, Greek letters should be used for angles and for specific quantities.

Subscripts.—The use of subscripts for components of vectors should be discouraged. As a general rule subscripts should be avoided.

Abbreviations for Names of Units.—Ordinary type should be used for the symbols of units, and not Clarendon.

NOMENCLATURE.

Terms in common use.	*Recommendation.*
Magnetic force, magnetic field..	None.
Intensity of magnetisation	,,
Magnetic induction, magnetic flux density	Retain both terms at present.
Inductance, coefficient of self-induction	Urge use of "inductance."
Permeability, inductivity	Urge use of "permeability."
Electric force, field, intensity ..	None.
Electric polarisation, displacement, electric flux-density	Disuse displacement.
Specific inductive capacity, dielectric constant, dielectric coefficient, permittivity	Urge use of electric inductivity and do not use permittivity.
Capacity, permittance	Do not use permittance.
Dielectric strength, dielectric rigidity and electric strength	Do not use dielectric rigidity.
Specific resistance, resistivity..	Urge use of resistivity.
Electromotive force, potential difference, voltage	No term preferred.

* Published by the Council with a view to a possible general discussion at a future date.

Symbols.

Quantity.	*Recommendation.*	
Base of Napierian logarithms	e rather than ε.	
Electric charge	Q, q	
,, force	None.	
,, inductivity	κ	
,, polarisation	P	
,, capacity	C, K	
Voltage	V, v, E, e	
Current	I, i	
Magnetic pole	m	For general use. Symbols used for terrestrial magnetism and atmospheric electricity given below.
,, force	H	
,, induction	B	
,, moment	M	
Permeability	μ	
Magnetic flux	F, Φ	
Intensity of magnetisation	I.	
Susceptibility	None.	
Self-inductance	L, l	
Mutual inductance	M, m	
Reactance	X, x	
Impedance	Z, z	
Resistance	R, r	
Resistivity	ρ	
Conductance	G	
Conductivity	γ	
Energy and work	W, w	
Power	P	
Period	T	
Frequency	n, f	
Decay coefficient	b	
Logarithmic decrement	δ	

To illustrate the meanings of these terms the following notes are adopted :—

The period or periodic time of a simple harmonic vibration is the time elapsing between two successive transits in the same direction through the same point. If the equation of the S.H.M. be taken, $v=V\cos\omega t$,

The period is $\quad T=2\pi/\omega$,

The frequency is $\quad n=\omega/2\pi$.

The simplest kind of damped vibration may be indicated by the equation $x=e^{-bt}\cos\omega t$.

The period is $T=2\pi/\omega$,
The frequency is $n=\omega/2\pi$,
The decay coefficient is b.

The logarithmic decrement is $\delta=bT$, or, in words, is the natural logarithm of the ratio of any value of the function x to its value when t is increased by T.

The damping factor is e^{δ}; it is the ratio of the values of the function at the times t and $t+T$.

Symbols for Multiples and Submultiples.

Multiple or submultiple.	*Name.*	*Symbol.*
10^{3}	Kilo	k
10^{-3}	Milli-	m
10^{-6}	Micro-	μ
10^{-9}	Millimicro-	$m\mu$
10^{-12}	Pico-	p or $\mu\mu$

The usage of $\mu\mu$ as an abbreviation for 10^{-9} metre is undesirable.

Symbols for Units.

Unit.	*Symbol.*	*Unit.*	*Symbol.*
Ampere	A	Watt-hour	Wh
Volt	V	Volt-ampere	VA
Ohm	Ω	Ampere-hour	Ah
Coulomb	C	Milliampere	mA
Joule	J	Kilowatt	kW
Watt	W	Kilovolt-ampere	kVA
Farad	F	Kilowatt-hour	kWh
Henry	H		

Printing of Large Numbers.

In printing numbers greater than 100 the method of marking thousands by spaces instead of by commas should be adopted. For example:

1 234 567 instead of 1,234,567.

Statement on the Definition of Capacity.

(*a*) *Capacity of a Conductor* (original definition).

The ratio of the charge to the potential of a conductor when

at a great distance from other conductors is constant, and is called its capacity.

For example, the capacity of a sphere is κr, and of two equal spheres in contact is $2\kappa r \log 2$, where κ is the electric inductivity of the homogeneous medium in which they are immersed and r is the radius of each sphere.

(*b*) *Capacity of a Conductor* (Maxwell).

The capacity of a conductor is the ratio of its charge to its potential when all neighbouring conductors are earthed. If the position of neighbouring conductors or insulators be altered, then, in general, the value of this capacity will also be altered.

(*c*) *Capacity between Two Conductors.*

In everyday work the capacity between two insulated conductors is measured by giving a positive charge to the first conductor and an equal negative charge to the second and finding the ratio of the charge on the first conductor to the difference of potential between them.

Provided that the two conductors and all neighbouring conductors are initially uncharged this ratio is constant. It is often of great value in practical work. In a good many cases its value can be computed.

The formal definition is as follows :—

Let two conductors be given in a field comprising dielectrics and other conductors which are uncharged. Then, if the two conductors have charges $+Q$ and $-Q$, and if the difference of their potentials is V, the ratio Q/V is independent of Q, and is called the *capacity of the field round the two conductors*, or simply the *capacity between the two conductors.*

The symbols C and K are indiscriminately used for (*a*) or (*c*).

Notes on Terrestrial Magnetism and Atmospheric Electricity. *By* CHARLES CHREE, *Sc.D., LL.D., F.R.S.*

THE following notes are not intended to advocate or criticise, but merely to call attention to notation in common use in terrestrial magnetism and atmospheric electricity, two branches of science which lie rather off the beat of the ordinary physicist.

Terrestrial Magnetism.—The notation in most common use when the earth's magnetic force is resolved into three rectangular components is X to the north, Y to the east and Z

vertically downwards. D (or δ) for declination, H for horizontal force, and V for vertical force are also in very common use; while T (also F and R) for total force, I (also θ) for inclination, X north and W west components are also not infrequently met with. The magnetic moment of a magnet is usually referred to some standard temperature and to an imaginary zero field. The former fact is usually represented by the use of a formula such as $m_t = m_0(1 - qt - q't^2)$, where m_0 refers to the standard temperature (usually 0°C.), m_t to the existing temperature, q and q' being "temperature coefficients" determined by experiment. When the horizontal force magnet is in the magnetic meridian at a place where H represents the horizontal field, its temporary moment is usually represented by an expression of the form $m + \mu$H. It should be noticed that μ is here used in a somewhat different sense to that usually employed; the volume of the magnet (which is not generally known accurately) is not explicitly introduced. What the user generally knows is the mass of the heterogeneous body composed of magnet, stirrup, glass scale and lens. μ is occasionally used also to denote pole strength—a quantity which there is seldom occasion to introduce explicitly. For the "pole distance," which is in more common use, 2λ seems the most usual notation. $2l$ (or l) is also sometimes used for pole distance, but more usually for the total length of the magnet. The force exerted by a bar or cylindrical magnet at a point situated in the prolongation of its axis at a distance r from its centre is generally expressed in the form $2mr^{-3}(1 + \mathrm{P}r^{-2} + \mathrm{Q}r^{-4})$, where P and Q are termed "distribution constants," of which Q is not infrequently assumed to be zero. The angle made with the magnetic meridian by one horizontal magnet deflected by another is generally called u. The moment of inertia of a magnet and its appurtenances is usually represented by K or I.

Owing to the practice of taking transit observations of the horizontal force magnet at intervals of 5 (sometimes 3 or 7) semi-vibrations, the use of T to denote *half* the period of a complete vibration is almost universal.

In connection with minor changes of magnetic force—*e.g.*, in the ordinary diurnal variation—the unit of force in almost universal use is $1\gamma \equiv 1 \times 10^{-5}$ C.G.S. It is also not infrequently used in connection with the complete value of a magnetic element—*e.g.*, the horizontal force at present in London is approximately 18 500γ.

When diurnal inequalities are expressed in Fourier series,

the two alternative forms are usually embodied in the following notation :—

$$c_1 \sin (t+\alpha_1)+c_2 \sin (2t+\alpha_2)+\ldots$$
$$a_1 \cos t+b_1 \sin t+a_2 \cos 2t+b_2 \sin 2t+\ldots,$$

where t—used as an abbreviation for $15°\times$ (time since midnight in hours)—represents the hour angle in the 24-hour term, $2t$ the hour angle in the 12-hour term, and so on.

The dependence of the range of the diurnal inequality on sun-spots is usually expressed by Wolf's formula $R=a+bS$, where R is the magnetic range, S the sun-spot frequency (after Wolf and Wolfer), and a, b constants.

Magnetic Observations at Sea.—A terminology of a very hybrid character peculiar to navigators has considerable vogue. ζ' represents the course of the ship as shown by the ship's own compass, ζ being the course that would be shown by an ideal compass unaffected by ship's iron.

The formula in common use is—

$$\text{Deviation of compass due to ship's iron}$$
$$=A+B\sin\zeta'+C\cos\zeta'+D\sin 2\zeta'+E\cos 2\zeta'+\ldots$$

The ordinary British navigator regards the coefficients A, B, &c., as improperly used unless attached as above. He also regards as natural symbols—

$\lambda\equiv$(horizontal component on board)/(what horizontal component would be if no iron present).

$\mu\equiv$(vertical component on board)/(what vertical component would be if no iron present).

θ' and θ are commonly used to represent the inclination as observed on board ship and as corrected for ship's iron respectively.

Special meanings are also attached to various other letters—*e.g.*, P, s, g, c—some of which are rather ambiguous in their dimensions owing to the old practice of taking magnetic force at Greenwich as unity. The use of magnetic " variation " for declination is still very common.

Atmospheric Electricity.—C (or K) for capacity, V for potential and i for current are in common use. The potential gradient is most often measured in volts per metre of height.

The use of suffixes + and —, referring respectively to plus and minus ions, is very usual. Thus we have ε_+ and ε_- (ε

being ionic charge), λ_+ and λ_- (λ being conductivity), n_+ and n_- (n being number of ions in the atmosphere per cubic centimetre).

The charge on free ions is most often given in electrostatic units, and then generally refers to a cubic metre of air. The notation varies, I (I_+ and I_-), J (J_+ and J_-), and E (E_+ and E_-) being all met with.

The notation Q for I_+/I_- is not unusual.

n is usually restricted to the light or mobile ions, having mobilities (*i.e.*, velocities corresponding to a potential gradient of 1 volt per centimetre) of the order of 1 cm. per second. The mobilities of these ions are denoted sometimes by u (u_+ and u_-), sometimes by v (v_+ and v_-).

N and V are occasionally used to denote the number and mobility of the Langevin (large) ions.

The notation a (a_+ and a_-), $\bar{a}=\frac{1}{2}(a_++a_-)$ and $q=a_-/a_+$ was at one time in very common use for electrical "dissipation" as measured by Elster and Geitel's apparatus.

There was also extensive use of A for the radio-activity of the atmosphere in the arbitrary unit employed by Elster and Geitel ($A=1$ when 1 metre of wire exposed under high negative potential for two hours to the atmosphere and then introduced inside special type of charged electroscope reduced the potential by 1 volt per hour).

XXXVII. *Production of Very Soft Röntgen Radiation by the Impact of Positive and Slow Cathode Rays. By* Sir J. J. Thomson, *O.M., F.R.S.*

[Abstract.]

Röntgen and his pupils had always held that light waves were identical in nature with electrical waves produced by mechanical means, but there was a gap, on which very little work had been done, between the longest infra-red radiation and the shortest electrical wave that could be mechanically produced. He believed the investigation of this gap to be essential to the proper study of the constitution of the atom. The work already done on X-rays had demonstrated the existence of two separate rings of electrons in the atom, one within the other. These rings were responsible for the K and L types of radiation respectively. The L radiation was so much softer than the K that if a third ring of electrons existed, the radiation from which was proportionately softer than that of the L type, this radiation would fall well within the gap already mentioned.

In the first experiment described a special form of discharge tube was employed. The positive rays passed through a tubular perforation in the cathode and impinged obliquely on a metal target. A photographic plate of the Schumann type was situated at the further end of a branch tube in such a position that no solid obstacle interposed between the target and the plate. When the discharge passed between the electrodes the photographic plate was affected. The application of an intense transverse electrostatic field between two metal plates situated between the cathode and the target completely stopped the effect, showing that this was not due to stray radiation *reflected* from the target, since, while charged particles would be swept to one side, radiation would not be affected by the field. Hence the passage of positive particles from the cathode to the target was essential. On the other hand, a strong transverse electrostatic field in the branch tube had no effect, showing that a radiation was passing between the target and the plate, which was not, therefore, merely affected by positive particles rebounding down the side tube after impact on the target.

The properties of this radiation were intermediate between ordinary X-rays and Schumann waves. They were susceptible to reflection by metal surfaces, and their penetrating power was very small. They were completely stopped by the finest collodion film obtainable.

It was shown that the quality of the radiation did not depend on the energy of the moving particles which gave rise to it, but on the velocity. Hence equally soft rays should be produced by cathode particles if these were travelling as slowly as the positive rays. A discharge tube was constructed in which the cathode rays, leaving the cathode with the ordinary velocity, could be subjected to a retarding electrostatic field of variable strength before impinging on the target. It this way the velocity of impact could be varied over a large range, and radiations were obtained varying in quality from ordinary hard X-rays to the so-called Schumann waves. It was hoped by the study of these radiations to be able to determine not only the number of rings of electrons within the atom, but the number of electrons in each ring.

DISCUSSION.

Sir OLIVER LODGE expressed the opinion that the work just described was of far-reaching importance, and he felt confident that the results warranted the anticipation that further work would confirm the explanation foreshadowed by the President. The full meaning of the experiments and the way in which they enabled us to estimate the number of rings in the atom was given to some extent in previous Papers by the President and others. Eventually we would understand the argument more fully, and in this way the unravelling of the secret of the atom would be materially advanced.

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Printed by GEO. TUCKER, 1, 2 and 3, Salisbury Court, Fleet Street, London, E.C.

THE PHYSICAL SOCIETY OF LONDON.

PROCEEDINGS,

VOLUME XXVI.—PART V.

AUGUST 15, 1914.

Price to Non-Fellows, **4s.** *net, post free* **4/3.**

Annual Subscription, **20/-** *post free, payable in advance.*

Published Bi-Monthly from December to August.

LONDON:
"THE ELECTRICIAN" PRINTING & PUBLISHING CO., LTD.,
1, 2 AND 3, SALISBURY COURT, FLEET STREET.

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